TRANSPORT COEFFICIENTS OF DENSE PLASMAS

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Résumé.- La théorie cinétique habituelle, basée sur l'idée que les particules d'un plasma sont faiblement couplées, est incapable de rendre compte des données récentes sur les coefficients de transport des plasmas denses. Nous discutons les approches récentes concernant la construction d'une théorie cinétique pour les plasmas fortement couplés. Les résultats pour la viscosité de cisaillement et le coefficient de self-diffusion d'un plasma à un composant obtenus à partir de cette théorie sont en bon accord avec les simulations sur ordinateur et ce dans toute la phase fluide du système.

Abstract.- Recent attempts to construct a kinetic theory for strongly coupled plasmas are discussed. The predictions of these theories for the shear viscosity and the self-diffusion coefficient of a one-component plasma are in good agreement with the available computer simulation data throughout the whole fluid phase.

1. Introduction.- The last sentence of Spitzer's famous paper on "Transport Phenomena in a completely Ionized Gas" states [1]: "In view of the lack of observational data in this field, development of a more refined theory does not seem worth the very considerable effort required". This was written more than 25 years ago and although some progress has been accomplished in the past decades the situation has remained the same for many years. Only very recently have things started to change. This is due partly to the fact that the use of sophisticated laboratory techniques has produced electrical conductivity data for plasmas outside the traditional weak-coupling region [2]. The main impetus however has come from the fact that the application of computer simulation techniques to ionized matter [3] has produced transport data for plasmas which are very dense, i.e. strongly coupled. It is our purpose here to summarize the main steps of the modern theories of transport in charged systems and to compare the few cases which have been worked out completely to the available numerical data.

2. Transport theory of weakly coupled plasmas.- Let us first recall some features of the traditional transport theory of plasmas. We will consider here only fully ionized plasmas and for the sake of presentational commodity we will always display expressions relevant to the one-component plasma, i.e. the theoretical model in which one of the charged species forms an uniform and inert neutralizing background in which the order specie moves according to classical dynamics, the interactions being restricted to the Coulomb forces. This model which looks rather theoretical can nevertheless yield realistic results for fully ionized matter in a restricted portion of the density-temperature plane including, for instance, typical white-dwarf conditions [4]. Besides the simplified dynamics this model has the advantage that the thermodynamic and transport data when suitably reduced, do depend only or a single parameter which we call the coupling parameter. For a system of mobile charges of electric charge e, number density n and inverse temperature (in energy units) $\beta = (k_B T)^{-1}$ this coupling parameter is obtained by estimating the ratio $\beta e^2/r_o$ of the average potential energy (e^2/r_o, r_o being a characteristic distance) to the average kinetic energy (β^{-1}). If r_o is taken to be the Debye length, $\lambda_D = (4\pi e^2 n\beta)^{-1/2}$ the coupling parameter becomes $\lambda/4\pi$ where $\lambda = (n\lambda_D^3)^{-1}$ is the inverse of the average number of particles in a "Debye cube" and as such λ has played an important role in weak-coupling expansions of equilibrium quantities. When discussing strong-coupling data however it is more convenient to us the "ion-sphere radius" a, with $n = \left(\frac{4\pi}{3} a^3\right)^{-1}$, as the characteristic distance r_o in which case the coupling para-

meter becomes $\Gamma = \beta e^2/a$. The relation between both parameters is $\lambda = 4\pi \sqrt{3}\ \Gamma^{3/2}$.
The early transport theory has been concerned exclusively with weakly coupled or dilute plasmas for which λ is small ($\lambda << 1$). In this case the kinetic equation, which is at the basis of the transport theory, can be written in standard notation [5]:

$$\frac{\partial f(\underline{r},\underline{p};t)}{\partial t} + \underline{v}\cdot\frac{\partial f}{\partial \underline{r}} + e\,\underline{E}(\underline{r},t)\cdot\frac{\partial f}{\partial \underline{p}} = J(f,f) \quad (1)$$

where $f(\underline{r},\underline{p};t)$ denotes the one-particle reduced distribution function, $\underline{E}$ the mean electric field and $J(f,f)$ the collision term which drives f towards its equilibrium value $f_0 = n\psi$, $\psi(f)$ being the Maxwellian. For weak-long range forces, small-angle scattering dominates and $J(f,f)$ is of Fokker-Planck type :

