Numerical methods for a reliable prediction of long water-wave phenomena : uncertainty quantification for tsunami runup

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Abstract: Aim of this study is to present robust numerical methods for shallow water equations permitting to correctly predict long water-wave phenomena. A semi-intrusive and polynomial-chaos based method are coupled with a residual based distribution scheme by considering several sources of uncertainties in the simulation of a long wave runup on a conical island. Stochastic results are assessed by comparing with Monte Carlo results. Numerical solutions are compared with experimental data by displaying a great sensitivity from physical and modelling uncertainties.

Key-words: numerical analysis, uncertainty quantification, shallow water flows, tsunami runup, friction

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Numerical methods for a reliable prediction of long water-wave phenomena

Résumé : Aim of this study is to present robust numerical methods for shallow water equations permitting to correctly predict long water-wave phenomena. A semi-intrusive and polynomial-chaos based method are coupled with a residual based distribution scheme by considering several sources of uncertainties in the simulation of a long wave runup on a conical island. Stochastic results are assessed by comparing with Monte Carlo results. Numerical solutions are compared with experimental data by displaying a great sensitivity from physical and modelling uncertainties.

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Nomenclature

J	Jacobian Matrix
f	Residual value vector
x	Variable value vector
F	Force, N
m	Mass, kg
Δx	Variable displacement vector
α	Acceleration, m/s^2

Subscript

i	Variable number

1 Introduction

In previous works, we presented an approach to discretize the shallow water equations on unstructured grids based on a stabilized nonlinear variant of a multidimensional Lax-Friedrichs [1, 2, 3]. The schemes proposed are conservative, well balanced, and second order accurate, capable in handling discontinuous flows and dry geometries without spurious oscillations. The preservation of the positivity of the water height is guaranteed under a time step constraint, without the need of a cutoff on the water height itself. With respect to other Finite Volume Godunov discretizations for Shallow Water simulations (e.g. [4, 5, 6] and references therein), this residual approach can be easily generalized to very high order of accuracy on unstructured grids, without losing any of its basic properties such as compactness, non-oscillatory behavior, and positivity of the water height.

Prediction of shallow water equations in realistic application depends on the level of complexity used for the the physical modelling (such as for example, for the fricton coefficient) and on a set of empirical coefficients that are usually chosen in order to fit the experimental data. Then, input environmental conditions, topography and modelling involve a certain degree of uncertainty. The capability to take into account these uncertainties in the numerical simulation is of great importance in order to correctly predict extreme flood events. Stochastic modeling of long-wave propagation demands a robust shallow-water model in order to characterize the physical processes.

Several stochastic methods have been proposed in literature, from Monte Carlo and sampling-based methods to perturbation methods and generalized polynomial chaos methods (for a more detailed review see for example [7]). Recently, Abgrall *et al.* [8] proposed a semi-intrusive scheme for taking into account the uncertainties, that displays a great flexibility and the capability to handle stochastic unsteady, shock-dominated flows. Concerning the shallow water equations, uncertainties have been taken into account in the work presented by Ge *et al.* in Refs [9, 10]. They proposed a spectral sampling scheme based on Galerkin projection, where a Godunov-type scheme mimics breaking waves as bores for accurate description of the energy dissipation in the runup process. This spectral sampling method generates an output statistical distribution using a much smaller sample of events comparing to the Monte Carlo method.

Objective of the present study is twofold. First, the coupling between the residual distribution scheme with some stochastic methods is presented. Robustness of the stochastic solution is analyzed in terms of numerical accuracy and comparison with experimental data. Secondly, the semi-intrusive scheme [8] is implemented in the residual distribution framework, showing its performances with respect to polynomial chaos and Monte Carlo solutions.

The outiline of this paper is as follows : in section 2 the shallow water system of equations and the numerical discretization based on residual approach is presented. In section 3, stochastic methods, *i.e.* semi-intrusive and polynomial chaos approach, are presented. In section 4, stochastic results obtained on the study of long wave propagation on a conical island are presented. Finally, in section 5 conclusions are drawn.

