Medium Access Games: The Impact of Energy Constraints

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***Abstract*—We consider random medium access schemes for devices that support sleep modes, i.e. turning off electronic compartments for energy saving. Due to hardware limitations, sleep mode transitions cannot occur at the medium access timescale. Each terminal can choose when to turn on/off and its probability to transmit on an arbitrary slot. Thus, we develop a two level model, consisting of a fast timescale for transmission scheduling and a slower timescale for the sleep mode transitions. We take a game theoretic approach to model the user interactions and show that the energy constraints modify the medium access problem significantly, decreasing the price of anarchy. Our results give valuable insights on the energy–throughput tradeoff for contention based systems.**

***Index Terms*—ALOHA, contention, energy saving, game theory**

I. Introduction

As mobile communications become part of our everyday life, new challenges for the system designers come to the foreground. First of all, the scarcity of bandwidth resources leads to extreme competition for the medium. Besides, the total energy dissipation by communication devices has been shown to amount to a significant portion of a nation's power profile, motivating efforts of per device energy economy. In an attempt to minimize their energy footprint and/or maximize the battery lifetime, existing wireless devices support radio *sleep modes*.

A generic wireless terminal consists of several circuit building blocks with the RF transceiver (radio) contributing significantly to the overall energy consumption. The RF transceiver itself consists of four subblocks. The transmit block that is responsible for modulation and up-conversion (i.e. transforms the baseband signal to RF), the receive block dedicated to the down-conversion and demodulation, the local oscillator that generates the required carrier frequency, and the power amplifier that amplifies the signal for transmission. Existing wireless devices support radio sleep modes that turn off specific subblocks, to minimize their energy consumption while inactive. For example, as shown in Table I the CC2420 transceiver ([1]) provides three different low power modes. In the deepest sleep mode, both the oscillator and the voltage regulator are turned off, providing hence the lowest current draw. However, this comes at the cost of the highest switching energy cost and the longest switching latency. On the other hand, the idle mode provides a quick and energy inexpensive transition back to the active state, but at the cost of higher current draw and consequently higher current consumption.

TABLE I
Switching time and energy consumption of a CC2420 radio

Power mode	**Switching time**(ms)	**Switching Energy** (μJ)	**Current Consumption** (μA)
Tx	0	0	10000
idle	0.1	1.035	426
power down	1.2	42.3	40
deep sleep	2.4	85.7	0.02

To address this tradeoff, the authors of [2] propose a scheme to dynamically adjust the power mode according to the traffic conditions in the network. They show that in a low traffic scenario, deep sleep should be preferred since most of the time the nodes tend to be inactive. In a high traffic setting though, a "lighter" sleep mode would be preferable, because frequent mode transitions incur high delay and energy costs, overshadowing the energy saving due to the low current draw. In a similar setting, [3] considers optimal scheduling of the sleep periods for the scenario of a mobile receiving data from a base station. The objective is to derive the sleeping strategy that balances the energy cost of frequent waking up to check for new packets and the retrieval latency.

In this direction, several energy aware MAC protocols have been proposed, either centralized or distributed ones, to resolve contention. However, most of them rely on the willingness of the nodes to comply with the protocol rules. Hence, they are vulnerable to selfish users that may deviate from the protocol in order to improve their own performance. Game theory comes as the ideal tool to model interactions among self–interested entities competing for common resources and it has also been considered recently for medium access.

In [4], the Nash Equilibrium Points (NEP) in a slotted ALOHA system of selfish nodes with specific quality-of-service requirements are studied. It has been observed that usually selfish behavior in medium access leads to suboptimal performance. For example, a prisoners dilemma phenomenon arises among selfish nodes using the generalized slotted Aloha protocols of [5]. A decrease in system throughput, especially when the workload increases due to the selfish behavior of nodes, has been observed in [6], [7].

The interplay of medium access contention and energy consumption was considered recently in [8]. In particular, a scenario of users selecting their back-off control parameters based on the measured collision rate and their power consumption was considered. Each user attempts to balance the utility acquired by transmitting and the disutility caused to him due to the induced energy consumption. The existence, uniqueness and stability of the equilibrium points of the arising game were investigated. In a similar framework and in an attempt to mitigate the effects of selfishness, the authors of [9] study the problem of minimizing the energy consumption for given throughput demands for a contention MAC. They show that whenever the demands are feasible, there exist exactly two Nash equilibrium points and derive a greedy mechanism that always converges to the best one.

