A Cancellation Operator Suitable for Identification of Nonlinear Volterra Models

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Abstract

We present a new identification method for nonlinear possibly singular Volterra models based on a suitable operatorial transformation of the problem with the property that some nonlinear terms are cancelled, allowing to get an equivalent formulation well adapted to least square identification.

Keywords

Nonlinear Volterra Equation, Diffusive Representation, Pseudo-Inverse.

1 Introduction

Consider a nonlinear Volterra model of the form:

$$H(\partial_t)X = F(X) + G(X)u, \tag{1}$$

where $u(t)$, $X(t) \in \mathbb{R}$, $t > 0$, F and G are possibly singular functions defined on $\mathbb{R}$, and $H(\partial_t)$ is an invertible causal convolution operator of diffusive type [3] such that $|\frac{H(i\omega)}{\omega}| \to 0$ when $|\omega| \to \infty$. We suppose that problem (1) is well-posed.

Let ψ_X the γ-diffusive representation of X, that is the unique solution of the following problem on $(t,\xi) \in \mathbb{R}^{+*} \times \mathbb{R}$ [3]:

$$\partial_t \psi_X = \gamma\, \psi_X + X,\ \psi_X(0,.) = 0, \tag{2}$$

with $\gamma : \mathbb{R} \longrightarrow \mathbb{C}^-$. With μ the γ-symbol associated with $H(\partial_t) \circ \partial_t^{-1}$, the γ-diffusive formulation of equation (1) is [3]:

$$\langle \mu, \gamma\, \psi_X + X \rangle_{\Delta'_\gamma, \Delta_\gamma} = F(X) + G(X)u. \tag{3}$$

The problem under consideration in the sequel is to build estimations of $H(\partial_t)$ via the identification of the γ-symbol μ of $H(\partial_t) \circ \partial_t^{-1}$ from (noised) data $(\tilde{X}, \tilde{u}) = (X^k + \eta^k, u^k)_{k=1:n}$, with (X^k, u^k) solution of (1) and η^k continuous measurement noises.

2 The $\mathfrak{D}_\varepsilon$-transformed problem

Given $\mathbf{X}, \mathbf{Y}$ two Banach spaces, let $(x,y) \in \left(C^0([t_0,T];\mathbf{X})\right)^n \times \left(C^0([t_0,T];\mathbf{Y})\right)^n$ and $\varepsilon \in \mathbb{R}^+$; we denote:

$$\Omega_{x,\varepsilon} := \bigcup_{i,j} \{(i,j)\} \times \Omega^{i,j}_{x,\varepsilon},\ \text{ where } \Omega^{i,j}_{x,\varepsilon} := \left\{(t,\tau) \in [t_0,T]^2;\ \left\|x^i(t) - x^j(\tau)\right\| \leqslant \varepsilon\right\}.$$

The *ε-cancellation operator* $\mathfrak{D}_\varepsilon$ is defined by:

$$(x,y) \mapsto \mathfrak{D}_\varepsilon\,(x,y) : \begin{array}{ccc} \Omega_{x,\varepsilon} & \to & \mathbf{Y} \\ (i,j,t,\tau) & \mapsto & y^i(t) - y^j(\tau). \end{array} \tag{4}$$

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Proposition 2.1. (1) $\mathfrak{D}_\varepsilon$ is continuous. (2) For any continuous $f : \mathbf{X}_0 \subset \mathbf{X} \to \mathbf{Y}$, we have the *cancellation property*: $\mathfrak{D}_0\,(x, f \circ x) = 0$.