$$J(f,f) = \int dp'\ \frac{\partial}{\partial f}\cdot\underline{\underline{M}}(f,f')\cdot\left(\frac{\partial}{\partial f} - \frac{\partial}{\partial f'}\right) f(\underline{r},\underline{p};t)\ f(\underline{r},\underline{p}';t) \quad (2)$$

where $\underline{\underline{M}}$ is a non-negative matrix which can be written :

$$\underline{\underline{M}}(\underline{p},\underline{p}') = n\int \frac{dk}{8\pi^3}\ \underline{k}\,\underline{k}\ \pi\delta(\underline{k}\cdot\underline{v} - \underline{k}\cdot\underline{v}')\left|\frac{V(k)}{\varepsilon_0(\underline{k},\underline{k}\cdot\underline{v})}\right|^2 \quad (3)$$

where $V(k) = 4\pi e^2/k^2$ is the Fourier-transform of the Coulomb interaction energy whereas $\varepsilon_0(\underline{k},Z)$ is the dielectric function of a weakly coupled system :

$$\varepsilon_0(\underline{k},Z) = 1 + V(k)\int d_{\underline{p}}\ \frac{1}{z - \underline{k}\cdot\underline{v}}\ \underline{k}\cdot\frac{\partial f}{\partial \underline{p}} \quad (4)$$

Equations (2-4) define the well-known Balescu-Guernsey-Lenard collision integral which yields the most refined weak-coupling theory and which will serve as a reference point for us. The linearized kinetic equation which contains all the information necessary for a transport theory can be obtained from eq.(1) by putting $f = n\psi + \delta f$ and neglecting $O((\delta f)^2)$ terms. In a form which will be usefull when comparing with the strong coupling results this linearized equation can be written :

$$(Z - \underline{k}\cdot\underline{v})\ \delta f(\underline{k},\underline{p};Z) - n\beta V(k)\,\underline{k}\cdot\underline{v}\psi(\underline{p})\int d\underline{p}'\,\delta f(\underline{k},\underline{p}';Z) = \int d\underline{p}'\,\Sigma_c(\underline{p},\underline{p}')\ \delta f(\underline{k},\underline{p}';Z) + i\delta f(\underline{k},\underline{p};t=0) \quad (5)$$

where $\delta f(\underline{k},\underline{p};Z$ denotes the Fourier transform with respect to $\underline{r}$ and the Laplace transform with respect to t of $\delta f(\underline{r},\underline{p};t)$ whereas the first term in the r.h.s. of eq. (5) is the Fourier-Laplace transform of the linearized version of $J(f,f)$, i.e. of $J(n\psi, \delta f) + J(f, n\psi)$.
A rough order of magnitude estimate of the collision term of eq.(5), say $\nu\,\delta f$, yields a collision frequency ν of the form $\nu \sim \lambda\,\omega_p$ λ being the coupling parameter and ω_p the plasma frequency of the mobile charges of mass m ($\omega_p^2 = 4\pi e^2 n/m$). This is easily understood since the collision term divided by δf has the dimension of a frequency (ω_p being the only characteristic frequency at our disposal here) while the pair correlation (say g) entering the collision term has been estimated to order λ (weak-coupling approximation). This weak coupling assumption for g is clearly in error for small interparticle distances since the two particle distribution (say $f_2 = ff(1+g)$) has to vanish for vanishing interparticle distances which in turn requires the two-particle correlations to be of order one ($g \simeq -1$) in this region instead of $O(\lambda)$. This underestimation of g leads then to the logarithmic large-k divergence of the r.h.s. of eq. (3) which, when cut off, leads in turn to the appearance of the so-called "Coulomb logarithm" in the collision frequency $\nu \sim \lambda\omega_p \ln(\alpha/\lambda)$.
A large number of authors [5] have tried to estimate the undetermined coefficient α. This is tentamount to pushing the small- λ expansion of ν one step further, viz. $\nu \sim \omega_p\ (\lambda \ln\lambda^{-1} + \lambda \ln\alpha + \ldots\ \lambda^2 \ln\lambda^{-1} + \ldots)$.
However we now know that a straightforward small-λ expansion of ν does not exist because of the feedback effect of hydrodynamic fluctuations onto the transport coefficients, a phenomen better known as the "long time tails" [7]. The dominant term $\nu \sim \lambda\ \omega_p\ \ln\lambda^{-1}$ for $\lambda << 1$ should however remain reliable. If we consider a transport coefficient, say the shear viscosity η, its reduced value $\eta/nm\omega_p\lambda_D^2$ becomes then

