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AEROSTRUCTURAL OPTIMIZATION WITH THE ADJOINT METHOD

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Abstract. This paper presents the current developments performed at ONERA to extend the aeroelastic adjoint method in the CFD software *elsA* towards an aerostructural adjoint. Because multiobjective and multipoint optimizations require an aerostructural design space, a tool for fully-flexible wings is created utilizing Python and Fortran. This structural module provides an equivalent beam model based on the CFD surface mesh of the wing, the internal structural geometry that meets the aerodynamic limit loads as well as an estimation of the wing weight. This module is integrated with the existing aeroelastic environment for adjoint-based optimization.

Key words: Adjoint, Multi-disciplinary optimization, Aeroelasticity, Fluid structure interaction, Drag, Sensitivity, Cost function, Shape.

1 INTRODUCTION

Since the end of the eighties¹, the efficiency of the adjoint method is far enough proved for optimizing rigid wing shapes regardless to the number of design parameters n ; $n \in \mathbb{N}^{*,2,3,4}$. A realistic wing design task requires to take in account interactions between aerodynamic and structures. Several activities are conducted at ONERA for the integration of the adjoint method in a multidisciplinary design process^{5,6,7}. From an aerodynamic point of view, many steps can be done to model correctly the structure and account for aeroelastic deformations, the ultimate approach being a complex process that couples CFD and CSM with fully featured finite element model. Onera is developing an automated aerostructural design tool⁸ laying on the adjoint method for the evaluation of the cost function gradients and on a structural package, coded principally in Python to predict structural behavior under aerodynamic loads. This approach aims at using multipoint fully-adjoint minimization of the weighted sum of drag and wing weight coefficients :

$$\mathcal{J} = C_D + \omega C_W \quad (1)$$

2 Computational aeroelasticity

In this study we consider a closely-coupled aeroelastic system. The RANS equations used to model the flow, are discretised using a finite volume method and solved by the ONERA CFD code *elsA*². The structural equations based on the Euler-Bernoulli Beam theory approach are solved by a Python module developed by Marcelet. M⁷. At each aeroelastic step, the flow solutions are converged and the computed aerodynamic loads are extracted and transferred to Python Beam-module. The structure deformations are calculated and transferred to the mesh deformation module. This process is repeated until the static equilibrium is reached.

3 Aeroelastic and Aerostructural adjoint system

The adjoint formulation calculates the gradients of any scalar cost function with respect to a large number of design parameters at very low computational cost. We first present the aeroelastic adjoint system, already available in *elsA*⁶. In this case the flexibility matrix F of the wing structure is kept constant.

Let J be the aerodynamic function to minimize, it depends on the aerodynamic field W , the aerodynamic mesh \mathcal{X} and the parametrization α .

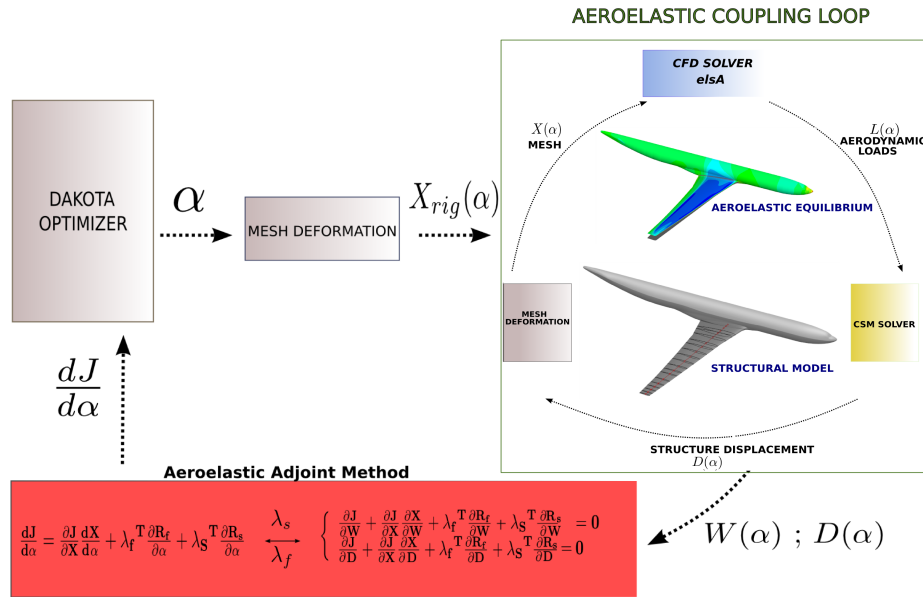


FIGURE 1: Aeroelastic adjoint-based optimization loop

The aeroelastic system is described by :

