On the truth judgments in informatics

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1 What is informatics?

One way to answer the questions: What is mathematics?, What is physics? What is informatics? is to investigate how a statement is judged true in these sciences.

At the ontological level, we may distinguish the *necessary* judgments, that a statement is true in all possible worlds, and the *contingent* ones, that it is true in a specific world: nature. At the epistemic level, we may distinguish the *a posteriori* judgments, where the truth of the statement is accessed through an observation, and the *a priori* ones, where it is not. Although part of this vocabulary is inherited from Kant, we shall sometimes depart from its Kantian use.

Investigating the place of informatics in the classification of sciences, we have defended elsewhere[1] that some judgments in informatics were necessary and *a posteriori*. In this note, we shall defend that, together with these necessary and *a posteriori* judgments, there are also other kinds of judgments in informatics and discuss how these different kinds of judgments are articulated.

2 Necessary and *a posteriori* judgments

Let us assume I have made a program that sorts lists of natural numbers and used it to the sort the list $[2, 3, 1]$, yielding the list $[1, 2, 3]$. The truth judgment of the statement "sorting the list $[2, 3, 1]$ yields the list $[1, 2, 3]$" is necessary: once the meaning of the word "sorting" has been defined, there is no possible world, where sorting the list $[2, 3, 1]$ would not yield the list $[1, 2, 3]$.

But at the epistemic level, this judgment is *a posteriori*, as we access to the truth of this statement through the observation of an object of nature: the screen of the computer on which the result is displayed.

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[1] Informatics in the classification of sciences, *International Conference on the History and Philosophy of Computing*, 2013. *Computation Proof Machine: mathematics enters a new age*, Cambridge University Press, 2015.

This remark raises a problem: with this observation, I can judge that the statement "sorting the list $[2, 3, 1]$, with this computer, here and now, yields the list $[1, 2, 3]$" is true in nature. How can I deduce that the statement "sorting the list $[2, 3, 1]$ yields the list $[1, 2, 3]$" is true in all possible worlds?

A solution to this problem is that I know *a priori* that there exists a list l such that the statement "sorting the list $[2, 3, 1]$ yields the list l" is true in all possible worlds. And the observation that, in one specific world, sorting the list $[2, 3, 1]$ yields the list $[1, 2, 3]$ permits to eliminate all the other lists, as candidates for being the list l.

But this raises another question, we shall discuss now: how do I know that the statement "sorting the list $[2, 3, 1]$, with this computer, here and now, yields the list $[1, 2, 3]$" is true in nature? In other words: how do I know that the program I am using sorts lists?

3 The maker's judgments

3.1 The maker

What kind of judgment is the truth judgment of the statement "the program I am using sorts lists"?

This judgment is obviously contingent, because I could use a program that does something else, such as reversing lists. But it is not, in general, by an observation that I know that this program sorts lists. So this judgment is not *a posteriori*.

I know that this program sorts lists because I have made it myself. We reach here an essential property of informatics, that is not only a science, but also a technology: informaticians also make things and this judgment is a *maker's judgment*. It looks contingent and *a priori*.

3.2 Contingent and *a priori* like ...

This maker's judgment has many common points with other contingent and *a priori* judgments, such as Descartes' *cogito* and the judgment that time passes.

In the three cases, there is, in the statement, a non-linearity: a repetition of the pronoun "I": *I* think, therefore *I* am, *I* know that time passes because *my* consciousness is temporal, *I* know this program sorts lists, because *I* have made it. The linear statement "you think, therefore I am" does not make sense.

In the three cases, only one person can judge this statement true: the non-linear statement "you think, therefore you are" does not make sense either. In fact, the first-person pronoun "I" being itself non-linear, as it expresses the identity of the subject and the enunciator of the statement, "I" is used three times in the statement "*I* think, therefore *I* am", as the subject of the verb to think, as the subject of the verb to be, and as the enunciator of the statement.

3.3 The maker's and the observer's judgments

On the other hand, the maker's judgments have many common points with the observer's contingent and *a posteriori* judgments.

In both cases, some information is copied: from the external object to the consciousness of the observer in one case, from the consciousness of the maker to the external object in the other. The flow of information is reversed, as the external object modifies the consciousness in one case and the consciousness modifies the object in the other. This is why the maker's judgments are not *a posteriori*. But in both cases there is a synchronization, an equalization, of an object and its image in someone's consciousness.

3.4 A third kind of judgments at the epistemic level

This remark leads us to distinguish, not two, but three kinds of judgments at the epistemic level: *a posteriori* judgments, that are based on a flow of information from the external object to the consciousness, maker's judgments, that are based on a flow of information from the consciousness to the external object, and *a priori* judgments, that do not involve any flow in information in either direction.

The previous definition of *a priori* was the negative of *a posteriori*: with no flow of information from an external object to the consciousness, including the maker's judgments. This more restricted definition, with no flow in information between an external object and the consciousness, in any direction, excludes them.

3.5 Relativizing the distinction between the *episteme* and the *techne*

As noticed by L. Floridi[2], taking the maker's judgments into account leads to relativize the distinction between science and technology.

When we define science by its goal to judge that some statements are true and technology by its goal to make objects, they look unrelated.

But, we may also define technology by its relation to truth: like science, technology is a way to judge that some statements are true, as a side effect of making them true: repainting the ceiling is one way, among others, to know what color it is.

