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Abstract—Wireless networks are composed of many users that usually have conflicting objectives and generate interference to each other. The system design is typically formulated as the optimization of the weighted sum of the users' utility functions. In an attempt to obtain distributed algorithms in the case this sum is nonconvex, researchers have proposed pricing mechanisms which however are based on heuristics and valid only for a restricted class of problems. In this paper we propose a general framework for the distributed optimization of the nonconvex sum-utility function. Our main contributions are: i) the derivation for the first time of a general dynamic pricing mechanism, ii) a framework that can be easily particularized to well-known applications, giving rise to very efficient practical algorithms that outperform existing methods; and iii) the solution to the currently open problem of social optimization for MIMO multiuser systems.

I. INTRODUCTION

Wireless networks are composed of users that may have different objectives and generate interference, when no multiplexing scheme is imposed a priori to regulate the transmissions; examples are peer-to-peer networks, cognitive radio systems, and ad-hoc networks. A usual and convenient way of designing such multiuser systems is by optimizing the (weighted) sum of the users' objective functions (also termed as social function), which is a measure of network performance. The main difficulty of this formulation lies in performing the optimization in a distributed manner with limited signaling among the users. Indeed, for advanced wireless and wired networks, centralized solutions are not implementable, as they require a central node gathering the knowledge of all the users' parameters (e.g., direct and cross-channels), performing the optimization, and then broadcasting the result to each of the users. When the social function is jointly convex, the problem reduces to the so-called Network Utility Maximization (NUM) for which many distributed algorithms have been derived, based on primal and dual decomposition techniques (see, e.g., [1]).

The nonconvex case is more involved and challenging as one cannot practically aim at finding global solutions anymore; instead, one seeks stationary (possibly local optimal) solutions. In this vein, some attempts follow classical optimization approaches based on distributed descent schemes such as gradient methods [2], [3]; the resulting algorithms, however, suffer from slow convergence. To improve the convergence speed, other works proposed best-response algorithms combined with pricing [4], [5], [6], [7]; state-of-the-art results of these techniques can be found in [8]. The common approach followed in these works is to interpret the users as players of a noncooperative game wherein the users' utility functions are perturbed by adding a proper (linear) pricing term, in order to take into account, to some extent, the social function [4], [5], [6], [7]. This point of view is somewhat odd, since it tries to analyze an optimization problem by using game-theoretic tools. The result is that, although the pricing best-response mechanisms have shown to be valuable—they exhibit rapid convergence in practice—so far only a restricted class of problems could be studied using these ad-hoc approaches, and, even for such problems, convergence of pricing algorithms is established under relatively strong assumptions. The state-of-the-art of pricing best-response algorithms applied to solve resource allocation problems over Interference Channels (ICs) is summarized as follows.

– **SISO networks**: convergence to a stationary solution of the (Shannon) sum-rate maximization problem by sequential best-response pricing algorithms is established [6]; studying convergence for simultaneous updates or different utility functions and constraints for which the underlying game is not supermodular remains an open problem [4], [5].

– **MISO networks**: Pricing algorithms are observed to perform well numerically; although convergence is typically observed in practice, a proof is available only for the two-user case [8].

– **MIMO networks**: Devising pricing best-response algorithms with provable convergence remains a challenging and open problem [8]; it is not even clear how to choose a suitable pricing mechanism; the two-user sum-rate maximization problem forcing beamforming has been studied in [7] but no proof of convergence is provided; the general case of more than two users remains unsolved.

By taking a more direct optimization point of view, in this paper, we propose a new general best-response distributed algorithm for solving the sum-utility problem along with its convergence framework. As a fundamental departure from existing works, our approach does not impose the use of

any pricing scheme a priori; instead, the resulting pricing mechanism has a very natural and simple interpretation. Our main results can be summarized as follows: i) we provide a formal derivation for the first time of a general dynamic pricing mechanism; ii) the proposed general framework can be readily particularized to well-known applications, such as [4], [5], [6], [7], giving rise in a unified fashion to algorithms that outperform existing methods both theoretically and numerically; and iii) as a direct application of our framework, we propose for the first time a dynamic pricing best-response algorithm for the sum-rate maximization problem over MIMO ICs with provable convergence.