$$\begin{cases} R_{fluid}(W, \mathcal{X}) = 0 \\ R_{structure}(D, L) = D - FL = 0 \end{cases}$$

Where D is the structural deformation and L the aerodynamic loads acting on the structure. One can prove that the variation of the cost function with respect to the design space is :

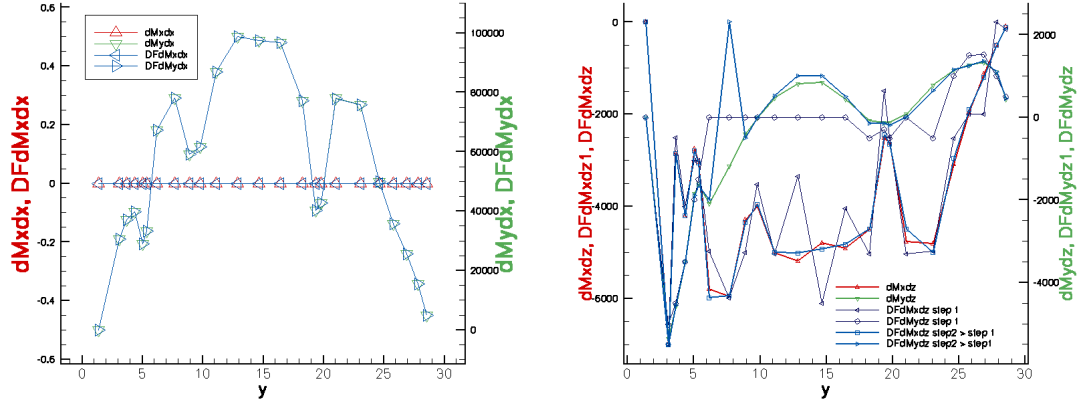
$$\frac{dJ}{d\alpha} = \frac{\partial J}{\partial \mathcal{X}} \frac{d\mathcal{X}}{d\alpha} + \lambda_f \frac{\partial R_f}{\partial \mathcal{X}} \frac{d\mathcal{X}}{d\alpha} + \lambda_s \left(\frac{\partial L}{\partial \mathcal{X}} \frac{d\mathcal{X}}{d\alpha} \right)$$

$\lambda_{structure}$ and λ_{fluid} are the adjoint vectors solution of the coupled adjoint system, see⁷ for details. To progress toward real aerostructural design where both aerodynamic and structural models are optimized, flexibility variations of the wing must be taken into account in the adjoint system formulation. Hence the gradient of J become :

$$\begin{aligned} \frac{dJ}{d\alpha} = & \frac{\partial J}{\partial \mathcal{X}} \frac{d\mathcal{X}}{d\alpha} + \lambda_f \frac{\partial R_f}{\partial \mathcal{X}} \frac{d\mathcal{X}}{d\alpha} \\ & + \lambda_s \left(\frac{\partial L}{\partial \mathcal{X}} \frac{d\mathcal{X}}{d\alpha} + \frac{\partial L}{\partial X_b} \frac{dX_b}{d\alpha} - L \frac{\partial F}{\partial [I; J]} \frac{d[I; J]}{d\alpha} \right) \end{aligned} \quad (2)$$

Where :

