Markov Decision Evolutionary Game for Energy Management in Delay Tolerant Networks

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Invited paper

Rachid El-Azouzi*, Habib Sidi*, Francesco De Pellegrini† and Yezekael Hayel*
*LIA, University of Avignon, 339, chemin des Meinajaries, Avignon, France
† CREATE-NET Via alla Cascata 56/D, Povo, Trento, Italy

***Abstract*—In this paper, we apply the concepts of Markov decision evolutionary games to non-cooperative forwarding control of Delay Tolerant Networks (DTN). Specifically, we rely on the design of mechanisms at the source node to study forwarding probability of the message in a DTN using the two-hop routing. We study the forwarding probability as a function of the competition within a large population of mobiles which need occasionally to make some action. In particular, for each message generated by a source, each mobile may take a decision that concerns the strategy by which the mobile participates to the relaying of the message from source to destination. A mobile that participates receives a unit of reward if it is the first to deliver a copy of the packet to the destination. The action taken by a mobile determine not only the immediate reward but also the transition probability to its next battery energy state. We characterize the Evolutionary Stable Strategies (ESS) for these games and propose a method to compute them. We also propose a mechanism design at the source in order to maximize the message delivery probability to the destination, given the equilibrium behavior (called Evolutionary Stable Strategy - ESS).**

I. Introduction

Delay tolerant networks (DTNs) emerged recently as a novel communication paradigm. Throughout this work, we focus on a specific class of DTNs where persistent connectivity cannot be guaranteed due to limited coverage and high mobility [1]. For such networks, forwarding strategies have been designed purposely to solve the problem of intermittent connectivity: a message is delivered to the intended destination leveraging the motion of a subset of nodes, i.e., relays, which carry copies of the message stored in their local memory. This is the so called *carry-store-and-forward* routing. The DTN paradigm has been validated by several experimental deployments [3], [4].

In order to reach the destination, a straightforward strategy is to disseminate multiple copies of the message in the network. This approach is known as epidemic forwarding [9], in analogy to spread of infectious diseases. The aim in doing so is to let some of such message copies reach the destination with high probability within some target deadline [5], [6]. We confine our analysis to the two hop routing protocol. In fact, it has two major advantages: first, compared to epidemic routing, it performs natively a better trade-off between the number of released copies and the delivery probability [6]. Second, forwarding control can be implemented on board of the source node. Under two hop routing, the source transmits a message copy to mobiles devices it encounters. Relays, conversely, forward to the destination only.

In this context, the higher the number of relays joining the forwarding process, the higher the success probability. However, battery lifetime of mobile devices may deplete due to continuous beaconing operations [8], which may be a critical factor discouraging the usage of mobile devices as relays for DTN-based applications. A solution is to design reward-based forwarding mechanisms where the probability of forwarding becomes function of the competition within a population of mobiles: a relay receives a unit of reward if it is the first to deliver the message to the destination. For each message generated by the source, a relay may choose two different actions that affect message relaying: full activation or partial activation, i.e., being active for a shorter time period and then go back to low power mode, thus saving batteries.

This paper extends a similar framework studied by El-Azouzi et al. [7]. The novelty here is that the strategy of a mobile relay determines not only the immediate reward but also the transition probability to its next battery energy state. The problem is formulated as a Markov Decision Evolutionary Game (MDEG), where each relay wishes to maximize the expected utility. We characterize the Evolutionary Stable Strategies (ESS) for these games and show a method to compute them. Once determined the possible equilibria for the game, the optimal forwarding control at the source node that maximizes the forwarding probability has been derived. We show that the success probability is not always increasing with the number of message copies, and may well *decrease* under some conditions, which is adding an intriguing novel facet to the control of forwarding in DTNs.

II. Basic notions on evolutionary games

We consider the standard setting of evolutionary games :

- There is one population of users. The number of users in the population is large.
- We assume that there are finitely many pure strategies or actions. Each member of the population chooses from the same set of strategies $\mathcal{A} = \{1, 2, \dots, I\}$.
- Let $M := \{(y_1, \dots, y_I) \mid y_j \geq 0,\ \sum_{j=1}^{I} y_j = 1\}$ be the set of probability distributions over the I pure actions. I can be interpreted as the set of mixed strategies. It is also interpreted as the set of distributions of strategies among the population, where y_j represents the proportion of users choosing the strategy j.

- The number of users interfering with a given randomly selected user is a random variable K in the set $\{0, 1, \ldots\}$.
- A player does not know how many players would interact with it.
- The payoff function of all players depends of the player's own behavior and the behavior of the other players. The expected payoff of a user playing strategy j when the state of the population is y, is given by $U_{av}(j, y) = \sum_{k \geq 0} \mathbb{P}(K = k) U(j, k, y)$, where $U(j, k, y)$ is the payoff of a user playing strategy j when the state of the population is y and given that the number of users interfering with a given randomly selected user is k. Hence the average payoff of a population in state y is given by $F(y, y) = \sum_{i=1}^{I} y_j U_{av}(j, y)$.
- The game is played many times and there are many local interactions at the same time

Evolutionary Stable Strategy Suppose that, initially, the population profile is $y \in M$. Now suppose that a small group of mutants enters the population playing according to a different profile $mut \in M$. If we call $\epsilon \in (0, 1)$ the size of the subpopulation of mutants after normalization, then the population profile after mutation will be $\epsilon\ mut + (1 - \epsilon)y$. After mutation, the average payoff of non-mutants will be given by $F(y, \epsilon mut + (1 - \epsilon)y)$ where $F(x, y) := \sum_{j=1}^{N} x_j U_{av}(j, y)$. Note that U_{av} need not to be linear in the second variable. Analogously, the average payoff of a mutant is $F(mut, \epsilon mut + (1 - \epsilon)x)$.