The rest of the paper is organized as follows. Section II introduces the sum-utility optimization problem along with two motivating examples, namely: the sum-rate maximization problems over SISO and MIMO ICs. Section III presents the new framework suitable for solving nonconvex sum-utility optimization problems based on dynamic pricing best-response algorithms. In Section IV we apply the developed methods to the sum-rate maximization problems over SISO and MIMO ICs previously introduced, and in Section V we report numerical results and compare the performance of our new algorithm with the state-of-the-art algorithms. Finally, Section VI draws some conclusions. The proofs of the main results can be found in [9] and are omitted here because of the space limitation.

II. PROBLEM FORMULATION

We consider the design of a multiuser system composed of Q coupled users, where each user q has a cost function $f_q(\mathbf{x}_q, \mathbf{x}_{-q})$ that depends on his own strategy vector $\mathbf{x}_q$, which belongs to the feasible set $\mathcal{K}_q \subseteq \mathbb{R}^{n_q}$, and the variables of the other users denoted by $\mathbf{x}_{-q} \triangleq (\mathbf{x}_r)_{q \neq r=1}^Q \in \mathcal{K}_{-q} \triangleq \prod_{r \neq q} \mathcal{K}_r$; the joint strategy set of the users is denoted by $\mathcal{K} \triangleq \mathcal{K}_1 \times \cdots \times \mathcal{K}_Q$. We are interested in the following social problem over the network:

$$\underset{\mathbf{x}}{\text{minimize}} \quad U(\mathbf{x}) \triangleq \sum_{q=1}^Q w_q \, f_q(\mathbf{x}_q, \mathbf{x}_{-q}) \tag{1}$$
$$\text{subject to} \quad \mathbf{x}_q \in \mathcal{K}_q, \quad \forall q = 1, \dots, Q.$$

where $w_1, \dots, w_Q$ is a given set of positive weights. We focus on (1) under the following assumptions.

Assumption 1: The functions f_q and the sets $\mathcal{K}_q$ are such that: *i)* The sets $\mathcal{K}_q$ are closed and convex; *ii)* The functions $f_q(\mathbf{x}_q, \mathbf{x}_{-q})$ are twice continuously differentiable on $\mathcal{K}$ with bounded second derivatives on $\mathcal{K}$, and convex on $\mathcal{K}_q$ for any given $\mathbf{x}_{-q} \in \mathcal{K}_{-q}$; and *iii)* The social function $U(\mathbf{x})$ is coercive [i.e., $\widetilde{U}(\mathbf{x}) \to +\infty$ as $\|\mathbf{x}\| \to +\infty$, where $\widetilde{U}(\mathbf{x}) = U(\mathbf{x})$ if $\mathbf{x} \in \mathcal{K}$, and $\widetilde{U}(\mathbf{x}) = +\infty$ otherwise]. □

The assumptions above are quite standard and are satisfied in many practical problems of interest. In particular, assumption ii) implies that the social function $U(\mathbf{x})$ is Lipschitz continuous on $\mathcal{K}$; we denote by L_U the Lipschitz constant of U. Condition iii) guarantees that the social problem has a solution, even when the feasible $\mathcal{K}$ is not bounded; if $\mathcal{K}$ is bounded then condition iii) is trivially satisfied. Note that,

differently from classical NUM problems, here we do not assume that the user objective functions are jointly convex, which leads to a nonconvex optimization problem (1).

Instances of the social problem above we are interested in are the sum-rate maximization over frequency-selective ICs

$$\underset{\{\mathbf{p}_q \geq \mathbf{0}\}_{q=1}^Q}{\text{maximize}} \quad \sum_{q=1}^Q w_q \left\{ \sum_{k=1}^N \log \left(1 + \frac{|H_{qq}(k)|^2 p_q(k)}{\sigma_q^2(k) + \sum_{r \neq q} |H_{qr}(k)|^2 p_r(k)} \right) \right\}$$
$$\text{subject to} \quad \mathbf{1}^T \mathbf{p}_q \leq P_q, \quad \forall q = 1, \dots, Q \tag{2}$$

