INCIPIENT SPALL, CRACK BRANCHING, AND FRAGMENTATION STATISTICS IN THE SPALL PROCESS

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JOURNAL DE PHYSIQUE
Colloque C3, Supplément au n°9, Tome 49, septembre 1988

INCIPIENT SPALL, CRACK BRANCHING, AND FRAGMENTATION STATISTICS IN THE SPALL PROCESS[1]

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Résumé

Des théories basées sur l'énergie de rupture d'écaillage et de fragmentation sont examinées. Un concept d'énergie limitée et de défaut limité d'écaillage émerge naturellement à l'intérieur d'une de ces théories.

Les conditions favorisant l'apparition de l'écaillage et la ramification des fissures dans le processus d'écaillage sont identifiées.

L'analyse statistique de la distribution de taille des fragments se produisant lors de l'écaillage est analysée.

Abstract - Energy based theories of spall fracture and fragmentation are examined. A concept of energy-limited and flaw-limited spall emerges naturally within one of these theories. Conditions favoring incipient spall and crack branching in the spall process are identified. The statistical distributions of fragment sizes occurring in spall fragmentation are investigated.

1 - INTRODUCTION

Impulsive loading and the subsequent interaction of stress waves in solids can lead to regions interior to a body being carried rapidly into tension, possibly exceeding the spall strength, causing fracture and fragmentation. Detrimental damage or failure of solid bodies and structures are a possible result of such loading. Consequently, dynamic spall has been a subject of considerable interest and active research over the past several decades.

The initiation of spall is intimately linked to material microstructure and considerable research has focused on the identification and behavior of the microstructural features leading to spall failure. Although much work remains, the field is mature and has been reviewed fairly thoroughly /1,2/.

From this research, a recognition of the importance of microstructure in dynamic spall, and some understanding of the response of various individual microstructural entities to transient tensile loading has emerged. The interaction and collective response of such microstructural entities leading to evolution of the global spall process, on the other hand, is not yet well understood.

An awakening recognition of this bias in pursuit of a complete theory of dynamic spall is aptly summarized by a similar shifting in emphasis in other fields of physics. It has been noted that, "Instead of reducing the universe to its smallest parts, a growing number of scientists... argue that physics can unearth more fundamental truths by looking at the big picture: discovering higher-level rules about how the parts interact."/3/ In the present work some concepts of dynamic spall are discussed which are thought to more closely mirror this latter direction of thinking. Recently, several theories of spall and fragmentation have been proposed which are based on energy considerations in the collective spall process.

(1)This work performed at Sandia National Laboratories supported by the U.S. DOE under contract DE-AC04-76DP00789

By pursuing these energy-based theories it is possible to draw some tentative conclusions concerning various features of dynamic spall phenomena such as the occurrence of incipient spall, observation of crack branching in the spall process, and spall fragmentation statistics.

2 - KINETIC ENERGY THEORY OF SPALL FRAGMENTATION

An energy-based theory which leads to a prediction of characteristic spacing and fragment size in the dynamic spall process follows from consideration of the kinetic energy associated with velocity divergence within the spall zone. /4,5,6/ By considering this characteristic spacing as a variable, a local kinetic energy relative to the center of mass of a domain of material with this dimension can be identified. If one assumes a model for energy absorption in the spall process, and further assumes that the local kinetic energy fuels the breakage event, then one can strike a balance between driving energy and fracture energy, yielding a prediction of the characteristic fragment size. Consideration of different simple geometric shapes for fragment regions leads to small differences in the magnitude of the local kinetic energy and different implementations of energy balance in various applications of the theory account for variations of as much as a factor of two in predicted fragment size. Experiments are not yet able to discern among the different approaches.

As an example, the local kinetic energy within a spherical region of diameter s assuming uniform velocity divergence is

$$T = \frac{\pi}{720}\rho\dot{\varepsilon}^2 s^5 , \tag{1}$$

where ρ is the density and $\dot{\varepsilon}$ is the rate of volumetric dilatation. Considering spall by ductile hole growth through plastic flow, the work dissipated in a similar volume is estimated to be /7,8/

$$W = \frac{\pi}{6}s^3(Y\varepsilon_c) \tag{2}$$

where Y is the appropriate flow stress and ε_c is the volume strain to hole linkup and failure ($\varepsilon_c \simeq 0.15$)/7/. Assuming a complete transfer of kinetic energy in (1) to plastic work in (2) provides an expression for the characteristic spall fragment size,

$$s = \sqrt{120Y\varepsilon_c/\rho\dot{\varepsilon}^2} . \tag{3}$$

Similar considerations for spall through crack growth and coalescence (brittle spall) yields

$$s = 2\left(\sqrt{45}K_c/\rho c\dot{\varepsilon}\right)^{2/3} \tag{4}$$

where c is the sound speed and K_c is a fracture-toughness characterization of the appropriate crack surface dissipation in the spall process.

