Extreme Value Based Adaptive Operator Selection

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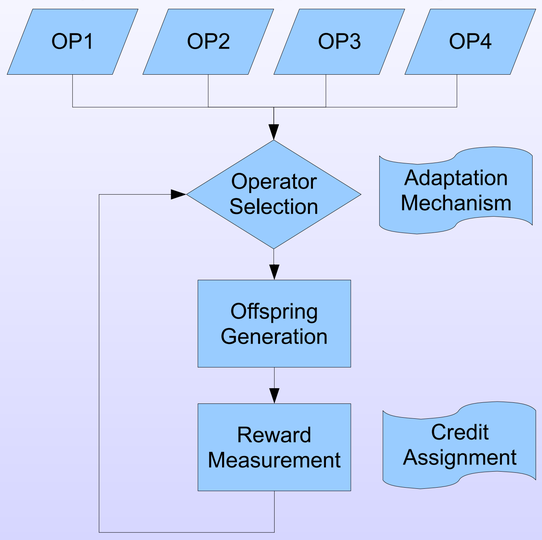
We address the question of how to estimate the quality of the operators used on an evolutionary algorithm. We argue that operators giving rare but **extreme** improvements are very valuable, and we propose a mechanism to reward them.

Adaptive Operator Selection

Dynamically select "best" operator during evolution



- **Credit assignment**: What reward?
- **Adaptation mechanism**: How to adapt operator selection depending on reward?

Credit Assignment

Previous work

- Reward $\propto$ fitness improvement,
- possibly averaged with decay factor,
- eventually distributed among offspring's back lineage

Bucket Brigade-like algorithm

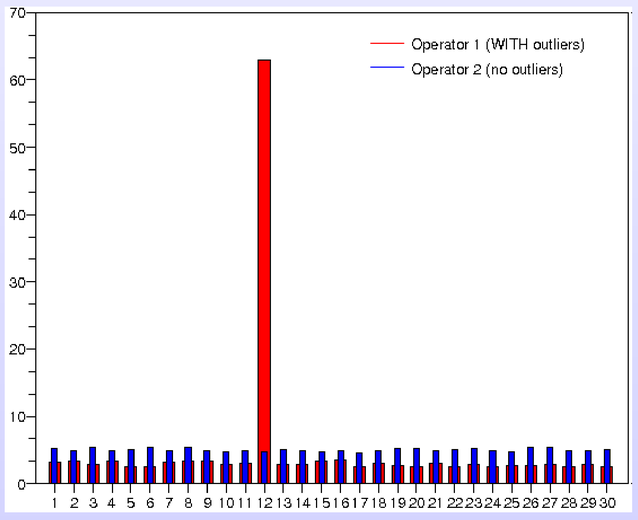
CLaim: Outliers matter

Very rare but extremely beneficial events can be more important than averages

- Indications from other disciplines

e.g. finance, rogue waves, ...

- Suggested in AOS context [Whitacre *et al*, 2006]



Two operators ("**red**" and "**blue**"), with same expectation. **Red** is extreme, **blue** is not

Proposed here: Extreme Reward

Best fitness improvement over a sliding window

Typically 50

Adaptation Mechanisms

Previous Work: Probability Matching and Adaptive Pursuit

see e.g. [Thierens, 2007]

Select an operator based on probabilities s_i ($\sum s_i = 1$)

- $\hat{Q}_{i,t}$ = estimated quality of operator i at time t
- Selecting operator $i \rightarrow$ reward $r_{i,t}$
- Estimated quality updated by relaxation

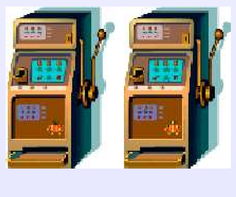
$$\hat{Q}_{i,t+1} = (1-\alpha)\hat{Q}_{i,t} + \alpha\, r_{i,t}$$

- **Probability Matching**: s_i proportional to Q_i
- **Adaptive Pursuit**: increase s_i for operator with best estimated quality, decrease all other s_j's evenly.
- Greediness: $s_i \geq P_{min}$ User-defined parameter

Dynamic Multi-Armed Bandits

proposed at GECCO 2008

- **Multi-Armed Bandit setting**



Reward for choosing arm j is

$$r_j = \begin{cases} 1 \text{ with prob } = p_j \\ 0 \text{ with prob } = 1 - p_j \end{cases}$$

- **Goal** Maximizze cumulated reward
- **UCB1 algorithm** [Auer *et al*, 2002]