- $\frac{\partial J}{\partial \mathcal{X}} \frac{\partial \mathcal{X}}{\partial \alpha}$: The change in aerodynamic function due to wing geometry modification through the CFD mesh
- $\frac{\partial R_f}{\partial \mathcal{X}} \frac{\partial \mathcal{X}}{\partial \alpha}$: The sensitivity of the residuals of CFD with respect to wing shape changes through the CFD mesh
- $\frac{\partial L}{\partial \mathcal{X}} \frac{\partial \mathcal{X}}{\partial \alpha}$: The effect of shape perturbation on the aerodynamic loads through the CFD mesh
- $\frac{\partial L}{\partial X_b} \frac{\partial X_b}{\partial \alpha}$: The effect of shape perturbation on the aerodynamic loads through the beam model (aerodynamic forces are not dependent on beam axis position X_b but aerodynamic moments are). The linearization of this term is now implemented in *elsA* and validated using finite differences [Fig.2]
- $\frac{\partial F}{\partial [I; J]} \frac{\partial [I; J]}{\partial \alpha}$: The sensitivity of the flexibility matrix with respect to design parameters through the torsion stiffness J and bending stiffness I



(a) Partial derivatives of flexion M_x and torsion M_y moments with respect to x-coordinate of the beam axis position (b) Partial derivatives of flexion M_x and torsion M_y moments with respect to z-coordinate of the beam axis position

FIGURE 2: $\frac{\partial L}{\partial X_b}$ by finite differences VS linearization

4 Adjoint gradient-based aerodynamic and aeroelastic optimizations

The configuration optimized is the XRF1 model. This model is a wide-body Airbus-type research configuration. We use 11 design variables, 10 twist section and angle of attack. The purpose of the optimization is to minimize the drag function without violation of the constraints, at a single cruise condition ($Mach = 0.83, Re = 10 * 10^8$).

Near-field drag optimization for rigid XRF1 configuration

The purpose of this aerodynamic optimization is to compare the results to the aeroelastic and aerostructural optimization in order to validate our approach. The objective function is the near-field drag under Cl constraint. We have chosen CFSQP algorithm based on Sequential Quadratic Programming (SQP). Each step of the gradient-based algorithm requires the computation of the aerodynamic flow and the evaluation of the adjoint system for each function (objective and constraint), the adjoint solution is fully converged after 2,5 hours on a 8 core bi-XEON 5570 2.93 GHz processors. The optimization reached a plateau [Fig.3,4] convergence after 40 evaluation of the function and 15 evaluation of the adjoint state.

The coefficient of drag decreased from 147.72 Drag counts (Dc) to 140.92 Dc, which is mainly due to wave drag reduction, the slight changes in spanload led to a small reduction of induced drag.

TABLE 1: Drag breakdown of the optimized and the initial configurations

	Baseline XRF1 configuration	Optimized XRF1 configuration
cd pressure	147.72	140.92
cd wave	15.40	9.53
cd induced	93.49	92.42
cd spurious	38.82	38.96

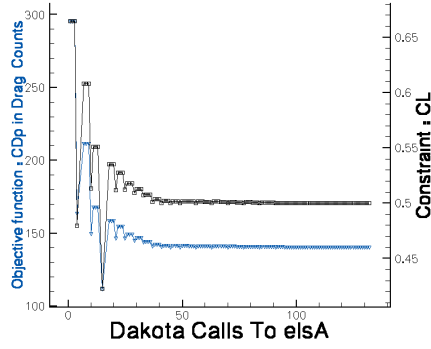


FIGURE 3: Optimization history of drag under lift constraint

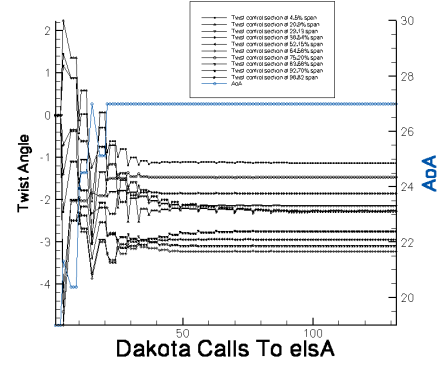


FIGURE 4: Optimization history of 10 section parametrized by twist angle and AoA

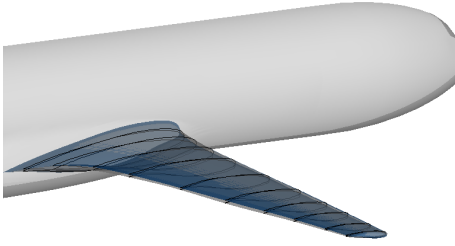


FIGURE 5: Initial and optimal design of CD_p minimization.