In this paper we introduce an additional level of decision making capturing the ON-OFF strategy of the terminals over the classic ALOHA game. We model contention for the medium as a game, where users with specific energy constraints *select both the proportion of time that they sleep and their medium access probabilities*. To the best of our knowledge, this is the first work that addresses the interplay between contention and energy consumption *for systems that support sleep modes*. The contributions of this paper can be summarized in the following:

- We characterize the throughput optimal strategy under energy constraints, which differs from traditional Aloha and serves as a reference point. We also provide a distributed counterpart which focuses on fairness.
- We formulate contention as a non-cooperative game and show that it has a unique NEP and bounded Price of Anarchy (PoA), i.e. contrary to intuition from prior works we find that energy constraints reduce the negative impact of selfish behaviour.
- Based on the rationality of the users we derive a modified strategy, which allows the users to observe competition. This policy is more efficient but has multiple NEPs.

The main characteristics of slotted ALOHA are also apparent in most contemporary contention-based systems, such as the IEEE 802.11. For example, all these systems exhibit a certain amount of inherent inefficiency; the total throughput in a common channel breaks down significantly, as the number of users and the message burstiness increase (see [10]). Despite its limited applicability and mainly due to its simplicity, Aloha remains a tractable insightful tool for studying such systems. Consequently, our results provide insights for other contention based systems.

II. System model

We consider a communication scenario of $N = |\mathcal{N}|$ mobile terminals transmitting to a common destination (e.g. uplink to a Base Station). Time is slotted and within each timeslot each user may select either to transmit or to stay silent. Medium access is performed probabilistically, according to a slotted Aloha protocol, where a collision occurs whenever two or more terminals transmit concurrently. Each terminal has always packets at its buffer for transmission (i.e. saturated queue), but it has limited energy resources. Each device i is characterized by an energy budget $\tilde{e}_i$, representing either its available battery power or the maximum energy it is willing to expend. In order to save energy it can turn into a sleep mode, where most of the circuits are turned off. For analytical tractability we assume that each terminal may be in one out of two possible operation modes, ON or OFF.

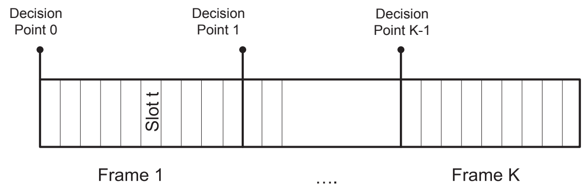

Fig. 1. The structure of a superframe

In general, a mode transition incurs significant energy and time (delay) costs. Besides, due to hardware limitations the time required for a mode transition is of the order of msec, much larger than the duration of a timeslot. Consequently, transition at the timeslot level is neither feasible nor desirable. Hence, we introduce a new timescale (we call it frame) where the mode switching takes place. Several timeslots constitute a frame. The beginning of each frame is a decision point, where a node may change its operation mode. Within a frame, the nodes keep their mode fixed and may access the medium randomly with probability p. This is also assumed fixed on a per frame basis. For decision reasons and without affecting the operation of the system, we introduce yet another timescale, that of the super frame, where an arbitrary number of frames, say K, forms a superframe (Fig. 1)

A binary vector $\boldsymbol{q}_i(j) = \{0, 1\}^K$, represents the ON or OFF state of user i on superframe j, and p_i its access probability (i.e. the access probability is selected only once per superframe). In practice the mobile terminals are autonomous and due to the limited knowledge available node level, synchronization of sleep modes is a difficult task. On the other hand, probabilistic ON-OFF has been deemed a feasible strategy ([11]). *Thus, we focus on a probabilistic version of the aforementioned problem*. Each user i is characterized by a probability of being ON, denoted with q_i and a medium access probability p_i. In matrix notation the strategy space can be written as $\mathcal{I} = \{\boldsymbol{p}, \boldsymbol{q}\}$, with $\boldsymbol{p} = [p_1, p_2, \ldots p_N]$ and $\boldsymbol{q} = [q_1, q_2, \ldots q_N]$.

If we consider the probability of user i successfully transmitting a packet in an arbitrary slot we can calculate throughput as

$$\bar{T}_i(\boldsymbol{p}, \boldsymbol{q}) = p_i q_i \prod_{j \in \mathcal{N} \setminus i} (1 - p_j q_j) \overset{p_i q_i \neq 1}{=} \frac{p_i q_i}{1 - p_i q_i} \prod_{j \in \mathcal{N}} (1 - p_j q_j) \quad (1)$$

Note that the throughput of user i is an increasing function of p_i and q_i, but decreasing in the number of terminals N contending for the medium. The latter is in compliance with the classic ALOHA, but in practice also holds for CSMA/CA protocols.

The energy cost of user i is a random variable with a mean value of $\bar{E}_i(p_i, q_i) = q_i(c_1 + c_2 p_i)$, where c_1 is the energy consumption while ON and c_2 the additional cost imposed by the transmission. Obviously, in order to transmit, the node has to be ON. Here, we do not consider the energy consumption of the transition itself.