Although a kinetic-energy based theory of spall fragmentation makes no explicit reference to fracture microstructure, some flaw structure sufficient to accommodate the predicted characteristic fracture spacing is tacitly assumed and a microstructural spall failure mechanism (hole growth or crack propagation) is considered in calculating the fracture energy. Dynamic material properties such as Y or K_c must be commensurate with the rates and scales experienced in the spall process and may, or may not, be consistent with quasistatic values.

3 - ENERGY-HORIZON THEORY OF DYNAMIC SPALL

More recently an alternative to the kinetic-energy theory of spall has been proposed /8/. It is also an end-state theory in the sense that transient states, or evolution of the spall process, are not predicted. The theory requires an additional premise. On the positive side, a broader set of spall properties is predicted including the spall strength, the energy absorbed, and the time to spall, in addition to the characteristic fragment size.

In this theory two physically reasonable propositions are assumed: The first is a horizon condition which establishes a domain of communication, consistent with the time to failure, within which spall must be independent of the surrounding environment. This condition, in particular, bounds from above the characteristic fragment size by the horizon scale at the time of failure. The second is an energy condition which requires that the elastic potential and kinetic energies associated with the tensile loading process exceed the fracture energy absorbed in the spall process. These two conditions, when combined with a kinematic expression describing constant dilatant strain-rate loading, lead to a set of inequalities which place constraints on the spall properties. In order of appearance these relations are /8/,

$$s \le 2ct \quad , \tag{5}$$

$$\frac{1}{2} P^2/\rho c^2 + \frac{1}{120} \rho \dot{\varepsilon}^2 s^2 \ge \Gamma \quad , \tag{6}$$

$$P = \rho c^2 \dot{\varepsilon} t \quad . \tag{7}$$

Here P is the tensile pressure, Γ is the fracture energy per unit volume, t is time, while other parameters have been alluded to earlier. The behavior of these equations is illustrated qualitatively in an energy versus time plot in Figure 1 which shows bounding curves for the driving elastic plus kinetic energy and for the fracture dissipation. Equalities in these relations correspond to energy-limited spall and provide specific analytic expressions for the spall properties. Inequalities imply microstructural flaw-limited spall and more detailed material property information is required before spall can be characterized.

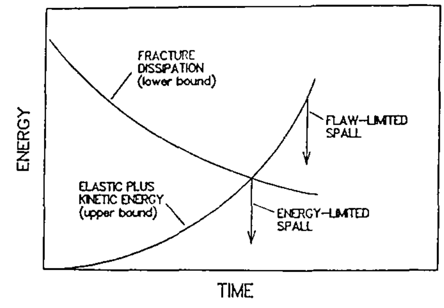

Figure 1: Energy-Horizon theory of spall.

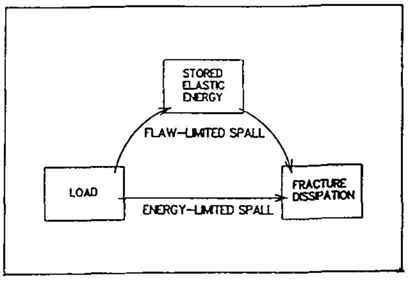

Figure 2: Energy flow in flaw- and energy-limited spall.

4 - ENERGY-LIMITED SPALL STRENGTHS

Assuming energy-limited spall in the energy-horizon theory (equalities in Equations 5-7), explicit expressions for the spall strength relating to more fundamental material properties can be determined. As before, in brittle spall it is assumed that failure occurs through the growth and coalescence of cracks with energy residing in new surface area characterized by a fracture toughness K_c. The surface energy density is then given by

$$\Gamma = 3K_c^2/\rho c^2 s \quad , \tag{8}$$

and the theory then provides an expression for a brittle spall strength of,

$$P_s = \left(3\rho c K_c^2 \dot{\varepsilon}\right)^{1/3} \tag{9}$$

Similarly for ductile spall based on an assumption of failure through plastic hole growth and coalescence, $\Gamma = Y\varepsilon_c$ and,

$$P_s = \left(2\rho c^2 Y \varepsilon_c\right)^{1/2} \tag{10}$$

Corresponding expressions can be obtained for the spall time and fragment size for brittle and ductile spall. Expressions for fragment size are dimensionally equivalent to those obtained from the kinetic energy theory, however the coefficients lead to predicted fragment sizes that are a factor of 3 to 4 smaller. This difference is principally due to inclusion of potential energy in the energy balance.