where Q is the number of users, N is the number of carriers, P_q is the power budget for the q-th user, $\sigma_q^2(k)$ is the variance of the thermal noise over carrier k at the receiver q, and $|H_{qr}(k)|^2$ is the gain of the channel between the r-th transmitter and the q-th receiver. Another instance is the more general MIMO formulation

$$\underset{\{\mathbf{Q}_q \succeq \mathbf{0}\}_{q=1}^Q}{\text{maximize}} \quad \sum_{q=1}^Q w_q \, \log \det \left(\mathbf{I} + \mathbf{H}_{qq}^H \mathbf{R}_q(\mathbf{Q}_{-q})^{-1} \mathbf{H}_{qq} \mathbf{Q}_q \right)$$
$$\text{subject to} \quad \text{tr}(\mathbf{Q}_q) \leq P_q, \quad \forall q = 1, \dots, Q \tag{3}$$

where $\mathbf{Q}_q$ is the covariance matrix of transmitter q, $\mathbf{H}_{qr}$ is the channel matrix between the r-th transmitter and the q-th receiver, $\mathbf{R}_q(\mathbf{Q}_{-q}) \triangleq \mathbf{R}_{n_q} + \sum_{r \neq q} \mathbf{H}_{qr} \mathbf{Q}_r \mathbf{H}_{qr}^H$ is the covariance matrix of the multiuser interference plus the thermal noise $\mathbf{R}_{n_q}$ (assumed to be full-rank), and $\text{tr}(\bullet)$ is the trace operator. Problems (2) and (3) have been shown to be NP hard [10]; there is then no hope to compute a globally optimal solution in polynomial time, even by using centralized methods. We are thus interested in devising *distributed* solution methods for computing stationary solutions (possibly local minima) of problem (1). Departing from standard algorithms suitable for (1), such as distributed gradient or Newton-based methods (see, e.g., [11]), we aim at devising *simultaneous best-response-like* schemes placing no step-size restriction in the updates of the users, so that we can obtain faster low-complex algorithms that converge even when current algorithms fail [8]. This is the subject of the next section.

III. DISTRIBUTED DYNAMIC PRICING ALGORITHM

We begin by introducing an informal description of our new algorithm that sheds light on the connection with classical descent gradient-based schemes, making clear why our scheme is expected to outperform current gradient descent methods.

A. A Quick look at conditional gradient algorithms: Why do they work poorly?

A classical approach to solve a nonconvex problem like (1) would be using some of the well-known gradient-based descent schemes. A simple way to generate a feasible, descent direction is given for example by the conditional gradient method (also called Frank-Walfe method) [11]: given the current iterate $\mathbf{x}^{(n)} = (\mathbf{x}_i^{(n)})_{i=1}^Q$, the next feasible vector $\mathbf{x}^{(n+1)}$ is generated according to

$$\mathbf{x}^{(n+1)} = \mathbf{x}^{(n)} + \gamma^{(n)} \mathbf{d}^{(n)} \tag{4}$$

where $\mathbf{d}^{(n)} \triangleq \overline{\mathbf{x}}^{(n)} - \mathbf{x}^{(n)}$, $\overline{\mathbf{x}}^{(n)} = (\overline{\mathbf{x}}_q^{(n)})_{q=1}^Q$ is the solution of the following set of convex problems (one for each user):

$$\overline{\mathbf{x}}_q^{(n)} = \underset{\mathbf{x}_q \in \mathcal{K}_q}{\operatorname{argmin}} \left\{ \nabla_{\mathbf{x}_q} U \left(\mathbf{x}^{(n)} \right)^T \left(\mathbf{x}_q - \mathbf{x}_q^{(n)} \right) \right\}, \quad (5)$$

for all $q = 1, \ldots, Q$, and $\gamma^{(n)} \in (0,1]$ is the step-size of the algorithm that needs to be properly chosen to guarantee convergence (see, e.g., [11]). Writing (5) we tacitly assumed that each linearized problem (5) has a solution.