simply :

$$\eta/nm\omega_p\lambda_D^2 = \omega_p/\nu \equiv 1/\gamma\lambda\ln\lambda^{-1} \qquad (\lambda<<1) \qquad (6)$$

while the remaining task of any weak-coupling kinetic theory (usually the hard part of the job !) consists solely in the determination of the proportionality constant γ ($\nu = \gamma\lambda\omega_p\ln\lambda^{-1}$). The BGL kinetic equation yields for instance $\gamma = 1 / 10\ \pi^{3/2}$ in the first Sonine polynomial approximation.

Turning now to the subject of our concern, one could be tempted to use the small- λ results like eq. (6) as an empirical expression for estimating transport coefficients outside the weak-coupling region. Recent measurements of the electrical conductivity of ionised inert gases [2] indicate however that a departure from the typical $(\lambda\ln\lambda^{-1})^{-1}$ behavior becomes detectable already for intermediate $\lambda(\sim 0.8)$. Moreover direct computersimulations [3] have revealed that for strong coupling ($\lambda > 1$) $\eta/nm\omega_p\lambda_D^2$ passes through a minimum as a function of λ after which it increases slowly with λ. For strong coupling eq. (6) clearly becomes inadequate and a new starting point is needed.

3. Transport theory of strongly coupled plasmas.- The main defect of the BGL kinetic theory, as far as its application to dense plasmas is concerned, stems from the fact that it is still too much of a perturbation theory with respect to the potential. The renormalizations (i.e. infinite order resummations of potential contributions) which have been performed to obtain the BGL-equation have led merely to the introduction of the dielectric function $\varepsilon_0(\underline{k},Z)$ of eq. (4) which describes collective effects in a weak-coupling approximation. The BGL equation as such still predicts ideal gas thermodynamics, conserves only the kinetic energy, while the transport coefficients derived from it barely differ [8] from those obtained from Landau's straightforward weak-coupling kinetic equation corresponding to eqs (1-3) with the approximation $\varepsilon_0(\underline{k},Z) = 1$. What we need here is to eliminate as much as possible the bare potential in favor of the exact equilibrium correlations.

A second feature of the BLG equation which prevents it from adequately treating dense media is that it describes a local collision process. In a dense medium a collision occuring at a given point will depend on the state of the system in the neighbourhood of that point. As a consequence the BLG theory neglects important potential contributions to the transport coefficients. What we would like is some kind of Enskog theory for plasmas in which delocalisation effects are taken into account together with the exact equilibrium correlations. Systematic methods which realize this program, leading to so-called "fully renormalized kinetic equations", have been developed in recent years for dense fluids of uncharged particles [9]. Here we will indicate how these methods can be taken over to dense plasmas [10].

3.1. Renormalized B.G.L. equation.- In the modern approach to transport theory the basic quantities are the equilibrium phase-space correlation functions (also called propagators or classical Green's functions) :

$$S(\underline{r}_1-\underline{r}_2,t_1-t_2;\ \underline{p}_1,\underline{p}_2) = \langle \delta N(\underline{r}_1,\underline{p}_1;t_1)\ \delta N(\underline{r}_2,\underline{p}_2;t_2)\rangle \qquad (7)$$

where $\langle\cdots\rangle$ denotes a canonical average, $\delta N = N - \langle N\rangle$ is the fluctuation of the phase-space density N :

$$N(\underline{r},\underline{p};t) = \sum_j \delta(\underline{r} - \underline{r}_j(t))\ \delta(\underline{p} - \underline{p}_j(t)) \qquad (8)$$