2 The shallow water system

2.1 Conservation law form

The Shallow Water Equations (SWE) model the behavior of shallow free surface flows under the action of gravity. In conservation law form they can be written as :

$$\frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot \mathsf{F}(\mathbf{u}) - \boldsymbol{S}(\mathbf{u}, x, y) = 0 \quad \text{on} \quad \Omega_T = \Omega \times [0, t_f] \subset \mathbb{R}^2 \times \mathbb{R}^+, \tag{1}$$

with conserved variables, flux, and source term given by

$$\mathbf{u} = \begin{bmatrix} H \\ H\vec{u} \end{bmatrix} \quad \mathsf{F} = \begin{bmatrix} H\vec{u} \\ H\vec{u} \otimes \vec{u} + g\nabla \dfrac{H^2}{2} \end{bmatrix} \quad \boldsymbol{S} = -gH \begin{bmatrix} 0 \\ \nabla B(x,y) + \mathrm{c}_f \vec{u} \end{bmatrix}, \tag{2}$$

where H denotes the relative water height, $\vec{u} = (u, v)$ the flow speed, g the (constant) gravity acceleration, and $B(x, y)$ the local bottom height. We also introduce the free surface level, or total water height η,

$$\eta(x, y, t) = H(x, y, t) + B(x, y). \tag{3}$$

The source term models the effects on the flow of variations of the bed slope, and the viscous friction on the bottom. In particular, c_f is the friction coefficient defined by the Manning formula :

$$\mathrm{c}_f = \frac{n^2 \|\vec{u}\|}{H^{4/3}} \tag{4}$$

with n the Manning coefficient. The SWE are endowed with an entropy pair associated to the total energy

$$E(\mathbf{u}) = H \left(\frac{1}{2} gH + gB + \frac{\vec{u} \cdot \vec{u}}{2} \right). \tag{5}$$

In particular, the total energy verifies the conservation equation

$$\frac{\partial E}{\partial t} + \nabla \cdot (\vec{u}E) + \nabla \cdot \left(\vec{u} \frac{gH^2}{2} \right) = -gH\mathrm{c}_f \|u\|^2 \leq 0, \tag{6}$$

Last equation shows the dissipative effects of the friction term, and allows the classical characterization of the hyperbolic model in terms of mathematical entropy [11, 12].

2.2 Discretization by means of Residual Distribution schemes

Discrete solutions of the SWE have been obtained by means of the Residual Distribution approach discussed in detail in Ref. [1]. Let τ_h be an unstructured tessellation of the computational domain composed by non-overlapping elements K, and $\mathbf{u}_0 = \mathbf{u}(t = 0, x, y)$ an initial state of the physical unknowns. For any $n \geq 0$, we obtain the values $\{\mathbf{u}_i^{n+1}\}_{i\in\tau_h}$ at the nodes of the mesh and at a next time level as follows :

- $\forall\, K \in \tau_h$ compute the unsteady element residual Φ^{K}

  $$\begin{aligned}\Phi^{\mathrm{K}} &= \int_K \left\{ \mathbf{u}_h^{n+1} - \mathbf{u}_h^n + \frac{\Delta t}{2}\left(\nabla\cdot\mathsf{F}(\mathbf{u}_h^{n+1/2}) - \boldsymbol{S}_h(\mathbf{u}_h^{n+1/2}, x, y)\right)\right\} \\ &= \int_K \left\{ \mathbf{u}_h^{n+1} - \mathbf{u}_h^n - \frac{\Delta t}{2}\boldsymbol{S}_h(\mathbf{u}_h^{n+1/2}, x, y)\right\} + \frac{\Delta t}{2}\oint_{\partial K}\mathsf{F}(\mathbf{u}_h^{n+1/2})\cdot\hat{n}\,dl\end{aligned}$$

  $\mathbf{u}_h$ denoting the continuous linear interpolation of the nodal values of $\mathbf{u}$ and $\mathbf{u}^{n+1/2} = (\mathbf{u}^n + \mathbf{u}^{n+1})/2$;

- $\forall\, K \in \tau_h$ compute weighted local nodal residuals by *distributing* fractions of Φ^{K} to the nodes of K :

  $$\Phi_i^{\mathrm{K}} = \beta_i^{\mathrm{K}}\Phi^{\mathrm{K}}\,, \quad \forall\, i \in K$$

  with β_i^{K} a *distribution matrix*

- Obtain the nodal values $\{\mathbf{u}_i^{n+1}\}_{i\in\tau_h}$ by solving the nonlinear algebraic system of equations

  $$\sum_{\mathrm{K}\in\tau_h|i\in\mathrm{K}} \beta_i^{\mathrm{K}}\Phi^{\mathrm{K}} = 0\,, \quad \forall\, i \in \tau_h$$

The key step of the procedure is the definition of the distribution matrices. An integral truncation error analysis can be used to show that second order of accuracy requires these matrices to be bounded [2]. The key point here is however not only to be able to obtain high order of accuracy but also to guarantee the non-oscillatory character of the solution and to preserve the positivity of the values of H. This is achieved by means of the following three step procedure [1] :