III. THE IMPACT OF CONSTRAINED ENERGY RESOURCES ON THE SYSTEM THROUGHPUT

First, we would like to find the ON–OFF and the medium access probabilities that maximize the collision–free utilization of the medium, and consequently the throughput of the system. This can be formally expressed as the following optimization problem:

$$\begin{array}{lll} \underset{\mathcal{I}=\{\boldsymbol{p},\boldsymbol{q}\}}{\text{maximize}} & \sum_{i=1}^{N} \bar{T}_i(\boldsymbol{p}, \boldsymbol{q}) & \\ \text{s.t.} & \bar{E}_i(p_i, q_i) \leq \tilde{e}_i & \forall i \\ & \{p_i, q_i\} \in [0,1]^2 & \forall i \end{array} \tag{2}$$

The intuitive explanation of the energy constraint is the fact that each battery cycle provides $\tilde{e}_i$ resource and thus, the maximum energy constraint is directly mapped to a minimum recharge time for the mobile. Throughout the paper, and without loss of generality, we assume that the users are ordered in decreasing energy budget, i.e. $\tilde{e}_1 \geq \tilde{e}_2 \geq \ldots \geq \tilde{e}_N$.

A. Throughput optimal scheduling in energy constrained ALOHA with sleep modes

In the classic ALOHA setting, where no energy constraints exist, the throughput optimal strategy would be the one that eliminates contention. Thus, if we could force only a single user, say user k, to access the medium with probability $p_k = 1$ in each frame, we would achieve the maximum total throughput. In our scenario though, due to the energy constraints, the users may not be able to stay continuously ON (i.e $q_k = 1$) or to transmit with $p_k = 1$. Then, what is the best way to exploit the available energy resources? For each user we need to find the portion of energy to spend for staying ON during the frames and the portion used for transmitting while being ON.

Lemma 1: Out of all the throughput optimal strategies the most energy efficient ones are of the form $\mathcal{I}^* = \{\mathbf{1}, \boldsymbol{a}\}$ with $\mathbf{1} = [1, 1, \ldots 1]$ and $\boldsymbol{a} \in [0,1]^N$. Thus, without loss of optimality we may restrict the strategy search space only to strategies where the nodes transmit continuously inside any ON frame.

Proof: Let $\hat{\mathcal{I}}$ be a feasible throughput optimal strategy, where $\{\hat{p}_i, \hat{q}_i\}$ the strategy of user i. From eq. 1 we may see that throughput depends only on the pq products. Let $a_i = \hat{p}_i \hat{q}_i$. The strategy $\mathcal{I}^* = \{\mathbf{1}, \boldsymbol{a}\}$ with $\boldsymbol{a} = [\hat{p}_1 \hat{q}_1, \hat{p}_2 \hat{q}_2, \ldots \hat{p}_i \hat{q}_i]$ achieves the optimal throughput, i.e. $\bar{T}(\mathcal{I}^*) = \bar{T}\left(\hat{\mathcal{I}}\right)$.

Then, we prove that $\mathcal{I}^*$ is also feasible, as the most energy efficient strategy.

Any other feasible throughput optimal strategy $\hat{\mathcal{I}}$ can be written as an expression of $\boldsymbol{a}$ as $\{\hat{p}_i = \frac{a_i}{a_i+\delta_i}, \hat{q}_i = a_i + \delta_i\}$, with $0 < \delta_i \leq \min\{1 - a_i, \frac{\tilde{e}_i - a_i(c_1+c_2)}{c_1}\}$. Thus, regarding the energy efficiency we have:

$$\begin{aligned} \bar{E}\left(\hat{\mathcal{I}}\right) &= \sum_{i=1}^{N} (a_i + \delta_i)\left(c_1 + c_2 \frac{a_i}{a_i + \delta_i}\right) \\ &= \sum_{i=1}^{N} \delta_i c_1 + a_i(c_1 + c_2) = \bar{E}(\mathcal{I}^*) + c_1 \sum_{i=1}^{N} \delta_i \\ &> \bar{E}(\mathcal{I}^*). \end{aligned}$$

This completes our proof. ■

Based on Lemma 1, the optimization problem described by eq. 2 can be simplified to an expression that depends only on $\boldsymbol{q}$ leading to the objective function $\bar{T}(\boldsymbol{q}) = \sum_{i=1}^{N} q_i \prod_{j \in \mathcal{N} \setminus i} (1 - q_j)$ and constraints $0 \leq q_i \leq \tilde{q}_i = \min\left\{\frac{\tilde{e}_i}{c_1+c_2}, 1\right\}$; this can be further simplified into a problem of binary integer programming.

Lemma 2: The optimal solution is of the form $\boldsymbol{q}^* = \boldsymbol{b}^* \text{diag}[\tilde{q}_1, \tilde{q}_2, \ldots \tilde{q}_N]$ where $\boldsymbol{b}^*$ is a binary row vector.