of the N particles of positions $\{\underline{r}_j(t)\}$ and momenta $\{\underline{p}_j(t)\}$. The fact that $S(\underline{r},t\ ;\ \underline{p},\underline{p}')$ is considered as the basic quantity instead of the one particle distribution function implies that we will be concerned from the start with a linearized kinetic theory whereas in the ordinary kinetic theory based on the particle distribution functions one starts from a nonlinear kinetic theory and linearizes it a posteriori. The present procedure turns out to yield a difinite advantage since progress in the ordinary nonlinear kinetic theory beyond the BGL approximation is known to be very difficult. Here one can write down immediately an exact kinetic equation for $S(\underline{r},t\ ;\ \underline{p},\underline{p}')$ or its Fourier-Laplace transform $S(\underline{k},Z\ ;\ \underline{p},\underline{p}')$:

$$(Z - \underline{k}\underline{v})\ S(\underline{k},Z\ ;\ \underline{p},\underline{p}') + C(k)\underline{k}.\underline{v}\ \psi(\underline{p}) \int d\underline{p}'\ S(\underline{k},Z\ ;\ \underline{p},\underline{p}_1) = \int d\underline{p}'\ \Sigma_c(\underline{k},Z\ ;\underline{p},\underline{p}')\ S(\underline{k},Z$$

$p', p_1) + iS(\underline{k}, t = o ; p, p_1)$ (9)

where the initial value of S is easily seen from eq. (7) to be :

$$S(k, t=o ; \underline{p}_1, \underline{p}_2) = \eta\psi \left[\delta(\underline{p}_1 - \underline{p}_2) + (p_2)\, h(k) \right] \quad (10)$$

In eqs. (9-10), C(k) and h(k) are, respectively, the Fourier transforms of the direct correlation function c(r) and of g(r)-1, g(r) being the equilibrium pair correlation function. These functions are further related by the Ornstein-Zernike relation : $1 + h(k) = (1 - C(k)^{-1}$. Several methods have been deviced to derive eq. (9) from first principles [9] but here we will skip this rather technical point and for us eq. (9) can serve merely as a definition of Σ_c, the exact "linearized collision operator", which in this context is usually designed as the "memory function". It is important to observe that without making any approximation we have already partly realized the above mentioned programme. Indeed comparing eq. (9) with eq. (5) we see that the bare potential (V(k)) appearing in eq. (5) has been eliminated in favor of an equilibrium correlation function ($\eta\beta V(k) \to - C(k)$) whereas the collision process described by eq. (9) is clearly nonlocal in space (and time) because of the $\underline{k}$ (and Z) dependence of Σ_c. To proceed we need an explicit expression for Σ_c. It can be shown [9] that Σ_c can be written as :

$$i\Sigma_c(1,2;t)\ \eta\psi(\underline{p}_2) = \int d1'd2'\ L(11')L(22')C'(11' ; 22' ; t) \quad (11)$$

where $\underline{1} \equiv (\underline{r}_1, \underline{p}_1)$, $d1 \equiv d\underline{r}_1 d\underline{p}_1$, etc ... while L(12) is the familiar two-body interaction operator :

$$L(12) = - \frac{\partial V}{\partial \underline{r}_1}(\underline{r}_1 - \underline{r}_2) \cdot \left(\frac{\partial}{\partial \underline{p}_1} - \frac{\partial}{\partial \underline{p}_2} \right) \quad (12)$$

The quantity C(11';22';t) which appears in eq. (11) is (apart from some technical irreducibility condition) the four-point generalization of the two-point function S(1,2;t) which is our basic quantity. Performing a cluster expansion we can write :

$$C(11';22';t) = S(1,2;t)\, S(1',2';t) + S(1,2';t)S(1',2;t) + C_c(11';22';t) \quad (13)$$

where C_c is the non-factorisable or "connex" part of C. The quantity C_c is known to describe close interactions and is extremely complicated except at t = 0 where it is known explicitly in terms of equilibrium correlation functions. A technique anologue to the "vertex renormalization" of the Green's function methods allows us to incorporate this known piece of information about C_c. Skipping the details we merely note here that it amounts to replace the bare vertex (V(k)) by a renormalized one ($-c(k)/\eta\beta$. Dropping C_c for later times leads then finally to the following expression for the linearized collision operator :

$$\Sigma_c(\underline{k}, t; p_1, p_2) = \frac{i}{(\eta\beta)^2} \int \frac{d\underline{k}'}{8\pi^3} \int dp_1, dp_2,\ C(\underline{k}')\, \underline{k}' \cdot \frac{\partial}{\partial \underline{p}_1} \quad (14)$$