Definition 1: A strategy $y^* \in M$ is an ESS if for any $mut \neq y^*$, there exists some $\epsilon_{mut} \in (0, 1)$, which may depend on mut, such that for all $\epsilon \in (0, \epsilon_{mut})$ one has

$$F(y^*, \epsilon mut + (1 - \epsilon)y^*) > F(mut, \epsilon mut + (1 - \epsilon)y^*) \quad (1)$$

which can be rewritten as $\sum_{j=1}^{N} (y_j^* - mut_j) U_{av}(j, \epsilon mut + (1 - \epsilon)y^*) > 0$. That is, y^* is ESS if, after mutation, non-mutants are more successful than mutants. In other words, mutants cannot invade the population and will eventually get extinct.

III. MODEL

Consider a network with several sources s_i, destination d_i and a large number of mobiles in the system. Each mobile is equipped with some form of proximity wireless communications. We assume that the message that is transmitted by a source, is relevant during some time τ. The network is assumed to be sparse, so that, at any time instant, nodes are isolated with high probability. Communication opportunities arise whenever, due to mobility patterns, two nodes get within mutual communication range. We refer to such events as "contacts". The time between subsequent contacts of a node with the source s_i or the destination d_i is assumed to follow an exponential distribution with parameter $\lambda > 0$. The validity of this model for synthetic mobility models has been discussed in [2].

The message contains a time stamp reporting its generation time, so that it can be deleted at all nodes when it becomes irrelevant. We do not assume any feedback that allows the source or other mobiles to know whether the message has been successfully delivered to the destination during time slot τ. We consider the two hop routing scheme [2] in which a mobile that receives a copy of the packet from the source can only forward it to the destination. We use evolutionary games to study the competition individual mobiles in a routing game in DTN.

We apply evolutionary games to non-cooperative 'live time' selection in delay tolerant networks.Specifically, we assume that each relay node can choose two different live times for the message: τ, i.e. full activation and τ' i.e., partial activation, where $\tau' < \tau$. At each slot, a mobile has to take a decision to be fully active or partially active, based on his battery energy sate. To simplify, we assume that the state can take three values : $\{F, A, E\}$ for Full, Almost empty or Empty. At state F only action τ is available, and at E participation on forwarding message is not possible any more. The life time of mobile is defined as the number of slots during which its battery is nonempty.

Local Interaction : Without loss of generality, we assume that in each local interaction, there is a source-destination pair in which the source has packet generated at each time $t_{i+1} - t_i = n\tau$ where $i = 1, 2..$ and $t_0 = 0$. Let N be the number of mobiles (possibly random) in an area which is assumed fixed during time slot. We denote by y (resp. $1 - y$) the fraction of mobiles sharing the strategy τ (resp. τ'). Consider an active mobile in a local interaction with source s_i, destination d_i and N opponents.

Some notation

We introduce the following notations:

- $P_i(a)$ is the probability of remaining at energy level i when using action a. Since at state F only action τ is available, we write P_F instead of $P_F(\tau)$
- $M_2 := \{(y, 1 - y)\}$ be the set of probability distributions over the 2 pure actions τ and τ'. M can be interpreted as the set of mixed strategies. It is also interpreted as the set of distributions of strategies among the population, where y (resp. $1 - y$) represents the proportion of mobiles choosing the strategy τ (resp. τ').

The probability that the tagged mobile relays the copy of the packet to the destination within live time τ is given by $1 - Q_\tau$ where Q_τ is given by $Q_\tau = (1 + \lambda\tau)e^{-\lambda\tau}$ and the probability that it relays the copy of the message if it chooses live time τ' is given by is given by $1 - Q_{\tau'}$ where $Q_{\tau'} = (1 + \lambda\tau')e^{-\lambda\tau'}$. Let $P_{succ}(\tau, N, y)$ (resp. $P_{succ}(\tau', N, y)$) be the probability that the tagged mobile receive the unit reward, if it chooses live time τ (resp. τ'). Now

$$\begin{aligned} P_{succ}(\tau', N, y) = &\quad (1 - Q_{\tau'}) \sum_{k=1}^{N} C_{k-1}^{N-1} \frac{(1-Q_{\tau'})^{k-1}(1-(1-Q_{\tau'}))^{N-k}}{k} \\ = &\quad \frac{1-(Q_{\tau'})^N}{N} \end{aligned}$$

The gain obtained by a mobile using live time τ' is given by

$$U(\tau', y) = \sum_{N=1}^{\infty} P(K = N) P_{succ}(\tau', N, y)$$

Now the probability that a mobile receives the unit award, if it chooses live time τ, is given by $P_{succ}(\tau, N, y)$

$$= P_{succ}(\tau', N, y) + (Q_{\tau'})^N \beta \sum_{k=1}^{N} C_{k-1}^{N-1} \frac{\beta^{k-1} y^{k-1} (1 - y\beta)^{N-k}}{k}$$

$$= P_{succ}(\tau', N, y) + (Q_{\tau'})^N \frac{1 - (1 - \beta y)^N}{Ny}$$

where $\beta = 1 - \frac{Q_\tau}{Q_{\tau'}}$. The utility for a mobile using live time τ

$$U(\tau, y) = \sum_{N=1}^{\infty} P(K = N) P_{succ}(\tau, N, y).$$

A general policy u is a sequence $u = (u_1, u_2, ...)$ where u_i is the strategy used at time t_i if the state is A. We shall use a pure stationary policy in which there exist two pure stationary policies ; the one always choose τ and the one that always choose τ'.