Proof: The partial derivative of the objective function is given by:

$$\begin{aligned} \frac{\partial \bar{T}}{\partial q_k} &= \prod_{j \in \mathcal{N} \setminus k} (1 - q_j) - \sum_{j \in \mathcal{N} \setminus k} q_j \prod_{l \in \mathcal{N} \setminus \{k,j\}} (1 - q_l) \\ &= \prod_{j \in \mathcal{N} \setminus k} (1 - q_j) - \sum_{j \in \mathcal{N} \setminus k} \frac{q_j}{1 - q_j} \prod_{l \in \mathcal{N} \setminus k} (1 - q_l) \\ &= \left(1 - \sum_{j \in \mathcal{N} \setminus k} \frac{q_j}{1 - q_j}\right) \prod_{j \in \mathcal{N} \setminus k} (1 - q_j) \end{aligned}$$

We have thus shown that the sign of the partial derivative with respect to the kth element depends only in the parameter $\sum_{j \in \mathcal{N} \setminus k} \frac{q_j}{1 - q_j}$. This leads to the following decision making:

$$q_k = \begin{cases} \tilde{q}_k, & \text{if} \sum_{j \in \mathcal{N} \setminus k} \frac{q_j}{1 - q_j} < 1, \\ 0, & \text{otherwise.} \end{cases} \tag{3}$$

■

Lemma 3: The optimal solution $\boldsymbol{b}^*$ is of the form $\boldsymbol{b}^* = [1, 1, \ldots 1, 0, 0, \ldots 0]$.

Proof: We will prove this by contradiction. Assume that $\tilde{\boldsymbol{b}} \neq \boldsymbol{b}^*$, a vector of k ones (i.e. any vector where the ones are not placed in the first k places), is the throughput optimal binary vector. We can construct a new vector $\hat{\boldsymbol{b}} = [1, 1, \ldots 1, 0, 0, \ldots 0]$, which has only the first k users activated and gives identical throughput, by deriving the ON-OFF vector $\hat{\boldsymbol{q}}$ from the corresponding indices of $\tilde{\boldsymbol{b}}$. To construct such a vector we need to move the rightmost 1, say from position l of the initial vector to an earlier zero position say m with $m < l$. Based on Lemma 2, we can show that by fully activating or deactivating users (i.e. meeting the constraint with equality or setting access probability to zero), we receive a new schedule

$\boldsymbol{b}^* = \hat{\boldsymbol{b}}$ and the corresponding $\boldsymbol{q}^*$ of increased throughput. This leads us to a contradiction regarding the optimality of $\tilde{\boldsymbol{b}}$. ∎

Based on the aforementioned lemmas we may derive the centralized Algorithm 1 that yields the throughput optimal probabilistic strategy and is of linear, in the number of users N, complexity. The main idea behind this algorithm is that contention may or may not be beneficial, depending on the energy constraints of the users. Namely, an additional user is useful if and only if the energy resources of the already active users are not sufficiently large, leaving thus the medium underutilized. An additional user introduces a gain due to the exploitation of the empty frames, but also a loss, due to the collisions whenever he is concurrently active within a frame with someone else. If the average gain is greater than the induced loss, it is beneficial for the system to be enabled.

Algorithm 1 Optimal probabilistic frame scheduling

1: Order users in decreasing $\tilde{e}_i$. Without loss of generality, we reassign the indices such that $\tilde{q}_1 \geq \tilde{q}_2 \geq \ldots \geq \tilde{q}_N$
2: $\boldsymbol{q} \leftarrow \mathbf{0}$
3: $j \leftarrow 1$
4: **while** $j \leq N$ **and** $\sum_{i=1}^{j} \frac{q_i}{1-q_i} < 1$ **do**
5: $\quad q_j \leftarrow \tilde{q}_j$
6: $\quad j \leftarrow j+1$
7: **end while**

Theorem 1: Algorithm 1 yields the throughput optimal probabilistic strategy.

Proof: The optimality of 1 comes directly from Lemmas 1, 2 and 3. ∎

Note that this algorithm gives also the throughput optimal scheduling for the constrained version of the Aloha problem, where each terminal has an individual constraint on its medium access probability p_i. This is a simplified version of the problem considered in this work, where no sleep mode support exists, i.e. $q_i = 1$.

The algorithm above promotes the k less energy constrained users ($0 \leq k \leq N$ depends on actual constraints) and suppresses the rest. It will only serve as a performance benchmark, since it is a centralized algorithm that introduces important coordination and fairness issues. In particular, it requires extensive coordination among the users and causes extremely unfair treatment of the users of low energy budget. Thus, in the following sections we develop algorithms that capture the autonomous nature of the users and fit to the dynamic distributed environments considered here.