1. $\forall i \in$ K compute low order local residuals by means of the Lax-Friedrich's distribution

   $$\Phi_i^{\mathrm{LO}} = \frac{|K|}{3}(\mathbf{u}_i^{n+1} - \mathbf{u}_i^n) + \frac{1}{3}\frac{\Delta t}{2}\left(\oint_{\partial K}\mathsf{F}(\mathbf{u}_h^{n+1/2})\cdot\hat{n}\,dl - \int_K \boldsymbol{S}_h(\mathbf{u}_h^{n+1/2}, x, y) + \alpha_{\mathrm{LF}}\sum_{j\in K}\left(\mathbf{u}_i^{n+1/2} - \mathbf{u}_j^{i+1/2}\right)\right)$$

   A positivity analysis can be performed to deduce a lower bound for α_{LF} guaranteeing the preservation of the positivity of the depth H in every node [1] ;

2. $\forall i \in \mathrm{K}$ limit the Φ_i^{LO} to get high order local residuals. In particular, let $\mathbf{P}$ be a matrix defining the transformation $\mathbf{u} \to \mathbf{p}$, that is $\mathbf{P} = \partial \mathbf{u}/\partial \mathbf{p}$, with $\mathbf{p}$ an arbitrary set of variables (characteristic variables, entropy variables, physical variables etc. etc., *cf.* [1]). The Φ_i^{LO} do not verify the accuracy condition since in general

$$\beta_i^{\mathrm{LO}} = \frac{\mathbf{P}\Phi_i^{\mathrm{LO}}}{\mathbf{P}\Phi^{\mathrm{K}}} \quad \text{is unbounded}$$

where the fraction denotes *componentwise division*. A high order scheme is obtained by setting

$$\beta_i^* = \frac{\psi(\beta_i^{\mathrm{LO}})}{\sum\limits_{j\in\mathrm{K}} \psi(\beta_j^{\mathrm{LO}})}$$

with $\psi(\cdot)$ a limiter function, and where the re-normalization by the sum of the limited betas is meant to guarantee that the consistency relation $\sum_j \beta_j^{\mathrm{K}} = 1$ is verified. Provided that $\psi(r), \psi(r)/r \geq 0$, and that the limiting is performed on the conserved quantities (*viz* $\mathbf{P}$ is the identity matrix), then one can prove that the positivity of the depth H in every node is preserved [1] ;

3. $\forall i \in \mathrm{K}$ add a linear upwind-biasing stabilization term. This term is basically meant to introduce an upwind bias, it is inspired by the consistency with the energy balance (6), and it bears a very close resemblance to the so-called streamline dissipation term [11]. In practice it leads to the final form of the distribution matrix

$$\beta_i^{\mathrm{K}} = \beta_i^* + \tau_{\mathrm{S}}\, \partial_u \mathsf{F} \cdot \vec{n}_i$$

with $\partial_u \mathsf{F}$ the Jacobian of the flux, $\vec{n}_i$ a nodal normal, and τ_{S} a *matrix time scale*. We refer the interested reader to [11, 13, 1] for further details.

A more detailed description of the schemes and of their implementation can be found in [1]. Results on a wide variety of flows involving dry areas are discussed in [1, 3]

3 Stochastic methods

Consider the following problem for an output of interest $u(\boldsymbol{x}, t, \boldsymbol{\xi}(\boldsymbol{\omega}))$[1]:

$$\mathcal{L}(\boldsymbol{x}, t, \boldsymbol{\xi}(\boldsymbol{\omega}); u(\boldsymbol{x}, t, \boldsymbol{\xi}(\boldsymbol{\omega}))) = \mathcal{S}(\boldsymbol{x}, t, \boldsymbol{\xi}(\boldsymbol{\omega})), \tag{7}$$

where the operator $\mathcal{L}$ can be either an algebraic or a differential operator (in this case we need appropriate initial and boundary conditions). The operator $\mathcal{L}$ and the source term $\mathcal{S}$ are defined on the domain $D \times T \times \Xi$, where $\boldsymbol{x} \in D \subset \mathbb{R}^{n_d}$, with $n_d \in \{1, 2, 3\}$, and $t \in T$ are the spatial and temporal dimensions. Randomness is introduced in (7) and its initial and boundary conditions in term of d second order random parameters $\boldsymbol{\xi}(\boldsymbol{\omega}) = \{\xi_1(\omega_1), \ldots, \xi_d(\omega_d)\} \in \Xi$ with parameter space $\Xi \subset \mathbb{R}^d$. The symbol $\boldsymbol{\omega} = \{\omega_1, \ldots, \omega_d\} \in \Omega \subset \mathbb{R}$ denotes realizations