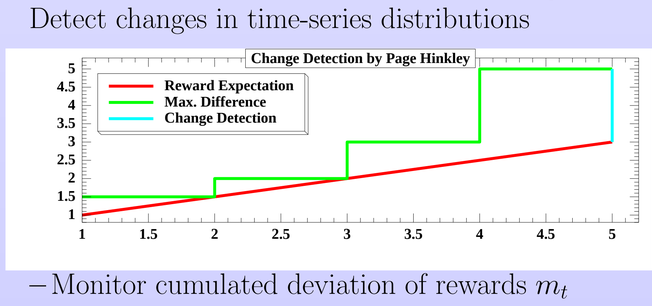
Be optimistic when facing the unknown:
At time t, choose arm j maximizing

$$\hat{r}_{j,t} + \sqrt{\frac{2\log\sum_k n_{k,t}}{n_{j,t}}},$$

$\begin{cases} \hat{r}_{j,t} : \text{ empirical reward for arm } j, \text{ favors exploitation} \\ n_{j,t} : \#\text{ times arm } j \text{ was chosen, favors exploration} \end{cases}$

- **Scaling** If $r_{j,t} \in [a,b] \neq [0,1]$: balance between exploitation and exploration requires scaling of $\hat{r}_{j,t}$
- **Dynamic rewards**
  - Operators' rewards likely to change during a run
  - UCB1 algorithm reacts slowly
  - $\rightarrow$ should be restarted in case of change in rewards
- **Page-Hinkley statistics** [Page, 1954]

Detect changes in time-series distributions



  - Monitor cumulated deviation of rewards m_t
  - Trigger restart when $|\text{Max}_{i \leq t}(m_i) - m_t| > \lambda$

λ user-defined parameter

Dynamic Multi-Armed Bandits (D-MAB):
UCB1 + Scaling + Page Hinkley

Winner Pascal EvE Challenge [Hartland *et al*, 2007]

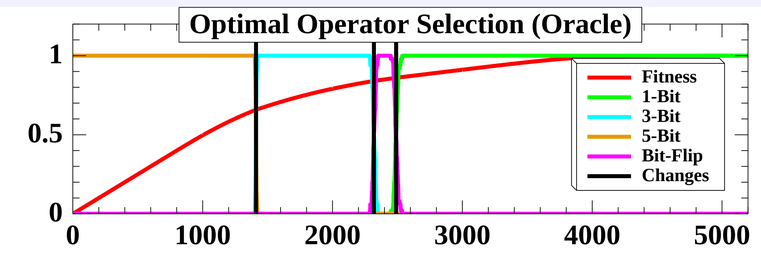
Experiments

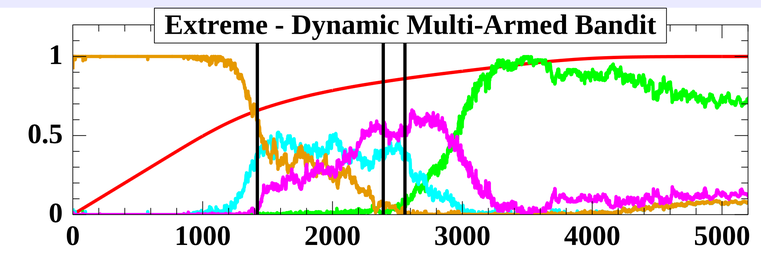
- **10000-bits OneMax** the drosophila of EC!
- **4 mutation operators**, (1+50)-EA
  - k-bit mutation (k=1,3,5) flips exactly k uniformly chosen bits
  - standard **bit-flip** mutation flips all bits with probability $\frac{1}{n}$
- Which is the best operator only depends on the fitness value

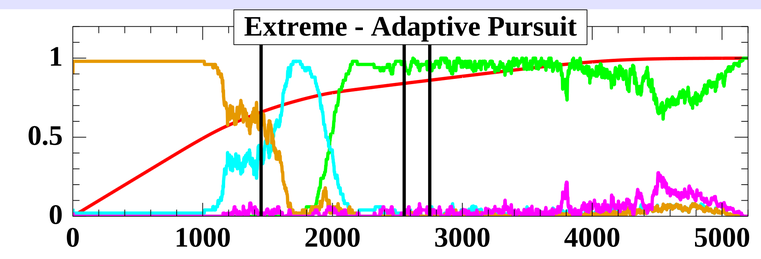
5-bit when low, 1-bit close to optimum, ...

Typical Behaviors:

Proportion of operators actually used, together with **fitness**







Extreme vs Average?

Dynamic-MAB	
Credit Assignment	Generations to Optimum
Extreme	5467 $\pm$ 513
Average	7727 $\pm$ 642

Adaptive Pursuit	
Credit Assignment	Generations to Optimum
Extreme	5478 $\pm$ 299
Average	5830 $\pm$ 324

Baseline	
Non-Adaptive Strategy	Generations to Optimum
Optimal according to Oracle	5069$\pm$ 292
Best Naive: $\mathcal{U}$(1-Bit+5-Bit)	6793 $\pm$ 625
Complete Naive: $\mathcal{U}$(all ops.)	7813 $\pm$ 708

Open Issues

- How much exploration are we really doing?
- Can those results be generalized? need a sound test suite
- Make some meta-parameters (self-)adaptive? e.g. λ in PH test
- What about a rank-based reward? $\rightarrow$scale-invariance

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