Thanks to proposition 2.1 and by application of operator $\mathfrak{D}_0(X, .)$ to both members of the equation, model (3) is rewritten ($n = 1$):

$$\left\{ \begin{array}{l} \langle \mu, \mathfrak{D}_0\,(X, \gamma\,\psi_X + X) \rangle = \mathfrak{D}_0\;(X, G(X)u) \\ \langle \mu, \gamma\,\psi_X(t_0, .) + X(t_0) \rangle - F(X(t_0)) = G(X(t_0))u(t_0). \end{array} \right. \tag{5}$$

The interest of this new formulation is that up to the quantity $F_0 := F(X(t_0))$, the γ-symbol μ can be identified *independently* of F by (for simplicity, G is here supposed to be known):

$$(\mu^*, F_0^*) = \mathcal{Y}_\varepsilon^\dagger \left(\mathfrak{D}_\varepsilon\,(\tilde{X}, G(\tilde{X})\tilde{u}), G(\tilde{X}(t_0))\tilde{u}(t_0) \right),$$

where $\mathcal{Y}_\varepsilon^\dagger$ is the pseudo-inverse, in a suitable sense, of the operator $\mathcal{Y}_\varepsilon$ defined by:

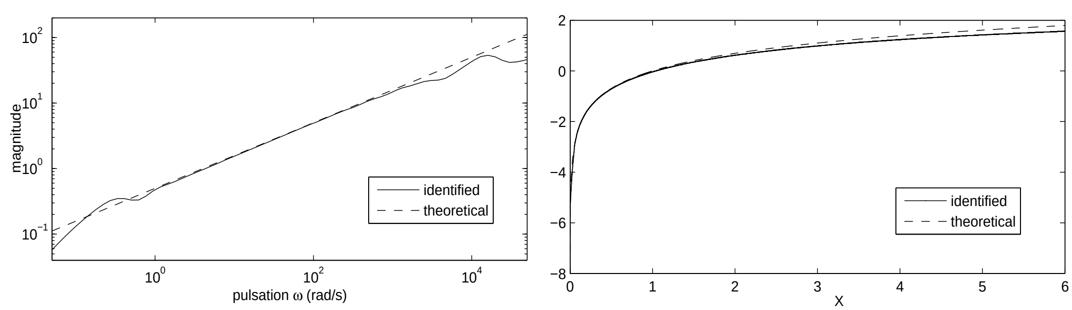
$$(\mu, F_0) \mapsto \left(\left\langle \mu, \mathfrak{D}_\varepsilon(\tilde{X}, \gamma\,\psi_{\tilde{X}} + \tilde{X}) \right\rangle, -F_0 + \left\langle \mu, \gamma\,\psi_{\tilde{X}}(t_0, .) + \tilde{X}(t_0) \right\rangle \right). \tag{6}$$

By means of standard regression methods, the function F can then be easily estimated from the "pseudo graph" of F:

$$\mathcal{G}_F = \bigcup_{k,j} \left\{ \left(\tilde{X}^k(t_j), \left\langle \mu^*, \gamma\,\psi_{\tilde{X}^k}(t_j, .) + \tilde{X}^k(t_j) \right\rangle - G(\tilde{X}^k(t_j))\tilde{u}^k(t_j) \right) \right\}. \tag{7}$$

3 A concrete example

This method has been tested on data elaborated from numerical simulation of the following model of spherical flames [2]: $X\,\partial_t^{1/2} X = X\,\ln(X) + u$, $u \geqslant 0$, $X \geqslant 0$. The identified symbol $H^*(i\omega) = \int \frac{\mu^*(\xi)}{i\omega - \gamma(\xi)} d\xi$ [3] and function F^* are given here-after. More details and extensions will be found in [1] and in a further publication.



(a) Frequency response of $H^*(i\omega)$.

(b) Identified F^*.

References

[1] C. Casenave, G. Montseny, *Diffusive Identification of Volterra Models by Cancellation of the Nonlinear Term*, IFAC Symposium on System Identification, (SYSID 2009), Saint-Malo, July 6-8, 2009.

[2] G. Joulin, *Point-source initiation of lean spherical flames of light reactants: an asymptotic theory*, Comb. Sci. and Tech., pp 99-113, 1985.

[3] G. Montseny, *Représentation Diffusive*, Hermes-Science, Paris, 2005.