[1]In the following the exposition is made for a scalar output variable (u) for brevity, but the extension to the multidimensional output case is straightforward

in a complete probability space $(\Omega, \mathcal{F}, P)$. Here Ω is the set of outcomes, $\mathcal{F} \subset 2^\Omega$ is the σ-algebra of events and $P : \mathcal{F} \to [0, 1]$ is a probability measure. In our case the random variables $\boldsymbol{\omega}$ are by definition standard uniformly $\mathcal{U}(0, 1)$ distributed. Random parameters $\boldsymbol{\xi}(\boldsymbol{\omega})$ can have any arbitrary probability density function $p(\boldsymbol{\xi}(\boldsymbol{\omega}))$, in this way $p(\boldsymbol{\xi}(\boldsymbol{\omega})) > 0$ for all $\boldsymbol{\xi}(\boldsymbol{\omega}) \in \Xi$ and $p(\boldsymbol{\xi}(\boldsymbol{\omega})) = 0$ for all $\boldsymbol{\xi}(\boldsymbol{\omega}) \notin \Xi$; we can now drop the argument $\boldsymbol{\omega}$ for brevity. The probability density function $p(\boldsymbol{\xi}(\boldsymbol{\omega}))$ is defined as a joint probability density function from the independent probability function of each variable: $p(\boldsymbol{\xi}(\boldsymbol{\omega})) = \prod_{i=1}^{d} p_i(\xi_i)$. This assumption allows an independent polynomial representation for every direction in the probabilistic space with the possibility to recover the multidimensional representation by tensorization. The aim is to find the statistical moments of the solution $u(\boldsymbol{\xi})$.

3.1 Semi-intrusive scheme

Let us introduce SI method for the $1D$ Euler equations

$$\begin{cases} \dfrac{\partial \mathbf{u}}{\partial t} + \dfrac{\partial \mathbf{f}(\mathbf{u})}{\partial x} = 0 & t > 0, x \in [0, L] \\ \\ \text{initial and boundary conditions} & , \end{cases} \tag{8}$$

where $\mathbf{u} = (\rho, \rho u, E)^T$ with ρ the density, u the velocity, $e^{\text{tot}} = \rho e + \frac{\rho}{2} u^2$ the total energy with e the specific internal energy. The physical flux reads $\mathbf{f}(\mathbf{u}) = (\rho u, \rho u^2 + p, hu)^T$ with p the pressure and $h = e^{\text{tot}} + p$ the total enthalpy. System (8) is closed by the equation of state $p = p(\rho, e)$; for the perfect gas flows considered in this study, $p = (\gamma - 1)\rho e$ with γ the ratio of specific heats. Let us suppose the initial condition is a random variable.

We consider a spatial discretisation for (8) with node points $x_i = i\Delta x$ where i belongs to some subset of $\mathbb{Z}$, a time step $\Delta t > 0$ and set $t_n = n\Delta t$, $n \in \mathbb{N}$. The control volumes are as usual the intervals $\mathcal{C}_i = [x_{i-1/2}, x_{i+1/2}]$ with $x_{i+1/2} = \frac{x_i + x_{i+1}}{2}$. We start from a finite volume scheme, and for the simplicity of exposure, we only consider a first order in time and space scheme. The generalisation to more accurate scheme is obvious. Thus we define the *deterministic scheme* as

$$\mathbf{u}_i^{n+1} = \mathbf{u}_i^n - \frac{\Delta t}{\Delta x} \Big(\mathbf{F}(\mathbf{u}_{i+1}^n, \mathbf{u}_i^n) - \mathbf{F}(\mathbf{u}_i^n, \mathbf{u}_{i-1}^n) \Big) \tag{9}$$

with $\mathbf{u}_i^0$ being an approximation of $\int_{\mathcal{C}_i} \mathbf{u}_0(x) \mathrm{d}x / \Delta x$ and $\mathbf{F}$ a consistent approximation of the continuous flux $\mathbf{f}$. In all what follows, $\mathbf{F}$ is the Roe flux.
When $\mathbf{u}_0$ or the flux $\mathbf{f}$ depends on a random variable ξ, we propose the following modifications. First the set $\Xi \subset \mathbb{R}^d$ is subdivided into non overlapping subset Ξ_j, $j = 1, \ldots, n_p$ and the variables are represented by their conditional expectancies in the Ξ_j subsets. More precisely, our set of variables is