A. Fitness

An active mobile during $[t_i, t_{i+1}]$, will receive a unit of reward r if it is the first to deliver a copy of the packet to the destination. Assume that y is fixed and does not change in time (Note that assuming that y is fixed in time does not mean that the actions of each player are fixed in time. It only reflects a situation in which the system attains a stationary regime due to the averaging over a very large population, and the fact that all mobiles choose an action in a given individual state using the same probability low). Then the expected optimal fitness of an individual starting at a given initial state can be computed using the standard theory of total-cost dynamic programming, that states in particular that there exist optimal stationary policy (i.e. a policy for which at any time that the individual is at state A, the action u_i of choosing T is the same). We shall therefore *restrict to stationary policies* unless stated otherwise.

Let $V_\tau(i, y)$ (respectively $V_{\tau'}(i, y)$) to be the total expected fitness of a user given that it is in state i, that it uses action τ and given the parameter y.

We proceed by computing the individual's expected total utility and remaining lifetime that correspond to a given initial state and a stationary policy. We have $V_\tau(A, y) = U(\tau, y) + P_A(\tau) V_\tau(A, y)$ which gives that

$$V_\tau(A, y) = \frac{U(\tau, y)}{1 - P_A(\tau)}$$

The total expected utility for a mobile starting from state F and using strategy τ, is giving by

$$V_\tau(F, y) \quad = \quad U(\tau, y) + P_F V_\tau(F, y) + (1 - P_F) V_\tau(A, y)$$

Thus

$$V_\tau(F, y) = U(\tau, y)\left(\frac{1}{1 - P_F} + \frac{1}{1 - P_A(\tau)}\right) \qquad (2)$$

Similarly, the total expected utility for a mobile starting from state F and using strategy τ', is giving by

$$V_{\tau'}(F, y) = \frac{U(\tau, y)}{1 - P_F} + \frac{U(\tau', y)}{1 - P_A(\tau')})$$

Our game is different and has a more complex structure than a standard evolutionary game. In particular, the fitness that is maximized is not the outcome of a single interaction but of the sum of fitnesses obtained during all the opportunities in the mobile's lifetime. Let H be the function defined as

$$\begin{array}{rcl} H : y \in (0, 1) \rightarrow & & V_\tau(F, y) - V_{\tau'}(F, y) \\ & = & \frac{U(\tau, y)}{1 - P_A(\tau)} - \frac{U(\tau', y)}{1 - P_A(\tau')}) \\ & = & \frac{U(\tau, y)(1 - P_A(\tau')) - U(\tau', y)(1 - P_A(\tau))}{(1 - P_A(\tau'))(1 - P_A(\tau))} \\ & = & \frac{\tilde{H}(y)}{(1 - P_A(\tau'))(1 - P_A(\tau))} \end{array}$$

thus $H(y) =$

$$\frac{\sum_{N=1}^{\infty} P(K{=}N) \left[(Q_{\tau'})^N \frac{1 - (1 - \beta y)^N}{Ny} (1 - P_A(\tau')) - \frac{1 - (Q_{\tau'})^N}{N} (P_A(\tau') - P_A(\tau)) \right]}{(1 - P_A(\tau'))(1 - P_A(\tau))}$$

B. Existence and Uniqueness of ESS

In this section, we are now looking at the existence and uniqueness of the ESS

Proposition 1: (1) The strategy τ dominates the strategy τ' if and only if $P_A(\tau') - P_A(\tau) \leq \sum_{N=1}^{\infty} P(K = N) \Big[Q_\tau'^N \Big((1 - \beta)^N (1 - P_A(\tau')) \Big) + 1 - P_A(\tau) \Big] \triangleq A_1$

(2 The strategy τ' dominates the strategy τ if and only if $P_A(\tau') - P_A(\tau) \geq \sum_{N=1}^{\infty} P(K = N) Q_{\tau'}^N \Big(N\beta(1 - P_A(\tau')) + P_A(\tau') - P_A(\tau') \Big) \triangleq A_0$

(3) If $P_A(\tau') - P_A(\tau) > A_1$ and $P_A(\tau') - P_A(\tau) < A_0$, then there exists an unique ESS y^* which is given by $y^* = \tilde{H}^{-1}(0)$

Proof

(1) The strategy τ dominates the strategy τ' if and only if $V_\tau(F, y) \geq V_{\tau'}(F, y)$ for all $y \in [0, 1]$. Since $\tilde{H}$ is decreasing function and $\tilde{H}(1) = A_1 - (P_A(\tau') - P_A(\tau)) \geq 0$, thus $\tilde{H}(y) \geq 0$ for all $y \in (0, 1)$. Then $V_\tau(F, y) - V_{\tau'}(F, y) = H(y) = \frac{\tilde{H}(y)}{(1 - P_A(\tau'))(1 - P_A(\tau))} \geq 0$ for all $y_T \in [0, 1]$. This completes the proof for (1)