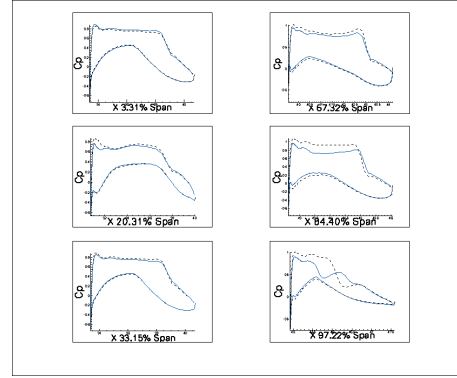


FIGURE 6: Initial and optimal design of CD_p minimization. Pressure distribution on the initial (pointed) and optimized (solid) configuration at different span location

Aeroelastic optimization

In this section, aeroelastic effects are considered without any modeling of the internal structure. The aeroelastic optimization of drag coefficient is conducted under the hypothesis of a constant stiffness matrix during the optimization process. The beam model is computed by Nastran. The optimization converged after 90 function evaluations and 11 gradient evaluations. 5 to 10 coupling iteration are performed to reach the aeroelastic equilibrium.

For each control section, the twist parameters vary independently from -5° to 5° , and the angle of attack vary in low-AoA range.

The results [Fig.7,8,9] show an optimizer strategy consisting of wing root unloading, angle of attack is increased and combined with a negative spanwise twist angle, the redistribution of circulation is plotted in [Fig.10]. For the optimized shape the wing tip bending is 1.6m, this highlights one more time the importance of accounting for wing flexibility.

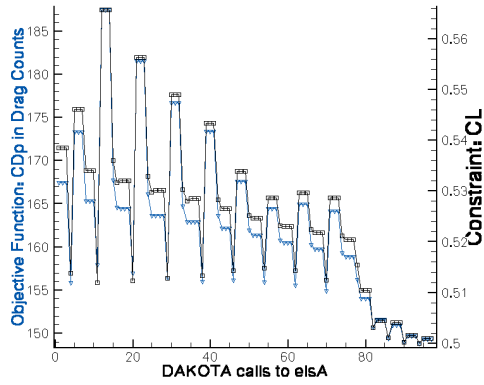


FIGURE 7: Aeroelastic optimization history of drag and lift

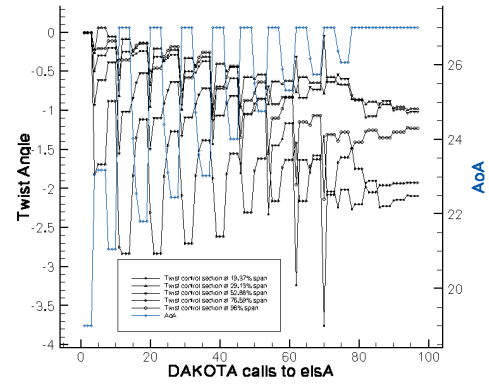


FIGURE 8: Aeroelastic optimization history of 5 section parametrized by twist angle and AoA

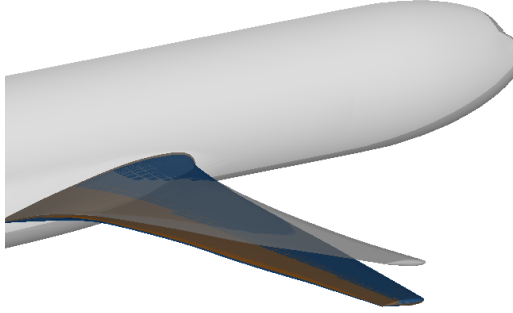


FIGURE 9: Initial (blue) design, optimized design (orange), optimized design at the elastic equilibrium (grey) of CD_p minimization using aeroelastic adjoint solver.