Looking at (5) one infers that gradient methods are based on solving a sequence of convex problems obtained by linearizing the *whole* utility function U, a fact that does not exploit any "nice" structure that the original problem may have. For instance, under Assumption 1, the objective functions $f_q(\mathbf{x}_q, \mathbf{x}_{-q})$ are convex in $\mathbf{x}_q$ for any given $\mathbf{x}_{-q}$; it turns out that it may be better to keep in each of the optimization problems (5) the "nice" part of the objective function unaltered—the convex part $f_q(\bullet, \mathbf{x}_{-q}^{(n)})$—while linearizing (convexifying) only the "bad" terms—the nonconvex part $\sum_{r \neq q} f_r(\mathbf{x})$. This motivates the introduction of the following mapping $\mathcal{K} \ni \mathbf{y} \mapsto \widehat{\mathbf{x}}_\tau(\mathbf{y}) \triangleq (\widehat{\mathbf{x}}_{\tau,q}(\mathbf{y}))_{q=1}^Q$, where $\widehat{\mathbf{x}}_{\tau,q}(\mathbf{y})$ is defined as

$$\widehat{\mathbf{x}}_{\tau,q}(\mathbf{y}) = \underset{\mathbf{x}_q \in \mathcal{K}_q}{\operatorname{argmin}} \left\{ \begin{array}{c} w_q\, f_q(\mathbf{x}_q, \mathbf{y}_{-q}) + \boldsymbol{\pi}_q(\mathbf{y})^T \mathbf{x}_q \\ + \dfrac{\tau}{2}\, \|\mathbf{x}_q - \mathbf{y}_q\|^2 \end{array} \right\} \quad (6)$$

where

$$\boldsymbol{\pi}_q(\mathbf{y}) \triangleq \sum_{r \neq q} w_r\, \nabla_{\mathbf{x}_q} f_r(\mathbf{y}) \quad (7)$$

and τ is a given positive constant. Note that, under Assumption 1 and given $\mathbf{y}$, each problem in (6) is strongly convex and thus has a unique solution, implying that $\widehat{\mathbf{x}}_{\tau,q}(\mathbf{y})$ is well-defined for any given $\mathbf{y} \in \mathcal{K}$. The best-response mapping $\widehat{\mathbf{x}}_\tau(\mathbf{y})$, with $\mathbf{y} = \mathbf{x}^{(n)}$, is clearly related to the solution of (5); the difference between the optimization problems in (5) and (6) is given by the fact that in the latter we linearized only the nonconvex part $\sum_{r \neq q} w_r \nabla_{\mathbf{x}_q} f_r(\mathbf{x})$ [resulting in the term $\boldsymbol{\pi}_q(\mathbf{x}^{(n)})^T \mathbf{x}_q$] rather than the whole function U, and we added the proximal regularization term $\frac{\tau}{2} \|\mathbf{x}_q - \mathbf{y}_q\|^2$, whose beneficial effects are well understood, see e.g. [11].

Therefore, the proposed candidate search direction $\mathbf{d}^{(n)}$ at point $\mathbf{x}^{(n)}$ in (4) becomes the vector $\widehat{\mathbf{x}}_\tau(\mathbf{x}^{(n)}) - \mathbf{x}^{(n)}$. The resulting algorithm is expected to perform better than classical gradient-based schemes (at least in terms of convergence speed) because the objective function structure is better preserved. Furthermore, thanks to the presence of the regularization term, we will show to get additional flexibility with respect to other gradient-based descent methods, that can be exploited to enhance the convergence speed.

B. Distributed Dynamic Pricing Algorithm (DDPA)

We provide now a formal description of the proposed algorithm. Preliminarily, we describe in Proposition 1 the main properties of the mapping $\widehat{\mathbf{x}}_\tau(\mathbf{y})$. We denote by $\nabla_{\mathbf{x}_i}^2 f_i(\mathbf{x})$ and $\lambda_{\min}(\mathbf{A})$ the Hessian matrix of f_i and the minimum eigenvalue of the symmetric matrix $\mathbf{A}$, respectively.