(2) The strategy τ' dominates the strategy τ if and only if $V_{\tau'}(F, y) \geq V_\tau(F, y)$ for all $y \in [0, 1]$. Since the function $\tilde{H}$ is decreasing function and $\tilde{H}(0) = A_0 - (P_A(\tau') - P_A(\tau)) \leq 0$, thus $\tilde{H}(y) \leq 0$ for all $y \in (0, 1)$. Then $V_{\tau'}(F, y) - V_\tau(F, y) = \tilde{H}(y) = \frac{\tilde{H}(y)}{(1 - P_A(\tau'))(1 - P_A(\tau))} \leq 0$ for all $\beta \in [0, (1 - Q_\tau]$. This completes the proof for (2)

(3) A strictly mixed equilibrium y^* is characterized $V_\tau(F, y^*) = V_{\tau'}(F, y^*)$. The function $\tilde{H}$ is continuous and strictly decreasing monotone on $(0, 1)$ with $\tilde{H}(0) > 0$ and $\tilde{H}(1) < 0$. Then the equation $\tilde{H}(y) = 0$ has a unique solution in the interval $(0, 1)$. This completes the proof.

C. Poisson distribution

We consider that nodes are distributed over a plan following a Poisson distribution with density γ. The probability that there is N nodes in local interaction is given by the following distribution : $\mathbb{P}(K = k) = \frac{\gamma^{k-1}}{(k-1)!}e^{-\gamma}$, $k \geq 1$. Considering those node distributions and from previous theorems, the unique ESS y^* is unique solution of the following equation:

$$\frac{e^{\gamma Q_{\tau'}} - e^{Q_{\tau'}(1-\beta y^*)\gamma}}{y^*} = (e^{\gamma} - e^{Q_{\tau'}\gamma})\frac{P_A(\tau') - P_A(\tau)}{1 - P_A(\tau)}$$

Thus, the equilibrium is given by

$$y^* = \frac{LambertW(-\frac{\alpha\beta e^{-\frac{\alpha(\beta e^{\alpha} - c)}{c}}}{c})c + \alpha\beta e^{\alpha}}{c\alpha\beta}$$

where $\alpha = Q_{\tau'}\gamma$ and $c = (e^{\gamma} - e^{Q_{\tau'}\gamma})\frac{P_A(\tau') - P_A(\tau)}{1 - P_A(\tau)}$

D. Dirac distribution

We consider that at a given time there is a fixed number of nodes in a local interaction. In this part, we suppose that the population of nodes is composed with many local interaction between n nodes where $N > 2$. The unique ESS y^* of this game is the unique solution of the following equation:

$$\frac{1 - (1 - \beta y^*)^N}{y} = \frac{1 - (Q_{\tau'})^N}{(Q_{\tau'})^N}$$

Since this polynome is of order N we can have an explicit expression only for $N \leq 5$ but we show some properties of the stable equilibrium by numerical computations in the next section.

For example:

$$N = 2 \implies y^* = \frac{(1 - (Q_{\tau'})^2)G - 2Q_{\tau'}(Q_{\tau'} - Q_{\tau})}{(Q_{\tau'} - Q_{\tau})^2}$$

with $G = \frac{P_A(\tau') - P_A(\tau)}{1 - P_A(\tau)}$. One can easily show that $y^* > 1$ thus $y^* = 1$.

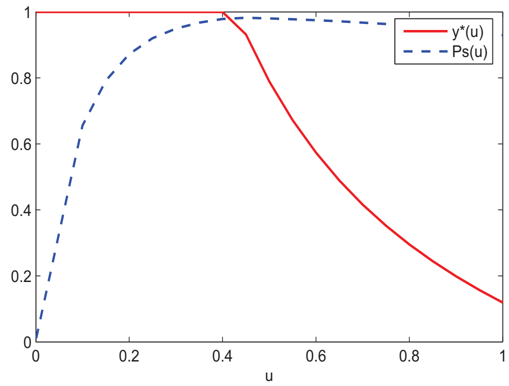
For $N = 3$, y^* is the solution of the equation

$$y^2 + \frac{3}{\beta}y - k = 0$$

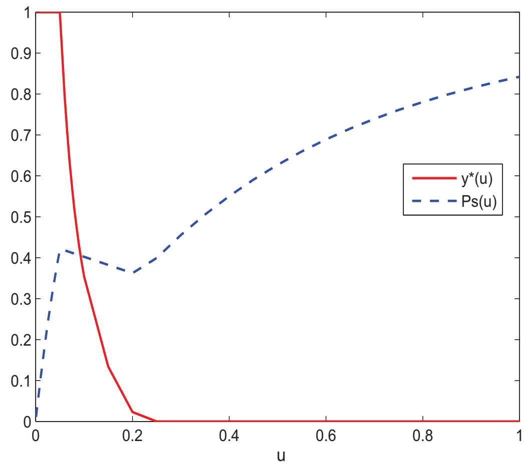
with $k = \frac{3}{\beta^2} - \frac{1 - (Q_{\tau'})^3}{\beta^3 (Q_{\tau'})^3}G$. $\delta = \frac{9}{\beta^2} + 4k$ with

$$k = \frac{3(Q_{\tau'})^2[Q_{\tau'} - Q_{\tau}]^2 + (Q_{\tau'})^4 - [Q_{\tau'} - Q_{\tau}]}{\beta^2 (Q_{\tau'})^2 [Q_{\tau'} - Q_{\tau}]^2} > 0$$

The realizable solution is $y^* = \frac{-\frac{3}{\beta} + \sqrt{\delta}}{2}$. In this case the solution y^* is positive when the conditions from proposition 1 are satisfied.