5 - A SPALL MECHANISM TRANSITION

Metals are typically characterized by both a fracture toughness K_c and a flow stress Y. Assuming that both K_c and Y are invariant material properties over the relevant rates of spall deformation, the energy-horizon theory in the energy-limited bound predicts a transition from spall failure by crack growth (K_c-dominated) to spall failure by hole growth (Y-dominated) with increasing loading rate /9/.

As stated earlier, the theory is an end-state theory in which a critical energy must be achieved before transition to the spalled state is allowed. Expressions for the separate energies for brittle and ductile spall are given above and it is readily shown that the brittle spall energy increases with loading rate whereas that for ductile spall remains constant. As a consequence, at lower loading rates, spall failure through crack growth (K_c-dominated) is energetically favored while at higher loading rates, failure through plastic hole growth (Y-dominated) is favored. The transition strain rate from K_c-dominated to Y-dominated spall is found to be, /9/

$$\dot{\varepsilon}_t = \sqrt{\frac{8}{9}\, \rho c^4 (Y\varepsilon_c)^3 / K_c^4} \tag{11}$$

This transition is predicted to occur in the 10^5/s - 10^6/s range for aluminum and titanium alloys /9/.

6 - CRACK PROCESS-ZONE CONSIDERATIONS

Metals which fail through ductile fracture exhibit a plastic process zone at the tip of a mature crack which extends over a characteristic length x_{pz}. Within this zone, plastic hole growth and ligament rupture occurs leading to full fracture separation. In a mechanistic theory of ductile fracture /10/ this characteristic length is related to K_c and Y through,

$$x_{px} = \frac{\pi}{72} (K_c/Y)^2 \tag{12}$$

Consideration of a plastic process zone leads to a degree of understanding of the K_c-dominated to Y-dominated spall transition. The transition strain rate (Equation 11) is found to correspond to the rate at which the spall horizon (characteristic fracture spacing) approaches that of the process zone length x_{pz}. Consequently, at loading rates exceeding ε_t, the spall horizon is too small to permit the formation of mature ductile cracks.

In this light the spall mechanism transition is relevant to the behavior of spall in brittle materials such as rocks and ceramics. Crack propagation in these materials also exhibits process zones in which extensive microfracturing preceeds crack separation. Measured values of K_c reflect this process zone dissipation. When loading rates leading to spall demand that fracture spacings become comparable to the process zone dimension, a transition in spall behavior would be expected and predictions of spall properties, based on K_c determined for lower rates, would no longer be valid. Intuitively one expects that the effective K_c would diminish for rates exceeding the transition rate.

7 - FLAW-LIMITED SPALL

Discussions to this point have focused on the nature of spall in the energy-limited bound. It was assumed that the spalling body was sufficiently flawed on all scales such that the evolution of spall is controlled by the rate of energy input to the body. Within the flaw structure a "survival-of-the-fittest" concept would be expected to apply with many potential fracture sites arresting or not nucleating due to a limited supply of energy. This behavior has been aptly demonstrated with computational studies of dynamic fracture /11/. The energy-limited bound is simpler in that a detailed description of the microstructure is not needed to make useful predictions concerning the spall behavior. Comparisons with experimental spall data suggest that many materials exhibit energy-limited spall behavior within the range of loading rates investigated /8/.

Energy-limited spall is nonetheless an assumption. It is easy to envision conditions under which energy-limited spall would not apply. A defect-free crystal would, in principle, achieve its theoretical strength before tensile failure occurred. At typical impact-induced spalling rates in quartz glass, energy-limited spall would predict a spall strength of about $P_s \simeq 0.1$ GPa, whereas experiments of Rosenberg et.al./12/ indicate a spall strength exceeding 5.0 GPa.