Proposition 1 ([9]): Given the mapping $\mathcal{K} \ni \mathbf{y} \mapsto \widehat{\mathbf{x}}_\tau(\mathbf{y})$, suppose that Assumption 1 holds. Then: (a) The function $\widehat{\mathbf{x}}_\tau(\mathbf{y})$ is continuous on $\mathcal{K}$; (b) The set of the fixed-points of $\widehat{\mathbf{x}}_\tau(\mathbf{y})$ coincides with the set of stationary solutions of the social problem (1); *therefore $\widehat{\mathbf{x}}_\tau(\mathbf{y})$ has a fixed-point; and (c) For every given $\mathbf{y} \in \mathcal{K}$, the vector $\widehat{\mathbf{x}}_\tau(\mathbf{y}) - \mathbf{y}$ is a descent direction of the social function $U(\mathbf{x})$ at $\mathbf{y}$ such that*

$$(\widehat{\mathbf{x}}_\tau(\mathbf{y}) - \mathbf{y})^T \nabla_{\mathbf{x}} U(\mathbf{y}) \leq -c \, \|\widehat{\mathbf{x}}_\tau(\mathbf{y}) - \mathbf{y}\|^2 \quad (8)$$

for some positive constant c with $c \geq c_\tau \triangleq \tau + \min_{i=1,\ldots,Q} \min_{\mathbf{z} \in \mathcal{K}} \lambda_{\min}(\nabla_{\mathbf{x}_i}^2 f_i(\mathbf{z}))$. □

Proposition 1 paves the way to the design of distributed best-response like algorithms for the social problem (1), making formal the idea introduced in Section III-A. Indeed, the inequality (8) states that either $(\widehat{\mathbf{x}}_\tau(\mathbf{x}^{(n)}) - \mathbf{x}^{(n)})^T \nabla_{\mathbf{x}} U(\mathbf{x}^{(n)}) < 0$ or $\widehat{\mathbf{x}}_\tau(\mathbf{x}^{(n)}) = \mathbf{x}^{(n)}$. In the former case, the vector $\mathbf{d}^{(n)} \triangleq \widehat{\mathbf{x}}_\tau(\mathbf{x}^{(n)}) - \mathbf{x}^{(n)}$ is a descent direction of $U(\mathbf{x})$ at $\mathbf{x}^{(n)}$; in the latter case, $\mathbf{x}^{(n)}$ is a fixed-point of the mapping $\widehat{\mathbf{x}}_\tau(\bullet)$ and thus a stationary solution of the social problem (1) [Proposition 1 (b)]. Large values of τ make c_τ large and thus permit to take $\gamma^{(n)} = 1$, resulting in a Jacobi best-response scheme (see [9] for more details). This suggests the descent-like algorithm described in Algorithm 1 below, whose convergence properties are given in Theorem 2, where c_τ is defined in Proposition 1; a more general version of the algorithm and Theorem 2 are given in [9].

Algorithm 1: Distributed Dynamic Pricing Algorithm

Data : Let $\tau > 0$; choose any $\mathbf{x}^{(0)} \in \mathcal{K}$; set $n = 0$.
(S.1) : If $\mathbf{x}^{(n)}$ satisfies a suitable termination criterion: STOP.
(S.2) : For each $q = 1, \ldots, Q$, compute the best-response $\widehat{\mathbf{x}}_{\tau,q}(\mathbf{x}^{(n)})$ in (6); set $\mathbf{x}^{(n+1)} \triangleq \widehat{\mathbf{x}}_\tau(\mathbf{x}^{(n)})$.
(S.3) : $n \leftarrow n + 1$, and go to **(S.1)**.

Theorem 2 ([9]): Given the social problem (1), *suppose that Assumption 1 holds. If τ is chosen so that $c_\tau \geq L_U/2$, then either Algorithm 1 converges in a finite number of interations to a stationary solution of* (1) *or every limit point of the sequence $\{\mathbf{x}^{(n)}\}_{n=1}^\infty$ is a stationary solution of* (1).□

The proposed algorithm is a distributed Jacobi best-response scheme: at each iteration n, all the users simultaneously update their strategies according to the best-response $\widehat{\mathbf{x}}_{\tau,q}(\mathbf{x}^{(n)})$; quite surprisingly, Theorem 2 states convergence under very mild assumptions (always satisfied in practice). This result is not trivial at all and represents along with Algorithm 1 a novel contribution even in the optimization literature; indeed classic best-response nonlinear Jacobi schemes applied to the sum-utility problem (1) (see, e.g., [11, Ch. 3.3.5]) do not converge under Assumption 1.