$$\{S(\underline{k}-\underline{k}', t; p_1, p_2)\ S(\underline{k}', t; p_1', p_2')\ c(k')\underline{k}' \frac{\partial}{\partial p_2}$$

$$- S(\underline{k}-\underline{k}', t; p_1, p_2')\ S(\underline{k}', t; p_1', p_2)\ C(\underline{k}-\underline{k}')$$

$$(k-k') \cdot \frac{\partial}{\partial p_2} \}\ (\eta\psi(p_2))^{-1}$$

which when Laplace transformed with respect to t and substituted back into eq. (9) constitutes the basic approximate kinetic equation we propose for dense plasmas [7,10,11].

3.2. Some properties of the renormalized theory.- The kinetic theory based on eqs (9, 10,14) has been obtained from the exact but formal results by neglecting dynamic close interactions. The static close interactions however are retained in the exact equilibrium correlations which appear in eqs (9,14). This kinetic equation can be shown [10] to conserve the total number, total momentum and total energy of the particles. Physically it corresponds to a fully renormalized version of the BGL theory in which the interactions proceed via an effective potential ($-C(k)/\eta\beta$) while the particles are propagated during the nonlo-

cal collision process with their exact propagator S instead if its Vlassov approximation. Notice that although we are concerned here with linear transport theory the kinetic equation is nonlinear in S. This is a typical feature of dense media as it allows for the feedback action of the medium onto itself. An explicit example of this effect, the so-called long-time tails of the integrands of the transport coefficients has been treated elsewhere [7] on the basis of eq. (14). The time non-locality of the collision operator of eq. (14) also allows for the description of finite frequency effects which are essential for a proper treatment of the plasma oscillations of dense plasmas [11]. Finally a slight modification of eq. (14) which guarantees that the short-time behavior of S (as characterized by its three first frequency moments) be rendered exactly has also been worked out [10].

As far as transport theory is concerned, the relation between eq. (14) and the linearized BGL collision operator, i.e. $\Sigma_c(\underline{p},\underline{p}')$ of eq. (5), can be seen as follows. If the collisions are weak the approximate nonlinear kinetic equation defined by eq. (9) and eq. (14) can be solved by iteration. As a first approximation we can compute eq. (14) by using for S its value obtained from the collisionless kinetic equation, i.e. eq. (9) with Σ_c deleted. This collisionless kinetic equation is nothing but a linearized Vlassov equation including however the exact statics through C(K). The BGL theory approximates moreover Σ_c by its local Markovian approximation, $\Sigma_c(k,Z;\underline{p},\underline{p}') \simeq \Sigma_c(k=o,\ z=o;\underline{p},\underline{p}')$, for which the above first-approximation becomes :

$$\Sigma_c(k=o,\ Z=o;\ \underline{p}_1,\underline{p}_2)\ \eta\psi(\underline{p}_2) = \frac{i}{\beta^2}\int \frac{dk'}{8\pi^3}\, k'.\frac{\partial}{\partial \underline{p}_1}$$

$$\frac{\underline{k}'.\frac{\partial}{\partial \underline{p}_2}}{\left|\frac{c(k')}{\varepsilon(\underline{k}',\underline{k}'.\underline{v}_1)}\right|^2}$$

$$\cdot\left[\pi\delta(\underline{k}'.\underline{v}_1-\underline{k}'.\underline{v}_2)\ \psi(\underline{p}_1)\ \psi(\underline{p}_2) - \delta(\underline{v}_1-\underline{v}_2)\ \psi\right.$$

$$\left.(\underline{p}_2)\int d\underline{p}_3\ \pi\delta\ (\underline{k}'.\underline{v}_1-k'\underline{v}_3)\ \psi(\underline{p}_3)\right] \qquad (15)$$

where $\varepsilon(k,Z)$ is defined as :

$$\varepsilon(k,Z) = 1 - \frac{C(k)}{\beta}\int d\underline{p}\ \frac{1}{Z - \underline{k}.\underline{v}}\ k.\frac{\partial\psi(\underline{p})}{\partial \underline{p}} \qquad (16)$$

If we moreover approximate the direct correlation function C(k) by its weak coupling value, i.e. $-\eta\beta V(k)$, then $\varepsilon(kZ)$ of eq. (16) reduces to the Vlassov result, $\varepsilon_o(k,Z)$ of eq. (4), while eq. (15) yields back the linearized BGL operator defined by eqs.(2-5). Hence, within the present framework, eq.(14) represents clearly a finite coupling, non-local (finite k) and non-Markovian (finite z) generalization of the BGL theory.