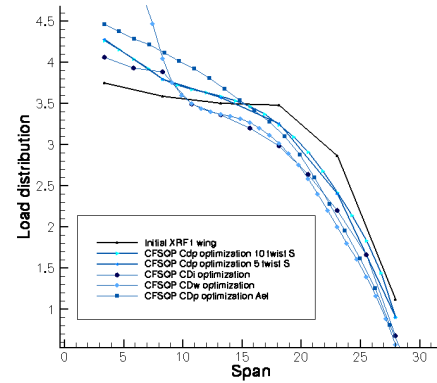


FIGURE 10: Load distribution of the initial and the optimal design of CD_p , CD_w , CD_i aerodynamic-only minimization and CD_p aeroelastic minimization

5 Structural modeling and sensitivity computation for the aerostructural adjoint solver

Wing Box design

FE models consist of thousands of elements. For an aerostructural optimization, that takes in account the variation of the wing structural rigidity, structural FE model-based adjoint will requests sensitivity calculation for each element with respect to the structural parameters (geometry of the internal components of the wing). FE models are well suited for advanced structural investigation such as aeroelastic tailoring^{19,20} but for preliminary design, beam models are a good physical alternative. The beam model [Fig. 11,12] needed for the aerostructural optimization is based on the structural module depicted in [Fig.13]. The goal of this work is to provide the aerodynamicist a simple python tool InAirSsi (INternal AIRcraft Structural SIzing) that can be easily integrated with any coupled optimization framework. The architecture of InAirssi is oriented for optimization processes only with high number of calls to aeroelastic analysis and weight estimation, for single aeroelastic computation much more sophisticated models exists and are used at Structures

Dynamics Department DADS at ONERA.

InAirSsi transforms aerodynamics inputs into structural outputs [Fig.13]. We choose to give the optimizer the role of sizing the structure, i.e there is no internal loop that sizes the structures, all the parameters are controlled by DAKOTA¹. At the iteration n of the optimization algorithm, α_{geom} produces the CFD grid. The primary structure of the wing that fits into the aerodynamic envelope is delimited by a front spar and rear spar and by the upper and lower wing skin and it is produced by α_{struct} . The parametrization of one control section is defined by

- Upper wing skin thickness
- Lower wing skin thickness
- Upper wing cap thickness
- Lower wing cap thickness
- Front spar thickness
- Rear spar thickness

It is possible to have a varying primary structure component thickness along the span to provide the locally required strength and stiffness with a minimum weight. The equivalent beam model is build and the stiffness matrix assembled to determine loads under 1-g and critical loads. Bending moment M_x , torsion moment M_y and shear force F_z are extracted for each load cases and for all structural model. A first estimation of the primary structure weight W_{ps} is computed by InAirSsi :

$$W_{ps} = 2 \sum_{\alpha_{struct}} \rho_i \int_{minspan}^{maxspan} S_i(y) dy \quad (3)$$

ρ_i and S_i are respectively the material density and the surface of the element i of the primary structure at span location y . The secondary structure that contains the high lift devices are computed either analytically or using a historical adjustment for Airbus aircrafts³². When the gradient is requested by the optimization algorithm, InAirSsi computes the sensitivities [Fig.14(b)] for the aerostructural adjoint assembly (eq.2)

Structural Design Constraints

Each element of the wing box is sized based on wing bending loads, torsion loads and shear forces. The structural module calculates the material stresses acting on caps, spars and skin which are assumed to be the principal load carrier at limit load cases and returns the ultimate strength in the structure to the optimizer. The design gradient-based algorithm handles the constraint on the maximum stress and on internal geometry that the material can withstand to stay in the elastic limit.

For bending loads, both tension and compression loads are used. The normal stress from the bending moment M_x at a section y is

$$\sigma(y) = \frac{M_x z}{I_x} \quad (4)$$

1. **Design Analysis kit for Optimization and Terascale Applications** developed by Sandia National Laboratories, contains optimization algorithms using gradient and nongradient-based methods.

The maximum stress induced by bending moments is located at the upper wing and lower wing interfaces located respectively at z_{uwing} and z_{lwing}

$$\begin{cases} \sigma_{max}^u = \frac{M_x(z_{uwing} - z_{beam})}{I_x} \\ \sigma_{max}^l = \frac{M_x(z_{lwing} - z_{beam})}{I_x} \end{cases}$$

For a wing box element of thickness t , the level of constraint due to torsional loads at a cross section of enclosed area A is given by

$$\tau = \frac{M_y}{2At} \quad (5)$$

The vertical force F_z (the integral of lift forces from the wing tip to current cross section) are used to size spar webs of thickness t_{web} . The shear constraint is

$$\tau = \frac{F_z}{2ht_{web}} \quad (6)$$

where h is the height of the wing box.