Note that Algorithm 1 implements naturally a pricing mechanism, given by the auxiliary (linear) term $\boldsymbol{\pi}_q(\mathbf{x}^{(n)})^T \mathbf{x}_q$ in the (modified) objective function of each player. Indeed, each $\boldsymbol{\pi}_q(\mathbf{x}^{(n)})$ represents a dynamic pricing that measures somehow

the marginal increase of the sum-utility of the other users due to a variation of the strategy of user q; roughly speaking, it works like a punishment imposed to each user for being too aggressive in choosing his own strategy and thus "hurting" the other users. But differently from all previous works [4], [5], [6], [7], [8], here pricing is not heuristically imposed a priori; it is instead the natural consequence of an optimization process. The result is that our framework can be applied to a very large class of problems, even when [4], [5], [6], [7], [8] fail as, e.g., the sum-rate maximization over MIMO ICs, as shown in the next section.

IV. DDPA FOR SISO AND MIMO PROBLEMS

With the developments of the previous section in mind, we can now go back to the SISO and MIMO rate maximization problems (2) and (3), and readily apply the DDPA described in Algorithm 1. More specifically, in the SISO case, the best-response $\widehat{\mathbf{x}}_{\tau,q}(\mathbf{y})$ in (6) reduces to $\hat{\mathbf{p}}_q(\mathbf{y})$ defined as [9]

$$\hat{\mathbf{p}}_q(\mathbf{y}) = \left[\frac{1}{2} \left(\mathbf{y}_q - \boldsymbol{\alpha}_q^{-2} \right) + \right.$$
$$\left. - \frac{1}{2\tau} \left(\tilde{\boldsymbol{\mu}} - \sqrt{\left[\tilde{\boldsymbol{\mu}} - \tau \left(\mathbf{y}_q - \boldsymbol{\alpha}_q^{-2} \right) \right]^2 + 4\tau} \right) \right]^+$$

where

$$\alpha_q^2(k) = \frac{w_q \, |H_{qq}(k)|^2}{\sigma_q^2(k) + \sum_{r \neq q} |H_{qr}(k)|^2 \, y_r(k)}$$

is the effective channel gain, $\boldsymbol{\alpha}_q^{-2} \triangleq (\alpha_q^{-2}(k))_{k=1}^N$, $\tilde{\boldsymbol{\mu}} \triangleq \boldsymbol{\pi}_q(\mathbf{y}) + \mu_q \mathbf{1}$ with the waterlevel μ_q chosen to satisfy the non-linear complementarity condition $0 \leq \mu_q \perp P_q - \mathbf{1}^T \hat{\mathbf{p}}_q(\mathbf{y}) \geq 0$, and the price vector $\boldsymbol{\pi}_q(\mathbf{y}) \triangleq (\boldsymbol{\pi}_{q,k}(\mathbf{y}))_{k=1}^N$ is defined as

$$\boldsymbol{\pi}_{q,k}(\mathbf{y}) \triangleq - \sum_{r \in \mathcal{N}_q} w_r \frac{|H_{rq}(k)|^2}{|H_{rr}(k)|^2} \frac{\alpha_r^2(k)}{\frac{1}{\alpha_r^2(k) \, y_r(k)} + 1} \quad (9)$$

where $\mathcal{N}_q$ denotes the set of neighbors of user q, i.e., the set of users r's which user q interferers with. According to Theorem 2, convergence of the DDPA is guaranteed provided that the proximal gain τ is sufficiently large, as quantified in the theorem (see [9] for more details). At the best of our knowledge, these are the weakest conditions available in the literature [8], [6]. Note that the proposed algorithm is fairly distributed. Indeed, given the interference generated by the other users over the carriers [and thus the effective channel coefficients $\alpha_q^2(k)$'s] and the current interference price $\boldsymbol{\pi}_q(\mathbf{y})$, each user can efficiently and locally compute the optimal power allocation $\hat{\mathbf{p}}_q(\mathbf{y})$ via a waterfilling-like expression. The estimation of the prices $\boldsymbol{\pi}_{q,k}(\mathbf{y})$ requires instead some (limited) signaling among nearby receivers. Quite interestingly, the pricing expression in (9) and thus the resulting overhead of the DDPA coincide with those of the state-of-the-art algorithms in [8], [6]. The DDPA however converges even when [8], [6] fail and appears to be faster than [8], [6] (see Figure 1 in Sec. V).