$$\mathbf{u}_{i,j}^n \approx \frac{\mathcal{E}(\int_{x_{i-1/2}}^{x_{i+1/2}} \mathbf{u}(x, t_n, \xi(\omega)) \mathrm{d}x \,|\, \omega \in \Xi_j)}{\Delta x P(\Xi_j)}.$$

where expectancy, of a generic function $g = g(\xi)$ is equal to

$$\mathcal{E}(g(\xi)) = \int_\Xi g(\xi) \mathrm{d}\xi$$

where dξ may or may not have a density. The scheme evolves as

$$\mathbf{u}_{i,j}^{n+1} = \mathbf{u}_{i,j}^{n} - \frac{\Delta x}{\Delta t}\left(\mathcal{E}(\mathbf{F}(\mathbf{u}_{i+1}^{n}, \mathbf{u}_{i}^{n})|\Xi_j) - \mathcal{E}(\mathbf{F}(\mathbf{u}_{i}^{n}, \mathbf{u}_{i-1}^{n})|\Xi_j)\right).$$

The scheme is fully defined provided the "flux" $\mathcal{E}(\mathbf{F}(\mathbf{u}_{l+1}^{n}, \mathbf{u}_{l}^{n})|\Xi_k)$ can be evaluated for any l and k.

3.2 Non-intrusive Polynomial Chaos technique

In this section we briefly sketch the non-intrusive PC technique first introduced by Wiener [14] . In this work we used the framework of the so-called generalized Polynomial Chaos (gPC) [?] in which the correct set of polynomials is chosen as an optimal basis for different (continuous) probability distribution types. First a sampling method is chosen to generate a discrete parameter space $\boldsymbol{\xi}_i \in \bar{\Xi} \subset \Xi$ with $i = 1, \ldots, N$ in which the model equation (7) is evaluated by a deterministic code determining a set of solution $u_i = u(\boldsymbol{\xi}_i)$. Finally it is necessary to reconstruct the variable $u(\boldsymbol{\xi})$ as a polynomial expansion in which the coefficients are computed evaluating d-dimensional integrals with an opportune quadrature techniques in which the u_i values are needed.

We can employ the orthogonal basis reported in the Askey scheme [?] to approximate the functional form between each random inputs and the stochastic response. The chaos (truncated) expansion reads

$$u(\boldsymbol{\xi}) = \tilde{u}(\boldsymbol{\xi}) + \mathcal{O}_T = \sum_{k=0}^{P} \beta_k \Psi_k(\boldsymbol{\xi}) + \mathcal{O}_T, \qquad (10)$$

where Ψ_k are the polynomials of total order n_o which form an Hilbert basis of $L_2(\boldsymbol{\xi}, p(\boldsymbol{\xi}))$ and the number of terms in the expansion (10) is

$$N_{tot} = \frac{(n_o + d)!}{n_o!\, d!} = P + 1. \qquad (11)$$

Recalling the definition of the inner product, the determination of the PC coefficients of the output expansion reduces to the evaluation of N_{tot} d-dimensional integrals

$$\beta_k = \frac{\int_{\Xi} u(\boldsymbol{\xi})\Psi_k(\boldsymbol{\xi}) p(\boldsymbol{\xi}) \mathrm{d}\boldsymbol{\xi}}{\langle \Psi_k \Psi_k \rangle}, \qquad (12)$$

The stochastic solution $u(\boldsymbol{\xi})$ is now reconstructed as $\tilde{u}(\boldsymbol{\xi})$ from which we can compute the expected value $\mathcal{E}(u)$ and the variance $\sigma^2(u)$:

$$\mathcal{E}(\tilde{u}) = \beta_0 \qquad (13)$$

$$\sigma^2(\tilde{u}) = \sum_{k=1}^{P} \beta_k^2 \langle \Psi_k^2(\boldsymbol{\xi}) \rangle. \qquad (14)$$

4 Results

4.1 Configuration case and sources of uncertainty

The model have been applied to the study of long wave propagation and runup on a conical island. The computed results have been compared with experimental data in order to study the interaction between the long waves and conical island in terms of water profile and wave runup height [15]. In the experiment, a conical island is set in a wave basin having the dimension of 30 m wide and 25 m long. The island has the shape of a truncated cone with diameters of 7.2 m at the base and 2.2 m at the crest. It is 0.625 m high with a side slope of 1:4. There is an absorbing materials placed at the four sidewalls in order to reduce wave reflection. The water depth is of $h_0 = 0.32$ m. Solitary waves with different heights are generated and the water level measured in different positions. A sketch of this experimental configuration, with the position of the gauges whose signal is available is depicted on figure 1. The wave gauge G1 is setup for the measurement of the incident waves; wave gauges G6 and G9 are for the waves in the shoaling area; and the wave gauges G16 and G22 are respectively, for waves on the right side and lee side of the island.