3.3. The renormalized interaction.- As stated above, the present theory is expressed in terms of a renormalized interaction which is proportional to the direct correlation function C(k) for which explicit data are needed. Originally [10] these data were taken from the Monte Carlo simulations [12]. As we now known, however, one can also predict theoretically very good data for C(k) by solving the hypernetted chain (HNC) equations for the Coulomb potential [4]. Recently it was shown [13, 14] that these equations can also be rewritten as an integral equation directly for C(k) :

$$C(k) = -\phi(k) + \frac{1}{n}\int \frac{dk'}{8\pi^3}\,\frac{k.k'}{k^2}\left\{\frac{[C(k-k')]C(k')}{1-C(k-k')}\right.$$

$$\left.\frac{+\ \phi(k)]\ -\ \phi(k')}{1\ -\ C(k')}\right\} \qquad (17)$$

where $\phi(k) = 4\pi e^2\eta\beta/k^2$. A less accurate but very simple model for C(k) has also been proposed recently [14]. This model is based on the ansatz that for small r, C(r) can be written as a polynomial in r^2 whereas for large r, C(r) reduces to the Coulomb potential. We will not dwell any further on these static aspects except for the observation that the above transport theory can be completed by computing C(k), for any value of the coupling parameter belonging to the system's fluid phase, directly from eq. (17)

3.4. Transport coefficients.- To proceed with this transport theory we still need an explicit expression for the transport coefficients. In the traditional theory one usually performs a Chapman-Enskog expansion of the kinetic equation. In the plasma case this expansion is hard to justify except in the limit of strong coupling [4, 5, 7, 11]. In the present scheme we can circonvent this difficulty and obtain ex-

pressions for the transport coefficients directly from eq. (9). Here we will sketch the procedure for the case of the shear viscosity which is very simple. Indeed from eq. (9) one can compute directly the transverse-momentum correlation function $G_\perp(k,Z) = \int d\underline{p}\, d\underline{p}'\, \underline{\varepsilon}.\underline{p}\, \underline{\varepsilon}.\underline{p}'\, S(\underline{k},Z;\underline{p},\underline{p}')$, $\underline{\varepsilon}$ being a unit vector othogonal to $\underline{k}$. Because of the rotational invariance of the various equilibrium quantities involved, $G_\perp$ decouples completely from the remaining "hydrodynamic" correlation functions and can be written exactly as $G_\perp(\underline{k}\, Z) = \frac{inm}{\beta}\, (Z - \Omega_\perp(\underline{k}Z))^{-1}$ where $\Omega_\perp(\underline{k}, Z)$ plays the rôle of a memory function for $G_\perp(\underline{k},Z)$. Because of momentum conservation we can write $\Omega_\perp(\underline{k}\, Z) = -i\frac{k^2}{nm}\, \eta(\underline{k}Z)$, $\eta(\underline{k}Z)$ being a non-local shear viscosity related to the ordinary shear viscosity η by $\eta = \eta(k=o, Z=o)$, whenever the latter limiting value exists which will be the case below. In this way we obtain an exact expression for the shear viscosity η in terms of the exact collision operator Σ_c :

$$\eta = n\, m \lim_{k\to o} \frac{1}{k} \Big\{ <\perp|\, i\Sigma_c(k,Z=o)\, |\perp> \quad (18)$$

$$+ <\perp|\, (\Sigma_o + \Sigma_c(k,o)\, Q(iQ\Sigma_c(o,o)Q)^{-1}\, Q(\Sigma_o + \Sigma_c(k,o))\, |\perp> \Big\} .$$