3.6 Are there maker's judgments for abstract objects?

So far, we have considered the notion of maker's judgment mostly for material objects. Computers are material objects, programs more or less, algorithms are not. But, we can make an algorithm, and know things about it as a side effect.

[2] Luciano Floridi, A Defence of Constructionism: Philosophy as Conceptual Engineering, *Metaphilosophy*, 42, 3, pp. 282-304, 2011.

So it seems that we can extend the notion of maker's judgment to abstract objects. However, this require to reconsider some ideas, ina particular the symmetry with the observer's judgements, that holds only in the case of material objects.

4 Other kinds of judgments in informatics

Besides the necessary and *a posteriori* judgments and the contingent and maker's judgments, there are more traditional judgments in informatics: necessary and *a priori* judgments, and contingent and *a posteriori* judgments.

4.1 Necessary and *a priori* judgments

The truth judgment of the statement "the halting problem is undecidable" or that of the statement "selection sort is quadratic" are necessary and *a priori*, like mathematical judgments.

There are also many such judgments in physics, for instance the truth judgment of the statement "Newton's laws imply Kepler's". It is because contingent and *a posteriori* judgments and necessary and *a priori* judgments are articulated in physics that physics is not a mere collection of empirical facts, not a mere "stamp collection".

4.2 Contingent and *a posteriori* judgments

The truth judgment of the physical Church-Turing thesis, that is the statement "it is impossible to build a physical machine that computes a non computable function" or that of the statement "it is impossible to build a physical machine that solves the travelling salesman problem in polynomial physical time" are contingent and *a posteriori*, hence falsifiable.

5 The articulation of the different kinds of judgments in informatics

5.1 Four kinds of judgments

We have identified four kinds of judgments in informatics:

1. necessary and *a posteriori*: what we judge from the observations of the result of a computation,

2. contingent and maker's: what I judge about programs, computers, etc. because I have made them,

3. necessary and *a priori*: mathematical judgments, like in other sciences,

4. contingent and *a posteriori*: because machines are physical objects.

Note that there seems to be no contingent and *a priori* judgments in informatics, and what we first took for such judgments were in fact contingent and maker's judgments.

There seems to be no necessary and maker's judgments, neither in informatics, nor elsewhere, as it seems that I cannot at the same time judge a statement true from my experience of making the object the statement speaks about, and judge that this statement is true in all possible worlds, as I could have made a different object.

For instance, the truth judgment of the statement "selection sort is quadratic" is necessary, but *a priori* and not maker's. The truth judgment of the statement "the program I have made contains two loops" is maker's, but contingent and not necessary, as I could have made another program. If I know that the halting problem is undecidable, the truth judgment of the statement "the program I have made does not solve the halting problem" is necessary, but *a priori* and not maker's, because it is something I could have judged, even if I had not made the program myself. In the case I did not know the halting problem to be undecidable and I decided to make a program that does something else than deciding if a program terminates, the truth judgment of the statement "my program does not solve the halting problem" would be maker's, but contingent and not necessary. The statement "my program does not solve the halting problem" could indeed have been judged true in all possible worlds, but it has not, as I did not know the halting problem was undecidable.

5.2 Different *rôles* within informatics

Because computers are truth judging machines, the goal of informatics is necessary and *a posteriori* judgments (1.): we write a program that computes the thousandth digit of π, because we ignore that it is a 9.

In the same way, there are many necessary judgments in physics, but they are not the goal of physics, that is to learn about nature: the contingent judgments.

To achieve this goal, we build computers and programs, that leads to contingent and maker's judgments (2.). To do so, we must take into account constraints imposed by nature. This way, we learn more about nature: contingent and *a posteriori* judgments (4.). In this respect, informatics is not different from others technologies.

These constraints are often annoying. The informaticians would be much happier in a world where the velocity of information were unbounded, the density of information were unbounded, and erasing information would not generate heat, that needs to be evacuated ($1/c = 0$, $h = 0$, $k = 0$).

Finally, there are different kinds of necessary and *a priori* judgments (3.) in informatics: the truth judgment of the statement "there is no algorithm to decide if an algorithm terminates", that of the statement "if the physical Church-Turing thesis holds then there is no machine that decides if an algorithm terminates", etc. They are similar to necessary and *a priori* judgments in physics. In this respect, informatics is not different from others sciences.

5.3 Maker's iudgments and judgments by definition

But there are also judgments about made objects that require a proof, such as the truth judgment of the statement "the program I have made is quadratic". There is a narrow line between the maker's truth judgment of the statement "the program I have made uses two loops", that requires no proof, and the truth judgment of the statement "the program I have made is quadratic", that requires a proof, that uses as axiom the statement "the program I have made uses two loops", that is judged true by a maker's judgment. The situation here can be compared with physics where proofs using empirical statements as axioms are many. In this respect, informatics is similar to other technologies. It seems nevertheless to differ by the extent of these proofs: there are many more proofs of properties of programs than proofs of properties of potteries.

This remark that the axioms in these proofs are judged true by maker's judgments leads us to the question of the status of axioms and definitions in other proofs, in particular in mathematical proofs. How do we know, for instance, that $\tan(x) = \sin(x)/\cos(x)$? The common answer is that this statement is judged true by definition of the tangent function. But is this different from judging it true with a second-hand maker's judgment, as we have made the tangent function this way. Following Hilbert and Poincaré, the axioms of Riemanian geometry are a definition of Riemanian geometry. Could not we say that we judge them true, because we have made Riemanian geometry this way?