(a) $N = 7$ users in local interaction



(b) $N = 60$ users in local interaction

Figure 1. Population profile and probability of success for different values of N using Dirac distribution

IV. Mechanism design

In sight of the characterization of the ESS, we are interested in controlling the system in order to optimize for the energy consumption and the delivery probability. Let us assume that the source node controls the forwarding of message copies: a copy of the message is relayed with constant probability u upon meeting a node with no message during a local interaction, i.e., using a static forwarding policy [6]. The main quantity of interest is denoted P_s and it is the success probability of a message at a local interaction; under the same assumptions of linearity in [6], the average energy expenditure at the source node is $\mathcal{E} = \varepsilon\Psi$, where $\varepsilon > 0$ is the source energy expenditure per relayed message copy and Ψ is the corresponding expected number of copies released.

A. Activation control

Let us consider first the activation control. Due to the forwarding control, the probability that the tagged mobile relays the copy of the packet to the destination within live time τ is given by $1-Q_\tau^u$ where Q_τ^u is given by $Q_\tau^u = \frac{e^{-\lambda u\tau}-ue^{-\lambda\tau}}{1-u}$ and the probability that it relays the copy of the message if it chooses live time τ' is given by is given by $1-Q_{\tau'}^u$ where $Q_{\tau'}^u$ is given by $Q_{\tau'}^u = \frac{e^{-\lambda u\tau'}-ue^{-\lambda\tau'}}{1-u}$ so that the success probability in a local interaction with N mobiles $P_s(u|N=k) = 1-\left(Q_\tau^u\right)^{ky_T(u,k)}$

$\Rightarrow P_s(u) = 1-\sum_{k=0}^{\infty} P(N=k)\left(Q_\tau^u\right)^{ky_T(u,k)}$ where $y_T(u,k)$ is the fraction of mobiles playing T when there are k nodes in the local interaction. At the equilibrium

$$P_s(u|N=k) = 1-\left[\left(Q_{\tau'}^u\right)^{k(1-y(u))}.\left(Q_\tau^u\right)^{ky(u)}\right]$$

$$\Rightarrow P_s(u) = 1-\sum_{k=1}^{\infty} P(N=k)\left[\left(Q_{\tau'}^u\right)^{k(1-y(u))}.\left(Q_\tau^u\right)^{ky(u)}\right]$$

Using the same notations for the ESS and the Poisson distribution, at the equilibrium we have :

$$y^*(u) = \frac{LambertW(-\frac{\alpha\beta e^{-\frac{\alpha(\beta e^{\alpha}-c)}{c}}}{c})c+\alpha\beta e^{\alpha}}{c\alpha\beta}$$

where $\alpha = Q_{\tau'}^u\gamma$, $c = (e^{\gamma}-e^{Q_{\tau'}^u\gamma})\frac{P_A(\tau')-P_A(\tau)}{1-P_A(\tau)}$ and $\beta = 1-\frac{Q_\tau^u}{Q_{\tau'}^u}$.

The probability of success is:

$$P_s(u) = 1-e^{-\gamma}\left[e^{\gamma\left(Q_{\tau'}^u\right)^{(1-y(u))}.(Q_\tau^u)^{y(u)}}-1\right]$$

The following theorem gives some results on the probability of success according to the behavior of the ESS when the controls change at the source.

Theorem 1: The maximum value of the probability of success is attained for $y* = argmax\{P_s(1), P_s(u_0)\}$, where u_0 satisfies $\bar{y}^*(u_0) = 1$ and $y^*(u_0+\delta) \leq 1,\ \delta > 0$.

To proove this theorem we need the two following lemma on the costs function $H(y,u)$, and other genral results.

Lemma 1: For a fixed number of users N in a local interaction, the function H is concave in u for a given y.

Lemma 2: Let $\tilde{y}^*$ the solution of the equation $H(y) = 0$. The following assertions are verified:

- $Q_{\tau'}^u$ is a decreasing function in u.
- Q_τ^u is a decreasing function in u.
- $\tilde{y}^*$ is a decreasing function in u .
- Under some specific conditions we have $\tilde{y}^*(\epsilon) \geq 1$ otherwise $\tilde{y}^*(\epsilon) \leq 0$ with ϵ a very small positive number.

Poofs of lemma 1 and 2 are given in the appendix of this paper. We give now the proof of the theorem 1.

Table I
PARAMETERS VALUES

-	λ	τ	τ'	$P_A(\tau)$	$P_A\tau'$	γ
Optimal between $[0,1]$	0.01	140	30	0.4	0.9	30
Optimal on the edge	0.01	100	40	0.3	0.8	30

Proof: Given the expression of the probability of success, maximizing $P_s(u)$ comes down to minimize the expression $(Q_{\tau'}^u)^{(1-y(u))}.(Q_\tau^u)^{y(u)}$. Let

$$f(u) = (1-y(u))\log(Q_{\tau'}^u)+y(u)\log(Q_\tau^u)$$

, we need to minimize $f(u)$.