For the simulations, the solitary waves have been approximated by long wave solutions that have the following analytical expression :

$$d\eta = d\eta_0(x - ct) \tag{15}$$

with c the celerity $c = \sqrt{gH} = \sqrt{g(h_0 + d\eta)}$ and with

$$d\eta_0(x) = A\ \mathrm{sech}^2 \left\{ \sqrt{\frac{3A}{4h_0^3}}(x - x_0) \right\} \tag{16}$$

with x_0 the center of the wave. The horizontal speed associated to this perturbation is

$$u(x) = \sqrt{\frac{g}{h_0}} d\eta_0(x - ct) \tag{17}$$

The results presented here refer to the case of amplitude $A = 0.2\, h_0$.

The system of equations constituted by Eqs. 1, 2, 4, 6 depend on Manning coefficient, *i.e.* n, on water depth h_0 and on the amplitude of the incoming wave A.

Uniform probability density function (pdf) are retained for each parameter, where a variation of 10% for h_0 and A are considered, while n vary between 0 and 0.1 .

4.2 Mesh convergence study

We have performed a mesh convergence study for two different values of the Manning coefficient n : $n = 0.01$ and $n = 0.05$. The results are displayed on figures 2 and 3 for gages 9, 16, and 22.

For the lower value of the friction coefficient, the model predicts quite well the shoaling, and its reflection after runup on the front side of the island. The runup on both the right and lee side are captured quite well. However, the model fails to completely capture the rundown on right and lee side, and the extra waves observed by gauge 9. This might be due to the analytical form of the friction model but also to a failure of the hydrostatic approximation underlying the SWE [16]. Further investigation is needed to asses this point.

For a higher value of the friction coefficient, while still capturing well the wave shoaling and relatively well the front side reflected wave, the model largely underestimates the runup on both the right and lee sides.

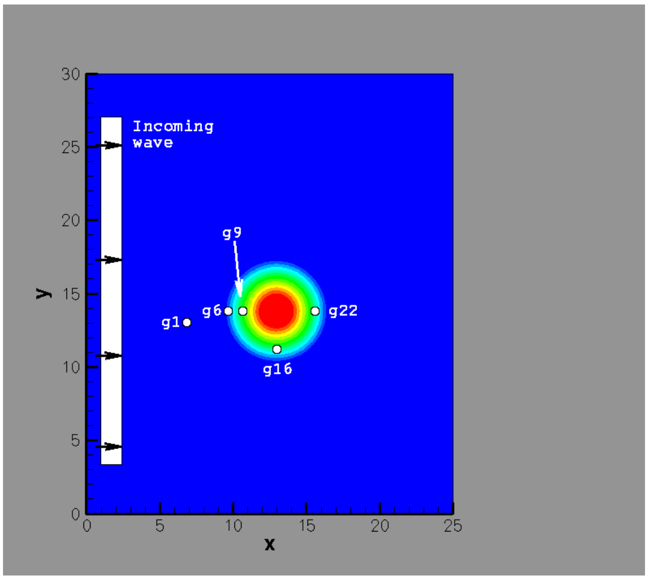


Figure 1: Sketch of the conical island in the wave basin.

4.3 Stochastic results

In this section, we present preliminary results obtained with the polynomial chaos method described in section 3.2. First, let us compare mean and deterministic solutions. As shown in figure 4, at gauge 9 (a) mean solution is coincident with the deterministic one, displaying a quasi-linear behavion in uncertainty propagation. On the contrary, at gauge 16 (b) and even more at gauge 22 (c), there are strong differences after the interaction phase.