B. A distributed fair algorithm

In order to capture the notion of proportional fairness we substitute the original objective function with the following: $U(\boldsymbol{p}, \boldsymbol{q}) = \sum_{i=1}^{N} w_i \log \bar{T}_i$. The multiplicative factor w_i can be used to balance the throughput among the users of the system at will. For example, the value $w_i = \frac{\tilde{e}_i}{\sum_{k\in\mathcal{N}} \tilde{e}_k}$ would allow us to split the throughput proportionally to the energy budget of the users. By proper reformulation, the objective function can be rewritten as $U(\boldsymbol{p}, \boldsymbol{q}) = \sum_{i=1}^{N} \log\left[(p_i q_i)^{w_i}(1-p_i q_i)^{1-w_i}\right]$. This is a separable per user function that leads to a fully distributed implementation, requiring minimal information exchange. Actually the only information required is the value of the total energy available in the terminals, namely $\sum_{k\in\mathcal{N}} \tilde{e}_k$, information that can be easily acquired. The solution to this optimization problem, in accordance to Lemma 1, is of the form $\mathcal{I}^* = \{\mathbf{1}, \boldsymbol{a}\}$ with $\boldsymbol{a} = [\min\{w_1, \tilde{q}_1\}, \min\{w_2, \tilde{q}_2\}, \ldots, \min\{w_N, \tilde{q}_N\}]$.

C. A modified strategy

Up to now we assumed that each user makes a decision once for his strategy and applies it forever. As a result, user k whenever active, transmits with $p_k = 1$, independently of the number of active users within a frame. Thus, whenever two or more users select to transmit within a frame they receive zero payoff, but consume energy. Based on these observations, a rational player would be expected to backoff whenever a collision is detected. Although, the terminal is not allowed to switch off in a crowded frame, due to the switching time overhead incurred, it may reduce its access probability. This way, it would avoid spending energy on useless transmission that always lead on collisions and could utilize these savings for pursuing further contention–free frames. Building on this idea we propose the following modified strategy.

Any active user attempts a transmission within the first timeslot of the current frame. If the transmission succeeds he selects a medium access probability of $p_i = 1$ and captures the whole frame successfully. Otherwise he adjusts his strategy, and reduces his transmission probability to $\tilde{p}_i$. It can be shown that this strategy always yields better throughput than the original one. The expressions for throughput and energy consumption for this case are respectively given by:

$$\bar{T}_i = q_i \left\{ (1-\tilde{p}_i) \prod_{j\in\mathcal{N}\setminus i} (1-q_j) + \tilde{p}_i \prod_{j\in\mathcal{N}\setminus i} (1-\tilde{p}_j q_j) \right\} \tag{4}$$

$$\bar{E}_i = q_i \left\{ c_1 + c_2 \left[\tilde{p}_i + (1-\tilde{p}_i) \prod_{j\in\mathcal{N}\setminus i} (1-q_j) \right] \right\} \tag{5}$$

The complicated throughput and energy expressions for the modified scenario make the problem of optimal scheduling analytically intractable. Thus, in order to get insight on the performance of the modified strategy, we consider a simplified version of the problem by restricting the feasible user strategies. In particular, we assume that all the users of the system can be classified in one of the following three groups; the aggressive, the conservative and the passive ones. The former capture the medium whenever they are active (ON), the second transmit only whenever they sense an empty frame and the last do not participate at all.

We consider Algorithm 2 that performs a search over the solution subspace. This algorithm is of exponential complexity and will be used only for comparison purposes. It can be shown that in the optimal scheduling for our restricted solution space, at least one aggressive and one conservative user exist. In all the simulation cases that we have tried, this categorization did not to limit the system performance; nevertheless a formal proof that the optimal solution lies on the restricted subspace could not be derived.

Algorithm 2 Modified optimal probabilistic frame scheduling

Search over $\mathcal{B} = \{\mathcal{A}, \mathcal{C}, \mathcal{P}\}$, i.e. the set of all the possible partitions of $\mathcal{N}$ of size 3, with $|\mathcal{A}| \geq 1$ and $|\mathcal{C}| \geq 1$ for the throughput optimal assignment:
for all $i \in \mathcal{A}$ (*% aggressive*) **do**
    $\{\tilde{p}_i, q_i\} = \{1, \min\left\{\frac{\tilde{e}_i}{c_1+c_2}, 1\right\}\}$
end for
for all $k \in \mathcal{C}$ (*% conservative*) **do**
    $\{\tilde{p}_k, q_k\} = \left\{0, \min\left\{\frac{\tilde{e}_i}{c_1+c_2\prod_{j\in\mathcal{N}\setminus k}(1-q_j)}, 1\right\}\right\}$
end for
for all $j \in \mathcal{P}$ (*% passive*) **do**
    $\{\tilde{p}_j, q_j\} = \{0, 0\}$
end for

IV. Game Theoretic Approach

Previously we derived probabilistic medium access protocols that require coordination of actions of the users involved. However, in an autonomous setting as the one considered here, individuals may not comply with the rules imposed by the protocol. Actually, users may exhibit selfish behaviour and select the strategy that maximizes their own utility, namely their individual throughput, at the expense of others. Thus, in this section we model the user interaction/contention as a non-cooperative game. We derive models for both the initial and the modified scenario.