It is also reasonable to expect that energy-limited spall may occur in some materials only over a partial spectrum of loading rates. The body should have a critical flaw size such that the spall strength would approach the static tensile strength, rather than zero as predicted for energy-limited brittle spall. A characteristic scale such as grain size in a polycrystalline material may lead to a transition from energy-limited to flaw-limited spall above a critical loading rate consistant with this characteristic size.

In materials where spall is controlled by the inherent flaw structure, a more detailed description of the microstructure will be needed to predict the spall properties. In a continuum representation, some set of internal state variables would be required to characterize the flaw structure leading to spall.

8 - ENERGY CONSIDERATIONS IN FLAW-LIMITED SPALL

Within the framework of the energy-horizon theory of spall, some qualitative conclusions regarding the path of energy transfer in flaw-limited spall, in contract to energy-limited spall, can be reached. During spall, work is performed on a region within the body at a rate characterized by the volumetric expansion rate ε. In energy-limited spall this work is transferred directly to fracture dissipation — it is stored in elastic energy only to a level needed to support the stresses driving fracture. As an idealization the path of energy transfer in energy-limited spall can be considered as from the load directly to fracture dissipation.

In contrast, for flaw-limited spall, tensile pressures significantly higher than energy considerations would require, are needed to activate the existing flaw structure. Thus, in flaw-limited spall, work is initially stored in elastic energy. Elastic energy is then transferred to fracture dissipation when fracture activation and growth initiates. So, the path of energy transfer in flaw-limited spall is considered to be from the load to stored elastic energy and then from stored elastic energy to fracture dissipation.

The paths of energy flow for energy-limited and flaw-limited spall are indicated in Figure 2.

9 - INSIPIENT SPALL

For some materials a transient tensile threshold can be achieved such that recovered specimens, after sectioning and polishing, show the residue of damage arrested at intermediate states of spall. The procedure is particularly useful in identifying microstructural features associated with spall failure. This precursor of complete spall, when it can be achieved, is commonly called incipient spall.

In energy-limited spall, energy transfer is directly from load to fracture dissipation. Thus, removal of the load immediately arrests the spall fracture process. In this sense, energy-limited spall is a stable failure mechanism. In contrast, for flaw-limited spall, energy transfer proceeds from load to stored elastic energy and then to fracture dissipation. Removal of the load is not sufficient to remove elastic energy stored in the microstructure and spall fracture continues. Consequently, flaw-limited spall, when initiated, is regarded as a catastrophic failure mechanism.

Thus incipient spall would appear to be a property of energy-limited spall where stable fracture dissipation can be arrested at intermediate states of spall damage. Incipient spall seems less likely in flaw-limited spall which, once initiated, proceeds catastrophically and would be much less easily arrested.

In practice neither limit is probably achieved. The trend seems consistent with observation, however. Incipient spall is achieved in a number of brittle and ductile metals, and in rocks where spall properties are consistent with energy-limited spall /2/. In contrast, incipient spall damage has not been observed in glass.

10 - CRACK BRANCHING

A feature of dynamic fracture which has not been seriously considered in spall failure is crack branching. Such a cooperative fracture process does not fit well within a picture of relatively simultaneous and isolated activation and growth of independent fracture sites. Crack branching, however, does not appear to be a common feature of incipient spall. Particularly clear micrographs of incipient brittle spall such as those of Shockey and co-workers /2/ on armco iron and novaculite suggest that spall failure is readily accommodated through a preponderance of independently activated cracks. Subsequent branching of cracks does not seem to be required in the collective response leading to coalescence and failure.

This behavior would appear consistent with the framework of energy-limited spall in which a thermodynamically sufficient collection of independent fractures activate and grow stably, leading to coalescence and failure of the body. In flaw-limited spall, however, material is loaded to tensile pressures well in excess of the minimum energy required for spall before a minimal fracture defect structure is activated. A large stored elastic energy would then be expected to fuel catastrophic fracture growth and crack branching as a means of absorbing the excess elastic energy. Although experimental spall studies supporting this conjecture are not available, the explosive failure of quasistatically compressed glass spheres /13/ is indicative of the mechanism expected.

Consequently, crack branching is expected to be an important fracture mechanism in the catastrophic failure process of flaw-limited spall. It would be less important or nonexistent in stable energy-limited spall.