This expression can be shown to be identical to the full Kubo formula for η. It results from a complete reorganization of the latter formula and has the advantage to be expressed directly in terms of the collision operator instead of the Liouville operator. It is instructive to inspect this general expression more closely. In eq.(18), the matrix element $<\perp|f|\perp>$ denotes $\int d\underline{p}\, d\underline{p}'\, \underline{\varepsilon}.\underline{p}\, \underline{\varepsilon}\underline{p}'\, f(\underline{p},\underline{p}')\psi(\underline{p}')$, while Σ_o denotes the free-flow term of eq. (9), i.e. $\Sigma_o(\underline{k}Z;pp') = \underline{k}.\underline{v}\, \delta(\underline{p}-\underline{p}')$. The operator Q which appears in eq. (18) projects onto the non-hydrodynamic momentum states. Its effect is identical to the well-known subsidiary conditions of the Chapman-Enskog method. It is seen that the non-Markovian character, i.e. the z-dependence, of $\Sigma_c(kZ)$ plays no role here. This is easely understood since we are concerned with a vanishingly low hydrodynamic frequency. Things are however different as far as the nonlocal character, i.e. the K-dependence, of the collision operator is concerned. Taking the indicated limiting values, the first term in the r.h.s. of eq. (18) is seen to depend on the second derivative of Σ_c with respect to k while the second term does depend on $\Sigma_c(k=o,o)$ and on the first k-derivative of $\Sigma_c(k,o)$ (notice that $\Sigma_o \sim O(k)$). Hence if one straightforwardly neglects the nonlocal character of the collision process by taking from the start $\Sigma_c \simeq \Sigma_c(k=o,Z=o)$ as is done in the standard BGL theory, one is left over with :

$$\eta = nm\, <\perp|\frac{\Sigma_o}{k}\, Q(iQ\Sigma_c(o,o)Q)^{-1}\, Q\, \frac{\Sigma_o}{k}|\perp> \quad (19)$$

i.e. a matrix element of the (non-hydrodynamic part of the) inverse collision operator. This will clearly be a good approximation to eq.(18) when Σ_c is weak, i.e. for weak coupling and indeed computing eq.(19) with the BGL expression of $\Sigma_c(o,o)$ yields (in the first Sonine polynomial approximation) back the well known Spitzer-type formula of eq.(6). When Σ_c is not weak, the first term in the r.h.s. of eq. (18) will however rise with the coupling and hence the approximation $\Sigma_c \simeq \Sigma_c(k=o)$ cannot be maintained for strong coupling. To proceed with the transport theory of dense plasmas we have thus to compute for ex. eq. (18) with the aid of our nonlocal collision operator.

3.5. Numerical results.- For the explicit evaluation of the transport coefficients we propose to use the approximate collision operator of eq. (14) evaluated with the collisionless approximation for S . This is a nonlocal generalization of the BGL theory including moreover the exact statics. As such our proposal is similar to the Enskog theory which also uses the collision process of Boltzmann's dilute gas theory, delocalizes this collision process and takes the equilibrium correlations into account. We also know that Enskog's theory yields good results for the transport coefficients of dense systems of neutral particles if the hard-sphere diameter is suitably adjusted. Notice that here we have some kind of Enskog theory for plasmas which is free of undetermined parameters. To test this theory we will compare its results with those

of the molecular dynamics simulations of a classical one-component plasma [3]. As far as we know only two coefficients, the shear viscosity and the self-diffusion, have been evaluated theoretically for dense plasmas [10]. Both calculations use a one-Sonine polynomial approximation in order to evaluate inverse collision operators such as the one appearing in eq. (19). This is generally (except for the electrical conductivity) a good approximation for dilute systems. In the dense plasma case the convergence of this expansion has not been tested yet. Another technical point is that for the shear viscosity the Landau approximation ($\varepsilon(kZ) = 1$) is a good approximation while this is not so in the case of the self-diffusion coefficient. Let us start thus with the simpler case of the shear viscosity.

3.5.1 Shear viscosity.- Consider the reduced viscosity $\eta^* = \eta / n\, m\, \omega_p\, a^2$ which is a fonction of the coupling parameter $\Gamma = \beta e^2/a$. For small Γ ($<< 1$) the results for η cannot be distinguished from those of the weak-coupling theories, i.e. the BGL theory and Landau's theory (BGL with $\varepsilon_0(kZ) = 1$). This is interesting since the latter theories require cut-offs in eq. (3). At $\Gamma = 0.1$ things begin to change slightly (see Figure) but our result ($\eta^* = 86$) still lies between the BGL value ($\eta^*_{BGL} = 83$) and the Landau value ($\eta^*_L = 93$). Beyond $\Gamma = 0.1$ the weak-coupling theories become invalid. Here η^* decreases rapidly and reaches a minimum of $\eta^* = 0.07$ around $\Gamma = 8$ after which it rises again up to $\eta^* = 0.3$ at freezing $\Gamma = 160$. On the figure we have also indicated the results of a phenomenological theory [3] based on a gaussian fit of the memory function of η^*. This theory does not posses the correct weak coupling limit and is invalid for $\Gamma<2$. From the molecular dynamics computation of η^* say η^*_{MD}, we find that the error slightly increases with Γ. More precisely we find $\eta / \eta_{MD} =$ 0.97, 0.86 and 1.22 for