A non-cooperative game is defined by a set of players, a set of strategies and a metric that indicates the preferences of the players over the set of strategies. In our case we have:

- **Players**: the N users
- **Strategies**: user's i set of feasible medium access and ON–OFF probabilities $\mathcal{I}_i = \{p_i, q_i : \bar{E}_i \leq \tilde{e}_i \text{ and } 0 \leq p_i, q_i \leq 1\}$
- **User preferences**: represented by a utility function $U_i(\mathcal{I}_i)$; peer i prefers strategy $\hat{\mathcal{I}}_i$ to $\mathcal{I}_i$ iff $U_i(\hat{\mathcal{I}}_i) > U_i(\mathcal{I}_i)$.

A. Our initial scenario as a non-cooperative game of perfect information

For the initial optimization problem (2) the utility function of user i is defined as $U_i(\mathcal{I}_i) = \bar{T}_i = p_i q_i \prod_{j\in\mathcal{N}\setminus i}(1-p_j q_j)$. It can be easily verified that the throughput maximizing strategy of each individual is independent of the actions of the other users and comes as the solution of the following optimization problem:

$$\begin{aligned} \underset{\mathcal{I}_i=\{p_i,q_i\}}{\text{maximize}} \quad & p_i q_i \\ \text{s.t.} \quad & q_i(c_1 + c_2 p_i) \leq \tilde{e}_i \\ & \{p_i, q_i\} \in [0,1]^2 \end{aligned} \tag{6}$$

Lemma 4: The throughput optimal strategy for user i is $\{p_i, q_i\} = \left\{1, \min\{\frac{\tilde{e}_i}{c_1+c_2}, 1\}\right\}$. The resulting game has a unique Nash Equilibrium Point, described by the strategy $\mathcal{I}^* = \{\mathbf{1}, \boldsymbol{q}^*\}$, with $q_i^* = \frac{\tilde{e}_i}{c_1+c_2}$.

Proof: Since the utility is an increasing function of both p_i and q_i the energy constraint should be satisfied with equality at the optimum.

The proof is similar in spirit to the one in Lemma 1. Let $\{\hat{p}_i, \hat{q}_i\}$ be the throughput optimal strategy of user i, with $\hat{p}_i < 1$. From the energy constraint we have $\hat{q}_i(c_1 + c_2\hat{p}_i) = \tilde{e}_i$. There exists the alternative strategy $\{\acute{p}_i, \acute{q}_i\} = \{1, \hat{p}_i\hat{q}_i\}$ of equivalent utility (individual throughput) but of lower energy consumption. In particular,

$$\acute{q}_i(c_1 + c_2\acute{p}_i) = \acute{q}_i(c_1 + c_2) = \acute{q}_i(c_1\acute{p}_i + c_2\acute{p}_i) < \tilde{e}_i \tag{7}$$

Consequently, there exists $\delta > 0$ such that $(\acute{q}_i + \delta)(c_1 + c_2) = \tilde{e}_i$. This strategy $\{p_i^*, q_i^*\} = \{1, \acute{q}_i + \delta\}$ obviously yields increased utility for the user, which contradicts the user optimality of $\{\hat{p}_i, \hat{q}_i\}$ and completes our proof. The same result can be derived through the KKT conditions.

Given that the optimal strategy of a user is independent of the actions of the other users and is determined only by his own energy constraint, deriving the resulting NEP is straightforward. ■

From the above, we may deduce that at the Nash equilibrium point we receive throughput, only when a single user is ON in a frame. Given the NEP of the game we may quantify the performance loss arising due to the selfishness of the individuals, by using so called *Price of Anarchy* (PoA) metric. This is the ratio of the value of the objective function at the global optimum to its value at the NEP and in our setting is given by:

$$\text{PoA} = \frac{\displaystyle\sum_{i\in\mathcal{S}} \frac{\tilde{e}_i}{c_1+c_2} \prod_{j\in\mathcal{S}\setminus i}(1 - \frac{\tilde{e}_j}{c_1+c_2})}{\displaystyle\sum_{i\in\mathcal{N}} \frac{\tilde{e}_i}{c_1+c_2} \prod_{j\in\mathcal{N}\setminus i}(1 - \frac{\tilde{e}_j}{c_1+c_2})} \geq 1, \tag{8}$$

where $\mathcal{S}$ is the set of enabled users at the global optimum.

Whereas in the classic Aloha games the PoA is unbounded, in our energy constrained Aloha setting the PoA is bounded, since the energy constraints impose a fictitious pricing scheme. The PoA grows unbounded only when at least two users have unconstrained energy resources. Then, these users (e.g. power plugged stations) would involuntarily act as jammers for each other and for all the others yielding hence zero system throughput.