Referring to the MIMO formulation (3), the best response in (6), denoted by $\hat{\mathbf{Q}}_{\tau,q}(\mathbf{Y})$, becomes

$$\hat{\mathbf{Q}}_{\tau,q}(\mathbf{Y}) =$$
$$\operatorname*{argmax}_{\mathbf{Q}_q \succeq \mathbf{0}, \, \mathrm{tr}(\mathbf{Q}_q) \leq P_q} \left\{ \begin{array}{c} w_q \, r_q(\mathbf{Q}_q, \mathbf{Q}_{-q}) - 2\, \boldsymbol{\Pi}_q(\mathbf{Y}) \bullet \mathbf{Q}_q \\ - \frac{\tau}{2} \left\| \mathbf{Q}_q - \mathbf{Y}_q \right\|_F^2 \end{array} \right\}$$

where $\mathbf{A} \bullet \mathbf{B} \triangleq \mathrm{tr}(\mathbf{A}^H \mathbf{B})$,

$$r_q(\mathbf{Q}_q, \mathbf{Q}_{-q}) \triangleq \log \det \left(\mathbf{I} + \mathbf{H}_{qq}^H \mathbf{R}_q(\mathbf{Q}_{-q})^{-1} \mathbf{H}_{qq} \mathbf{Q}_q \right)$$

and

$$\boldsymbol{\Pi}_q(\mathbf{Y}) \triangleq \sum_{r \in \mathcal{N}_q} w_r \, \mathbf{H}_{rq}^H \widetilde{\mathbf{R}}_r(\mathbf{Y}_{-r}) \mathbf{H}_{rq} \quad (10)$$

with

$$\widetilde{\mathbf{R}}_r(\mathbf{Y}_{-r}) \triangleq \left[\left(\mathbf{R}_r(\mathbf{Y}_{-r}) + \mathbf{H}_{rr} \mathbf{Y}_r \mathbf{H}_{rr}^H \right)^{-1} - \mathbf{R}_r(\mathbf{Y}_{-r})^{-1} \right].$$

Note that, once the price matrix $\boldsymbol{\Pi}_q(\mathbf{Y})$ is given, the best-response $\hat{\mathbf{Q}}_{\tau,q}(\mathbf{Y})$ can be computed locally by each user solving a convex optimization problem. As for the SISO case, at each iteration of the algorithm, the users need to exchange the current value of the pricing matrix $\boldsymbol{\Pi}_q(\mathbf{Y})$; convergence is guaranteed for sufficiently large values of τ (see [9] for details). At the best of our knowledge, this is the first best-response Jacobi algorithm based on pricing with provable convergence for MIMO systems.

V. NUMERICAL RESULTS

In this section, we compare the proposed DDPA with the state-of-the-art algorithms proposed in the literature for solving the SISO sum-rate maximization problem (2), namely: the Modified Asynchronous Distributed Pricing (MADP) algorithm [6] and the Jacobi Gradient Projection Algorithm (GPA) [2], [11]. To quantify the tradeoff between signaling and performance we also included the simultaneous Iterative Waterfilling Algorithm (IWFA) proposed in [12] that solves the associated noncooperative game, without requiring any signaling among the users. To stress the comparison in terms of convergence speed, we used for the GPA and the MADP the largest step-size (less than one) under which the algorithms are experimented to converge, even though such a step-size violates the theoretical convergence conditions, as given in [11] and [6], respectively.