$f'(u) = y'(u)\Big[\log(Q_\tau^u) - \log(Q_{\tau'}^u)\Big] + (1-y(u))\left(\frac{(Q_{\tau'}^u)'}{(Q_{\tau'}^u)}-\frac{(Q_\tau^u)'}{(Q_\tau^u)}\right)+\frac{(Q_\tau^u)'}{(Q_\tau^u)}$.

For u small, using lemma 2, we have: $y* = 1$ or $y* = 0$

If $y* = 0$, given that $y*$ is decreasing in u then $y* = 0\ \forall\ u$. $f'(u) = \frac{(Q_{\tau'}^u)'}{(Q_{\tau'}^u)} \leq 0$ thus, f is decreasing and $P_s(u)$ is always increasing on $[0,1]$.

On the other hand, if $y* = 1$ for u small, we need to prove that if $P_s(u) = P_{max}$ then $u \in [u_0,1]$. Since y^* is a decreasing function of u, we have, $\bar{y}^*(0) = 1 \implies \exists u_0$ s.t. $\bar{y}^*(u_0) = 1$ and $\bar{y}^*(u_0+\delta) \leq 1,\ \delta > 0$.Where $\bar{y}^*$ is the projection of y^* on the interval $[0,1]$.

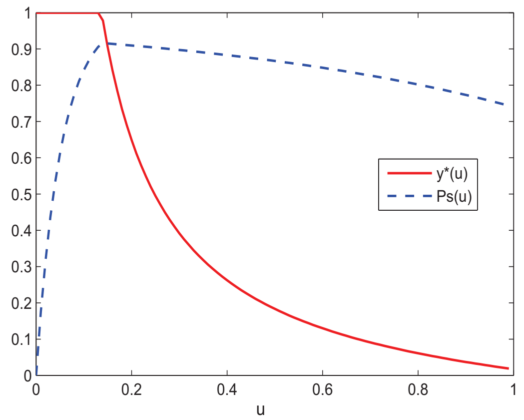
$y* = 1$ for u small $\Rightarrow f$ is decreasing and P_s is always increasing $for\ u \in [0,u_0]$. ■

Figure IV-A shows the shape of the ESS using the Poisson distribution for different set of values for the parameters. Values are set in table I for each cases. As we can see from the figures, the optimal value of the source control (maximizing the probability of successful delivevry) is sometime obtained, according to the notations of theorem 1, inside the interval $[u_0,1[$. This means that increasing the control at the source does not alway insure a higher probability of success given that the ESS $y*$ changes accordingly. A similar observation can also be deduced from Figure III-D for the Dirac distribution.

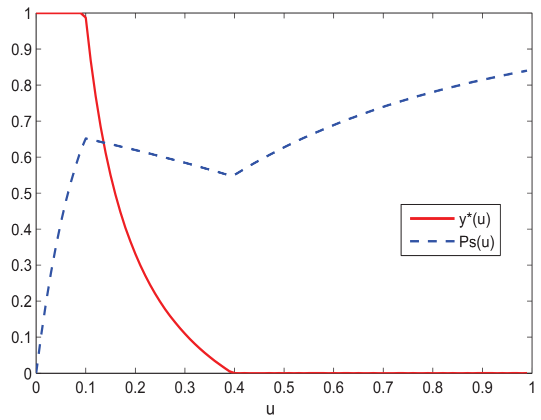
V. CONCLUSION

In this paper Markov decision evolutionary games are used to model competition between individual mobiles acting as relay nodes in a DTN routing game. The objective of the source node is to maximize the probability of success of delivering a message to destination. However, mobiles decide to join message relaying based on their current energy state, which in turn is influenced by the forwarding control used by the source, in trade for reward.

Under this framework, we studied a source-controlled evolutionary game aimed at optimizing the energy consumed by relays. We observed a clear trade-off, where the optimal solution in general does not correspond to forwarding at full rate at the source node, and we showed cases where such a greedy strategy is well sub-optimal in maximizing the probability of success at the equilibrium.



(a) Optimal value in the interval



(b) Optimal on the edge

Figure 2. Population profile and probability of success using Poisson distribution: (Two main observed cases)

VI. APPENDIX

A. Activation control

We recall lemma 1. We state that: For a fixed number of users N in a local interaction, the function H is concave in u for a given y.