11 - SPALL FRAGMENTATION STATISTICS

Whereas the theories discussed earlier in this report predict a characteristic fragment size from energy considerations in the spall event, it is well recognized from both intuition and experience that the consequence of such an event is a complex statistical distribution of particles. The predicted characteristic size is, at best, some measure of the mean of the observed distribution.

The factors leading to size randomization in the spall fracture process are complex. First, the weakness or failure strength is a complicated topological variable which depends on numerous microstructural and molecular features including vacancies, impurities, dislocations, grain boundaries, and second phases to name a few. Second, the local stress state, as the material is carried into tension, will be a random variable due to spatial fluctuations of the modulus in structural materials.

Attempts to account for statistical features in the spall process have so far met with mixed success. A treatment of microstructural variability in detail is unduly complicated and most studies have approached the problem through various statistical formulations. A review of much of this work is provided in Grady and Kipp /14/. A popular approach which relies on various applications of geometrical statistics has been shown to be generally unsatisfactory due to strong dependence on the assumed construction algorithm /14/.

A promising approach to the problem of statistical fragmentation may lie in emerging theories based on the maximum entropy principle as recast in the more general terms of information theory /6,14/. One application predicts that for a statistically random homogeneous fragmentation process, the fragment number will be exponential in mass /14/.

In a body carried uniformly into tension a statistically random activation of fractures will be Poisson-distributed over space. A Poisson process is implicit in the maximum entropy analysis leading to an exponential distribution. The energy-limited bound of the energy-based theory of spall is expected to result in an exponential distribution of fragments in that the stable growth and coalescence of fracture should maintain a correspondence between the Poisson distributed activation sites and the fragment distribution.

Experimental verification of this deduction is difficult because spall events are not statistically homogeneous process. Recently, however, molecular dynamics calculational methods have been used to simulate a homogeneous spall process on high-speed computers /15/. In these calculations a large number of molecular particles, characterized by a mass and an intermolecular potential, are initialized at a specific density and temperature consistent with the liquid state such that subsequent motion carries the system into tension at a specified loading rate. In these computer experiments, spall fragmentation occurs with fracture initiating at defects brought about by random thermal motion in the system.

First, fragmentation from these molecular dynamics calculations has been found to be consistent with energy-limited spall in the sense that the fragment size, and dependence on strain rate, are adequately predicted by the theory. Second, fragment size distributions resulting from the computer simulations are found to be exponential in mass. Thus, the molecular dynamics computer experiments, although preliminary, provide compelling support for the maximum entropy prediction of an exponential distribution in energy-limited spall.

That an exponential distribution will be obtained in a flaw-limited spall event seems less likely. Although random fracture activation would still be a Poisson process, the subsequent catastrophic crack propagation and branching would be expected to undo the correspondence between activation sites and fragment sizes. One would anticipate that intense fracturing at crack tips would lead to excessive fine fragments and broadening of the distribution. Particle size distributions from impact fragmentation experiments on crystalline quartz indicate such broadening of the distribution /16/. The quasistatic compression of glass spheres mentioned earlier /13/ results in similar distributions.

12 - SUMMARY

We have examined energy-based theories of the spall fracture and fragmentation process. Two theories are discussed — one is based on a balance of kinetic energy and fracture dissipation, while the second incorporates energy balance with a communication horizon within the spall event. Although the latter provides a richer predictive capability, the two have not been reconciled.

Within the latter theory a concept of energy-limited and flaw-limited spall emerges. When spall is energy-limited specific expressions for spall stress, fragment size and failure time can be derived. Based on microstructural models for brittle spall and ductile spall, explicit expressions for spall stress are obtained. The theory further predicts a transition from brittle (K_c-dominated) to ductile (Y-dominated) spall behavior with increased loading rate. This transition is shown to occur when the spall communication horizon achieves a comparable dimension with the crack tip process zone.

From further consideration of the energy-horizon theory it is concluded that energy-limited spall leads to a stable failure process, conducive to incipient spall and devoid of crack branching phenomena. In contrast, flaw-limited spall is concluded to be an unstable catastrophic failure event. Crack branching is a dominant effect and incipient spall would be difficult to achieve.

Finally, it is suggested that energy-limited spall will lead to relatively centered (perhaps exponential) fragment size distributions, whereas flaw-limited spall is expected to provide broadly distributed particle sizes. Recent molecular dynamics simulations of homogeneous spall indicate exponential mass-number distributions for energy-limited spall, supporting maximum entropy theories of dynamic fragmentation /15/.

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