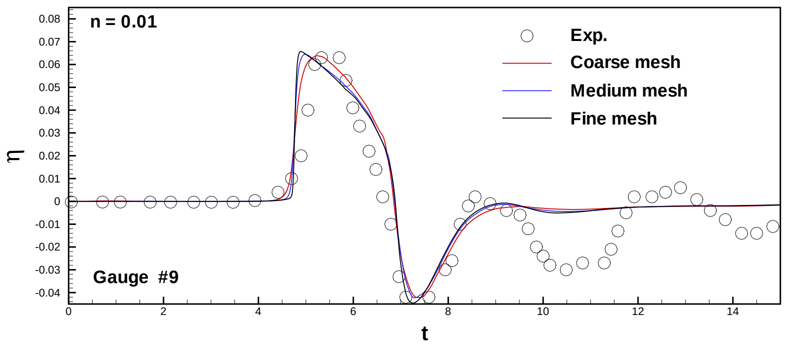
Larger bars confirm what part of the physics most influenced by the parameters :

- In the shoaling area especially on the reflected wave.
- on the right side on runup phase, rundown level, and water level after interaction.
- on the lee side in the run up region and in the post-interaction. Finer meshes computations could give nicer bars probably getting closer to the data. This point will be clarified in the final paper.

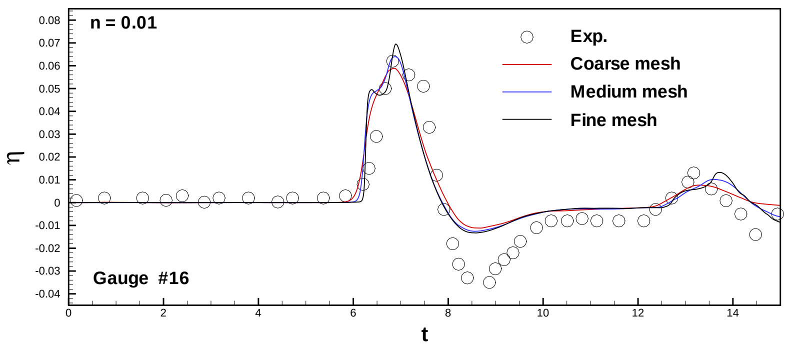
In the final paper, these results will be validated againt Monte Carlo computations, and results obtained with SI method will be presented and discussed in terms of accuracy and computational cost.

5 Conclusion

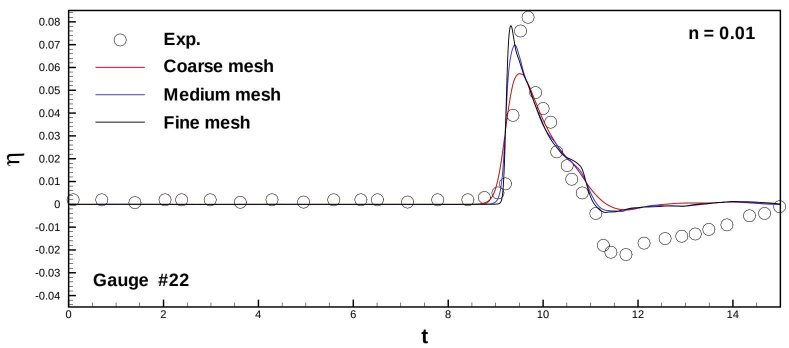
In this work, a residual distribution scheme for shallow water equations have been coupled with some stochastic methods in order to take into account uncertainties in the numerical simulation. Preliminary results showed that influence of uncertainties is stronger after the phase interaction indicating the need for a



(a)

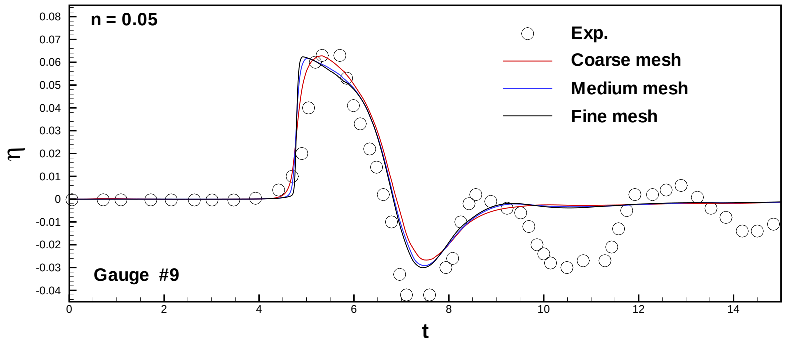


(b)