We examined the behavior of the above algorithms under the following setup. We considered an ad-hoc network composed of 30 active users; the (cross-)channels among the links are simulated as FIR filter of order $L = 10$, where each tap is a zero mean complex Gaussian random variable with variance equal to $1/L$; the channel transfer functions are the FFT of the corresponding impulse responses over $N = 256$ points (carriers). we considered an high interference scenario corresponding to $\mathrm{SNR}_q \triangleq P_q/(d_{qq} \sigma_q^2(k)) = 2\mathrm{dB}$ and $\mathrm{INR}_q \triangleq P_r/(d_{qr} \sigma_q^2(k)) = 5\mathrm{dB}$ for all q and r, where d_{qr} denotes the distance between the transmitter r and the receiver q (d_{qq} is the intra-pair distance). All the algorithms are initialized by the same starting point, chosen randomly in the feasible set of the users, and are terminated when
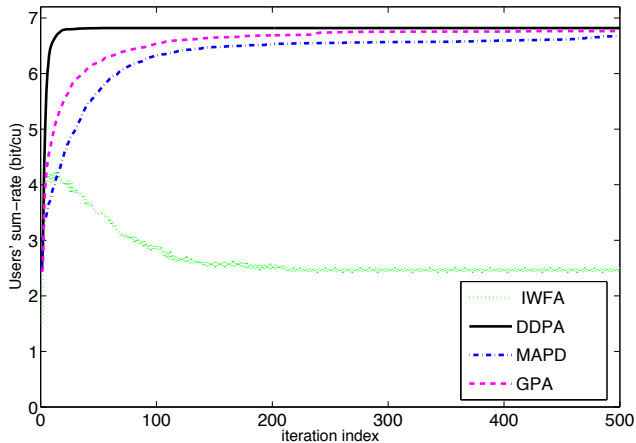
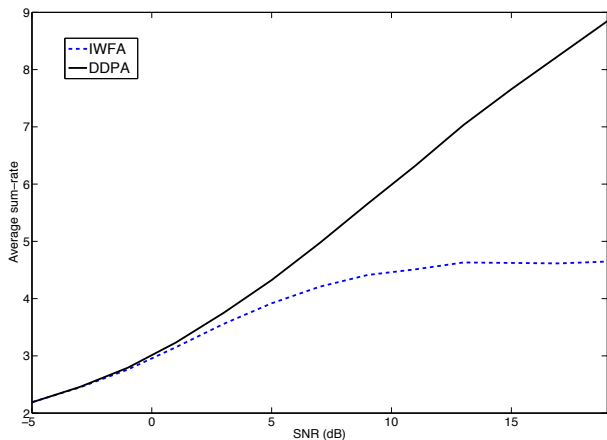Figure 1. Sum-rate versus iterations: IWFA, DDPA, GPA, and MADP.

Figure 2. Average Sum-rate versus SNR: IWFA and DDPA.

the Euclidean norm of the error in two consecutive iterations becomes smaller than 10^{-6}.

In Figure 1, we plot the users' sum-rate $\sum_q r_q(\mathbf{p})$ versus the iterations achieved by the aforementioned algorithms. The picture clearly shows that our DDPA is much faster than the GPA and MADP, while requiring the same signaling and even less complexity (the best-response $\hat{\mathbf{p}}_q(\mathbf{y})$ is available in closed form). We observed similar behaviors also in the MIMO scenario (see [9] for details).

Figure 2 shows the average performance of the proposed algorithm. We plotted the average sum-rate versus the $\text{SNR}_q \triangleq P_q/(d_{qq} \sigma_q^2(k)) = \text{SNR}$ for all q and k, achievable at the NE reached by the simultaneous IWFA and the DDPA. The curves are averages over 5000 random channel realization; the rest of the parameters are the same as in Figure 1. Figure 2 confirms the superior performance of the proposed algorithm with respect to the IWFA, especially in high interference scenarios. This gain however comes at the cost of some signaling among the users.

VI. CONCLUSIONS

In this paper we proposed a novel distributed best-response algorithm for solving the sum-utility problem along with its convergence framework. The algorithm is a mixture of a best-response algorithm and a descent-based method wherein a dynamic pricing mechanism has a very natural and simple interpretation. In [9], we show that (a slight variation of) the proposed algorithm is also robust against stochastic errors on the price estimates, due to an imperfect communication scenario (random link failures, noisy estimate, quantization, etc...). The proposed framework can be readily applied to solve the sum-rate maximization problem over MIMO ICs giving rise, in a unified fashion, to best-response based algorithms with pricing that outperform existing methods both theoretically and numerically.

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