B. The modified strategy as a non-cooperative game of perfect information

Here, we consider the game arising from the modified strategy. In this setting, we derive a non–cooperative game of perfect information, where at each iteration, given the parameters $\alpha = \prod_{j \in \mathcal{N} \setminus i} (1 - q_j)$ and $\beta = \prod_{j \in \mathcal{N} \setminus i} (1 - \tilde{p}_j q_j)$ a user selects its best response. By *best response* we mean that each user updates his decision variables, so as to maximize its utility function, in response to the others' actions.

Theorem 2: The best response strategy of user i is given by:

$$\tilde{p}_i = \begin{cases} 1, & \text{if } [c_1 + c_2\alpha]\,\beta > (c_1 + c_2)\alpha, \\ 0, & \text{otherwise.} \end{cases} \tag{9}$$

$$q_i = \frac{\tilde{e}_i}{c_1 + c_2\left[\tilde{p}_i + (1 - \tilde{p}_i)\alpha\right]} \tag{10}$$

The arising modified game has multiple NEPs.

Proof: Since the utility is an increasing function of $\tilde{p}_i$ and q_i, the constraint needs to be satisfied with equality. Thus, we may replace q_i from eq. 5 into the throughput expression, namely eq. 4. Then, the partial derivative of the objective function is given by the following expression:

$$\begin{aligned} \frac{\partial \bar{T}}{\partial \tilde{p}_i} &= \tilde{e}_i \frac{[\beta - \alpha]\left[c_1 + c_2\alpha\right] - c_2\alpha[1 - \alpha]}{\left\{c_1 + c_2\left[\tilde{p}_i + (1 - \tilde{p}_i)\alpha\right]\right\}^2} \\ &= \tilde{e}_i \frac{\left[c_1 + c_2\alpha\right]\beta - (c_1 + c_2)\alpha}{\left\{c_1 + c_2\left[\tilde{p}_i + (1 - \tilde{p}_i)\alpha\right]\right\}^2} \end{aligned}$$

The sign of this expression depends only on $\alpha = \prod_{j \in \mathcal{N} \setminus i} (1 - q_j)$ and $\beta = \prod_{j \in \mathcal{N} \setminus i} (1 - \tilde{p}_j q_j)$. As a result given the actions of the other users, the objective function is either a strictly increasing or a strictly decreasing function of $\tilde{p}_i$. Thus, the best response strategy of user i is given by eq. 10. ■

V. Numerical Results

In order to quantify the throughput performance of the proposed schemes we perform extensive simulations of a small, easy to follow system. We consider a scenario of $N = 5$ terminals with energy constraints given by $\tilde{e} = [30, 25, 15, 10, 5]$ and $\{c_1, c_2\} = \{50, 70\}$ units. By abusing slightly the definition of PoA, we use the modified optimal as the performance benchmark and for each scheme we derive the performance ratio metric defined as:

$$\text{Performance\_ratio}_X = \frac{\text{Throughput of modified optimal}}{\text{Throughput of scheme X}} \tag{11}$$

Thus, all the figures depict the performance degradation in comparison to the modified optimal. Since the modified game has several NEPs we depict the PoA i.e. the ratio of the throughput at the optimum to the throughput at the worst NEP, the price of stability (PoS) i.e. the ratio of the throughput at the optimum to the throughput at the best NEP, and the mean performance at the NEPs. Regarding the initial setting, we depict the performance degradation of the original optimal, the fair approach and the initial game theoretic scheme.

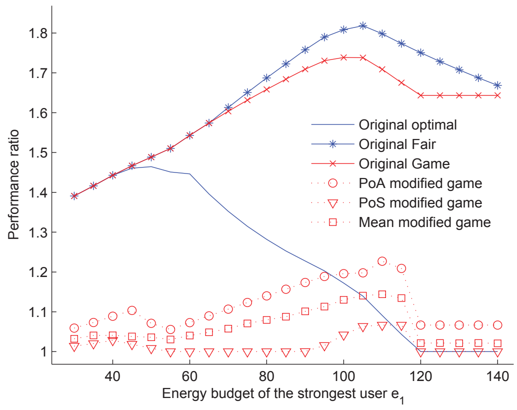

Fig. 2. The throughput performance of the system as an expression of the less energy constrained user

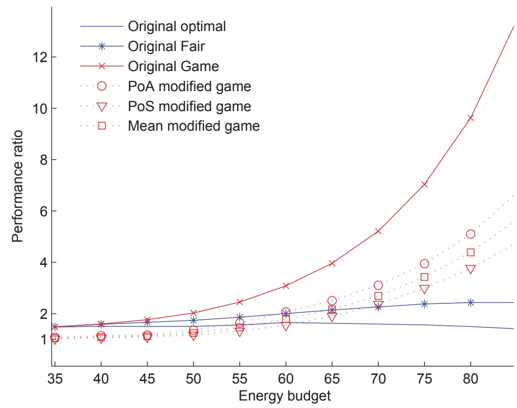

Fig. 3. The throughput performance of the system as the total energy budget of the system increases