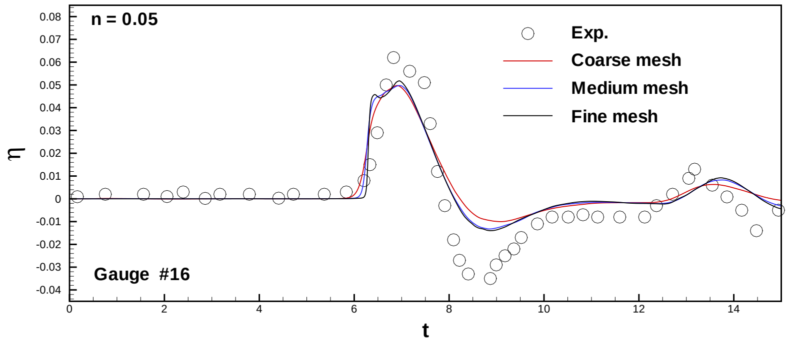


(c)

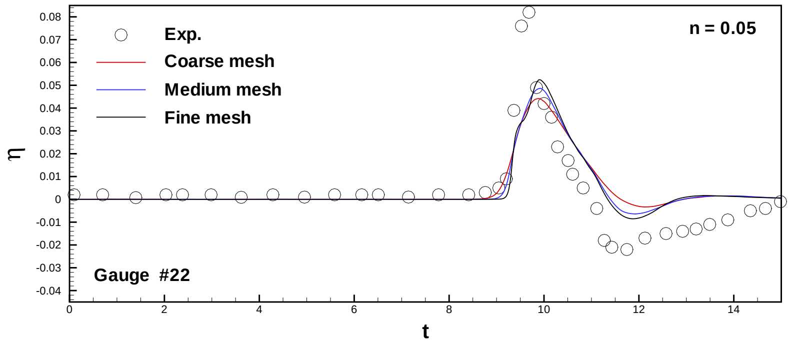
Figure 2: Time evolution of η at gauge 9 (a), 16 (b) and 22 (c) for $n = 0.01$



(a)

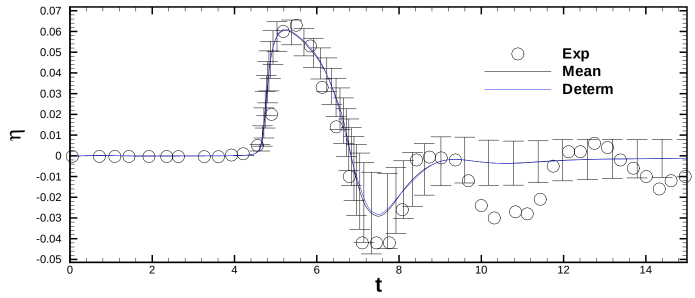


(b)

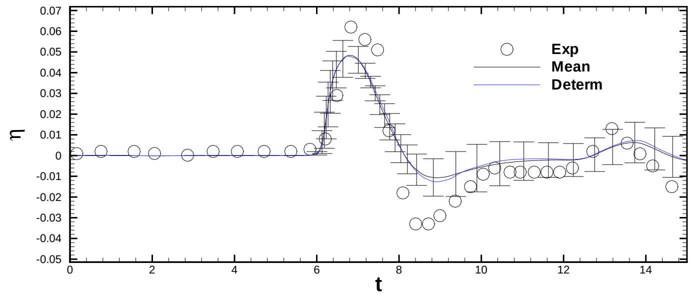


(c)

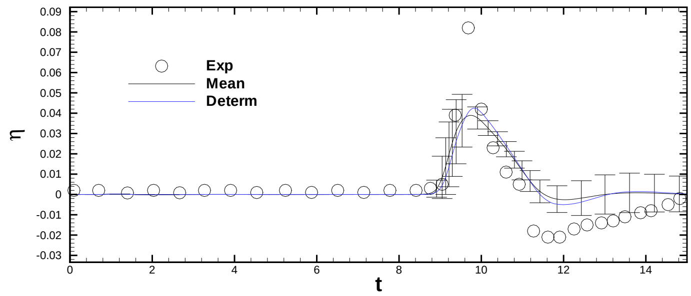
Figure 3: Time evolution of η at gauge 9 (a), 16 (b) and 22 (c) for $n = 0.05$



(a)



(b)



(c)

Figure 4: Time evolution of η at gauge 9 (a), 16 (b) and 22 (c) for the deterministic solution ($n = 0.05$) and the stochastic solution with uncertainty bars

stochastic simulation in order to have a correct prediction of the numerical solution. In the final paper, stochastic results obtained with semi-intrusive scheme will be presented and compared with Monte Carlo solution to assess their validity, and with polynomial chaos solutions in order to estimate their performances in terms of computational cost.

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