Proof: For N fixed and a given y, let

$$h^N(u) = (Q^u_{\tau'})^N \frac{1-(1-\beta y)^N}{Ny} A - \frac{1-(Q^u_{\tau'})^N}{N}\eta$$

with $A = (1 - P_A(\tau'))$ and $\eta = (P_A(\tau') - P_A(\tau))$, from the expression of $H(y,u)$. If h is concave in u for any N then $H(y,u)$ is also concave. Using $\beta(u) = 1 - \frac{Q^u_\tau}{Q^u_{\tau'}}$ we have,

$$\begin{aligned} h^N(u) &= (Q^u_{\tau'})^N \frac{1-\left[1-(1-\frac{Q^u_\tau}{Q^u_{\tau'}})y\right]^N}{Ny} A - \frac{1-(Q^u_{\tau'})^N}{N}\eta \\ &= \frac{(Q^u_{\tau'})^N - \left[Q^u_{\tau'} - (Q^u_{\tau'} - Q^u_\tau)\right]^N}{Ny} A - \frac{1-(Q^u_{\tau'})^N}{N}\eta \\ &= \frac{-1}{Ny}\left[(Q^u_{\tau'}(1-y) + Q^u_\tau y)^N A - (Q^u_{\tau'})^N (A + y\eta)\right] - \frac{\eta}{N}. \end{aligned}$$

Let $B = A + y\eta$; we express the derivative of $h^N(u)$:

$$\begin{aligned} \frac{dh^N(u)}{du} &= \frac{-1}{y}\Big[A[Q^u_{\tau'}(1-y) + Q^u_\tau y]^{N-1}[(\dot{Q}^u_{\tau'})(1-y) + (\dot{Q}^u_\tau)y] \\ &\qquad - (Q^u_{\tau'})^{N-1}(\dot{Q}^u_{\tau'})\Big] \end{aligned}$$

Let $f(y,u) = Q^u_{\tau'}(1-y) + Q^u_\tau y$ then $f(0,u) = Q^u_{\tau'}$.

$$\begin{aligned} \frac{dh^N(u)}{du} &= \frac{1}{y}\Big[Af^{N-1}(y,u)(-f'(y,u)) - B(f(0,u))^{N-1}(-f'(0,u))\Big] \\ &= \frac{1}{y}\Big((f(0,u))^{N-1}[A(-f'(y,u)) - B(-f'(0,u))] \\ &\qquad + A(-f'(y,u))[f^{N-1}(y,u) - f^{N-1}(0,u)]\Big) \end{aligned}$$

We know that $(\dot{Q}^u_\tau) \le (\dot{Q}^u_{\tau'})$, f is decreasing in u and $(\dot{Q}^u_{\tau'}) = (\dot{Q}^u_{\tau'})(1-y) + (\dot{Q}^u_{\tau'})y$ then $(\dot{Q}^u_{\tau'}) \ge (\dot{Q}^u_{\tau'})(1-y) + (\dot{Q}^u_\tau)y$ $\Longrightarrow$ $f'(0,u) \ge f'(y,u)$ and $f^{N-1}(y,u) - f^{N-1}(0,u)$ is decreasing.

Let's show that $A(-f'(y,u)) - B(-f'(0,u))$ is also decreasing.

$$A(-f'(y,u)) - B(-f'(0,u)) = A[f'(0,u) - f'(y,u)] + y\eta f'(0,u)$$

$\Rightarrow$

$$\begin{aligned} \frac{d(A(-f'(y,u)) - B(-f'(0,u)))}{du} &= Ay((\ddot{Q}^u_{\tau'}) - (\ddot{Q}^u_\tau)) + y\eta(\ddot{Q}^u_{\tau'}) \\ &= \frac{B\left[e^{-\lambda u\tau'}((\lambda\tau')^2(1-u)^2 + 2) - 2ue^{-\lambda\tau'}\right]}{(1-u)^3} \\ &\quad - \frac{A\left[e^{-\lambda u\tau}((\lambda\tau)^2(1-u)^2 + 2) - 2ue^{-\lambda\tau}\right]}{(1-u)^3} \end{aligned}$$

Though the negativity of this last expression has been observed for all the several set of values we experimented, it is not obvious to see here. We will then assume that it is negative and conclude that the function H is concave in u.

■

We recall lemma 2. We state that: Let $\tilde{y}^*$ the solution of the equation $H(y) = 0$. The following assertions are verified:

- $Q^u_{\tau'}$ is a decreasing function in u.
- Q^u_τ is a decreasing function in u.
- $\tilde{y}^*$ is a decreasing function in u .
- Under some specific conditions we have $\tilde{y}^*(\epsilon) \ge 1$ otherwise $\tilde{y}^*(\epsilon) \le 0$ with ϵ a very small positive number.

Proof: We proove here the two last points.

- To show that $\tilde{y}^*$ is a decreasing function in u, we first show that H is a non-increasing function of y. Indeed, We have,
$H(y) =$
$$\frac{\sum_{N=1}^{\infty} P(K{=}N)\left[(Q_{\tau'})^N \frac{1-(1-\beta y)^N}{Ny}(1-P_A(\tau'))-\frac{1-(Q_{\tau'})^N}{N}(P_A(\tau')-P_A(\tau))\right]}{(1-P_A(\tau'))(1-P_A(\tau))}.$$
Only the term $f(y) = \frac{1-(1-\beta y)^N}{Ny}$, in the expression of H is dependent on y. For every parameters (other than y) fixed, we have:

$$\begin{aligned}\frac{df(y)}{dy} &= \frac{N\beta\,(1-\beta y)^{N-1}\,Ny - N\left(1-(1-\beta y)^N\right)}{(Ny)^2}\\ &= \frac{N\beta y\,(1-\beta y)^{N-1} - 1 + (1-\beta y)^N}{Ny^2}\\ &= \frac{(1-\beta y)^{N-1}\,(N\beta y + 1 - \beta y) - 1}{Ny^2}\\ &= \frac{(1-\beta y)^{N-1}\,((N-1)\,\beta y + 1) - 1}{Ny^2}\end{aligned}$$