Initially, we consider how the energy budget of the less energy constrained user affects the performance of the system as a whole. In Figure 2 we see that the additional power budget increases the performance degradation due to the additional collisions caused. The system stabilizes for $\tilde{e}_1 = c_1 + c_2$, where user 1 has sufficient energy budget to capture medium entirely on his own. We notice also that the modified strategy, of backing off whenever a collision is detected, provides significant performance benefits. The improvement is becoming more significant as the available energy budget increases.

In Figure 3 we depict the system performance as we relax the energy constraints of every terminal. We start from the energy budget vector of $\tilde{e} = [30, 25, 15, 10, 5]$ and increase each dimension by 5 within each iteration. The x axis refers to the $\tilde{e}_1$ value, whereas the whole vector is given by $\tilde{e} = [\tilde{e}_1, \tilde{e}_1 - 5, \tilde{e}_1 - 15, \tilde{e}_1 - 20, \tilde{e}_1 - 25]$. In this scenario, one

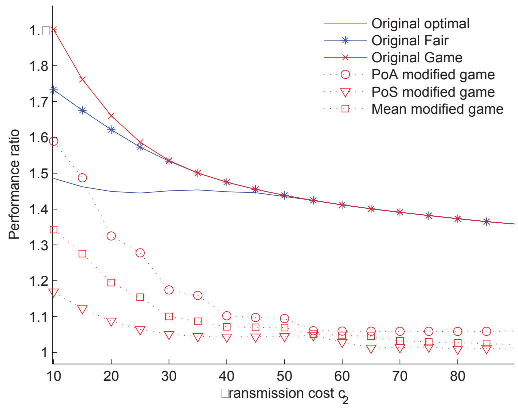

Fig. 4. The throughput performance of the system as an expression of the transmission cost

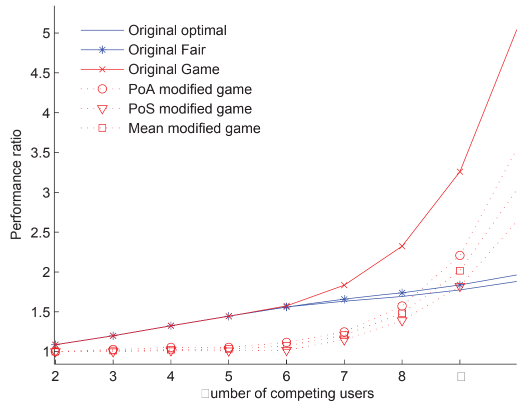

Fig. 5. The throughput performance of the system an expression of the number N of competing terminals

may notice the benefits that arise from the coordination among the terminals. The coordinated approaches are robust again the increased availability of energy, whereas in the game theoretic approaches this leads to an increased number of collisions, due to the increased aggressiveness of the users.

Next, we consider the impact of the transmission cost c_2. We notice in Figure 4 that in a scenario of low energy constraints the increased transmission cost makes the users less aggressive, leading thus to reduced collisions and consequently the performance gap among the coordinated and the game theoretic approaches diminishes. Nevertheless, the performance gap of the initial and the modified strategy remains more than 40% throughout.

Finally, we investigate the impact of the number of competing users on the performance at the resulting equilibria (Figure 5). Starting from a system of only two competing users with energy constraints $\tilde{e} = [5, 10]$ we keep adding in each iteration a single terminal of increasing energy budget. Thus, newcomer i is characterized by energy constraint $\tilde{e}_i = 5i$. As expected the introduction of additional users increases competition for the medium and leads to more collisions. Consequently, the performance of the game theoretic approaches deteriorates very fast, whereas the coordinated approaches exhibit significant robustness.

VI. Conclusion

This work is a first step towards characterizing the energy–throughput tradeoff for mobile devices that support sleep modes and operate according to contention medium access schemes. We characterized the throughput optimal strategy under energy constraints and we developed game theoretic models that capture the autonomous nature of the terminals. We showed that energy constraints may reduce contention and lead to better exploitation of the medium. Finally, we developed a simple modified medium access scheme, where the contention state of the medium can be sampled within a frame by the competing stations. This scheme can lead to more efficient decision making even without any cooperation.

In this work, we have assumed non–cooperative games of perfect information. The scenario where each involved entity has only a subjective belief on its opponents' strategies is an interesting topic of future study. Besides, mechanism design could be considered as a means of driving the system to more efficient equilibrium points. This could be achieved either in the form of penalizing the users that exhibit extremely aggressive behaviour or by incentivizing the individuals to adopt socially optimal strategies.

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