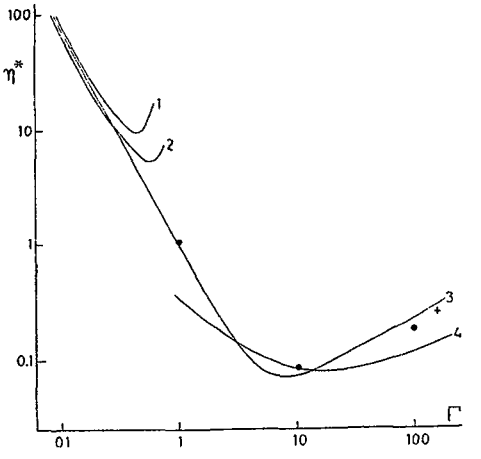

Fig. η^* as a function of Γ from : (1) the Landau theory [8], (2) the BGL theory[8], (3) the kinetic theory of Wallenborn and Baus [10], (4) the phenomenological theory of Vieillefosse and Hansen [3]. Also shown are the molecular dynamics results (dots) of Bernu and Vieillefosse [3] and an independent estimate (cross) by Hansen et al [3].

respectively $\Gamma = 1.0$, 10.4 and 100.4. The following empirical expression :

$$\eta^* = \frac{1}{45}\left(\frac{3\Gamma}{\pi}\right)^{1/2} + \frac{\left[1 + (3\Gamma)^{3/2}(0.49 - 2.23\Gamma^{1/3})\right]^2}{\left(\frac{3\Gamma}{\pi}\right)^{1/2}\ (0.96\Gamma^{10/3})}$$

where the different terms correspond to those of eq. (18), fits the theoretical strong coupling data ($2 < \Gamma < 160$) to within a few percent.

3.5.2. Self-diffusion.- In the case of the self-diffusion coefficient D an additional difficulty arises due to the fact that the values of $D^* = D/\omega_p\, a^2$ are obtained theoretically from a time integral of a rapidly oscillating function. In the case of the shear viscosity similar oscillations did appear but they did not occur around zero and are easier to treat. This explains why the Landau approximation ($\varepsilon = 1$ hence no oscillations !) for D is not as good as for η. In the Table below we compare the molecular dynamics values (D^*_{MD}) and the theoretical ones, with (D^*_1) and without (D^*_2) Landau approximation. Using improved statics (D^*_2) is seen to improve upon the earlier theoretical values (D^*_{GM}). Finally, using an improved short-time behavior (D^*_3) as required by the above mentioned oscil-

lation-problem is also seen to improve the situation although further investigation is required here. In any case D is seen, both from theory and experiment, to be a rapidly decreasing functions of Γ.

Γ	D^*_{MD}	D^*_{GM}	D^*_1	D^*_2	D^*_3
1	2.01	-	2.03	3.22	2.04
20	0.060	0.092	0.015	0.067	0.055
110	0.0051	0.0090	0.00098	0.0065	0.0056

Table. D^* as a function of Γ from : the molecular dynamics (D_{MD}) of Hansen et al [3], the kinetic theory of Gould and Mazenko (D_{GM}) [10]. Also shown are the results of [15] based on the theory of Gould and Mazenko [10] but with improved statics and with (D_1) and without (D_2) the Landau approximation and with improved short time behavior (D_3).

4. Conclusions.-

It appears to us that kinetic theories like those described above give a reasonable description of the transport coefficients of a classical one-component plasma both in the dilute gas and in the dense liquid-like situations. Hence, although the amount of work involved will be rather formidable, it should be interesting to extend these results to more realistic situations than those covered by the one-component plasma model.

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