Let's show that $(1-\beta y)^{N-1}\,((N-1)\,\beta y+1) - 1$ is negative.
$(1-\beta y)^{N-1}\,((N-1)\,\beta y+1) - 1 \leq 0 \Rightarrow (N-1)\,\beta y + 1 \leq \frac{1}{(1-\beta y)^{N-1}}$ which is true since $\frac{d((N-1)\beta y+1)}{dy}\,|_{(y=0)} = \frac{d\left(\frac{1}{(1-\beta y)^{N-1}}\right)}{dy}\,|_{(y=0)} = \beta(N-1)$ and $\frac{d^2\left(\frac{1}{(1-\beta y)^{N-1}}\right)}{d^2 y} = \frac{\beta^2(N-1)(N+1)}{(1-\beta y)^{N+2}} > 0$. Thus H is a non-increasing function of y.
Using lemma 1 we have $if\,\exists\;\; u\;\; s.t\;\; H(u) = 0$ for a given y then, u is unique in $[0,1]$.
$\forall y_1 \leq y_2,\;\; H(y_1,u) \geq H(y_2,u)$, then $if\,\exists\;\; u_1,\, u_2\;\; s.t.\;\; H(y_1,u_1) = H(y_2,u_2) = 0$ then $u_1 \geq u_2$ and $\tilde{y}^*$ is a decreasing funtion of u.

- We have, $H(y,u) = V_\tau(F,y) - V_\tau(F,y)$, let's find a condition on τ and τ' so that $H(y,u) \geq 0$,
$\Rightarrow\; V_\tau(F,y) \geq V_\tau(F,y),\;\; \Longleftrightarrow$

$$\begin{aligned}U(\tau,y)\left(\frac{1}{1-P_F}+\frac{1}{1-P_A(\tau)}\right) &\geq \frac{U(\tau,y)}{1-P_F}+\frac{U(\tau',y)}{1-P_A(\tau')}\\ \frac{U(\tau,y)}{1-P_A(\tau)} &\geq \frac{U(\tau',y)}{1-P_A(\tau')}\\ \frac{1-P_A(\tau')}{1-P_A(\tau)}\frac{U(\tau,y)}{U(\tau',y)} &\geq 1\\ T\frac{U(\tau,y)}{U(\tau',y)} &\geq 1\end{aligned}$$

with $T = \frac{1-P_A(\tau')}{1-P_A(\tau)} \leq 1$. When u is taken very small, we have,

$$\begin{aligned}\frac{U(\tau,y)}{U(\tau',y)} &= \frac{\sum_{N=1}^{\infty} P(K{=}N)(P_{succ}(\tau',N,y)+(Q_{\tau'})^N\frac{1-(1-\beta y)^N}{Ny})}{\sum_{N=1}^{\infty} P(K{=}N)P_{succ}(\tau',N,y)}\\ &= 1+\frac{\sum_{N=1}^{\infty} P(K{=}N)(Q_{\tau'})^N\frac{1-(1-\beta y)^N}{Ny}}{\sum_{N=1}^{\infty} P(K{=}N)\frac{1-(Q_{\tau'})^N}{N}}\\ &= 1+\frac{\sum_{N=1}^{\infty} P(K{=}N)\frac{d}{du}\frac{1-(1-\beta y)^N}{Ny}}{\sum_{N=1}^{\infty} P(K{=}N)\frac{d}{du}(-(Q_{\tau'}))}\end{aligned}$$

Considering u small, we have, $\frac{d}{du}(-(Q_{\tau'})) = \lambda\tau' + e^{-\lambda\tau'} - 1$ and

$$\begin{aligned}\frac{d}{du}\left(\frac{1-(1-\beta y)^N}{Ny}\right) &= \frac{\frac{d}{du}[1-(1-\beta y)^N](Ny-(N\dot{y}(1-(1-\beta y))))}{Ny^2}\\ &= \frac{(1-\beta y)^{N-1}\frac{d}{du}(\beta y)}{y}\end{aligned}$$

assuming that $\dot{y}$ is bounded

$$\begin{aligned}\frac{d}{du}\left(\frac{1-(1-\beta y)^N}{Ny}\right) &= \frac{\beta\dot{y}+\dot{\beta}y}{y}\\ &= \dot{\beta}. \qquad (3)\end{aligned}$$

Thus,

$$\frac{U(\tau,y)}{U(\tau',y)} = 1+\frac{\dot{\beta}}{\lambda\tau'+e^{-\lambda\tau'}-1} = \frac{\lambda\tau+e^{-\lambda\tau}-1}{\lambda\tau'+e^{-\lambda\tau'}-1}$$

and $H(y,u) \geq 0 \iff \frac{1}{T} \leq \frac{\lambda\tau+e^{-\lambda\tau}-1}{\lambda\tau'+e^{-\lambda\tau'}-1}$.
  - If $\frac{1}{T} \leq \frac{\lambda\tau+e^{-\lambda\tau}-1}{\lambda\tau'+e^{-\lambda\tau'}-1}$ then $H(y,u) > 0$ and $y* = 1$.
  - If $\frac{1}{T} > \frac{\lambda\tau+e^{-\lambda\tau}-1}{\lambda\tau'+e^{-\lambda\tau'}-1}$ then $H(y,u) \leq 0$ and $y* = 0$.

■

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