To validate InAirSsi, the module was first tested to size the XRF1 internal structure using a maneuver load case 2.5g for yield strength. The loads of this case were computed taking the simple assumption of load/g-factor proportionality. 132 parameters are controlled (6 geometry-parameter for 22 control section) and the results of structural weight of primary structure component are showed in Table 2. To have an order of idea, the primary

TABLE 2: Structural sizing of the internal structure of XRF1 configuration

Primary structure component	W (kg)
Skin	1487
Upper wing caps	9470
Lower wing caps	4846
Front spar web	3615
Rear spar web	3615
<i>Wps</i>	23033

structure weight of the A330-300 and B 747-100.wings can be found in literature³² : around 23000 kg and 22000 kg, respectively.

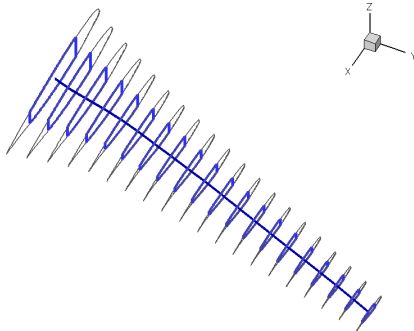


FIGURE 11: Model of the wing computed by InAirSsi



FIGURE 12: Primary structure elements : front spar, rear spar and skin

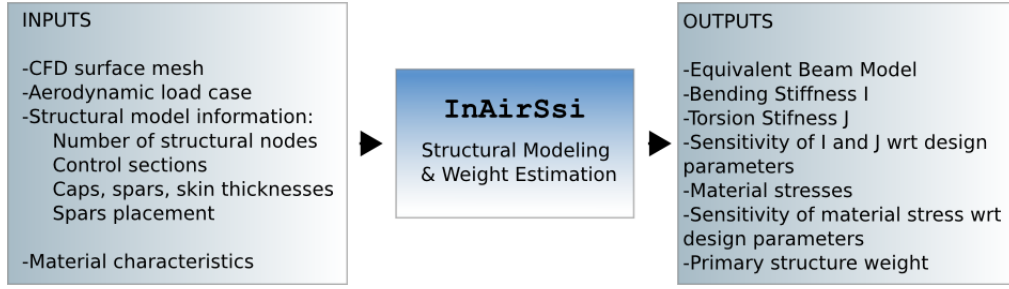
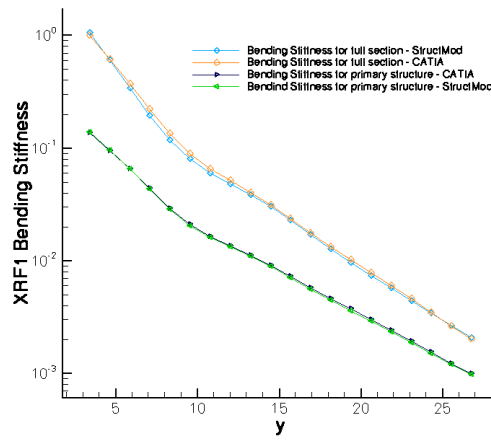
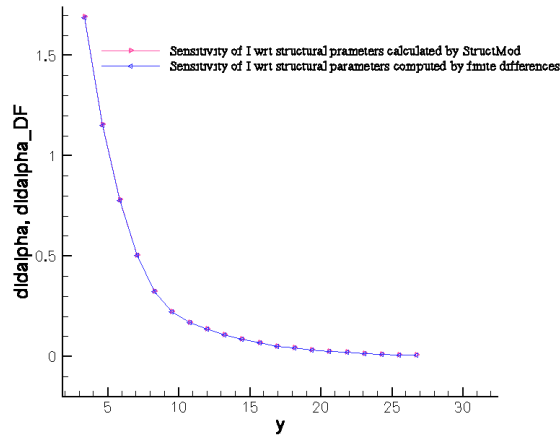


FIGURE 13: Input-output system of structural modelling tool



(a) Bending stiffness coefficient computed by InAirSsi and CATIA of the XRF1 configuration



(b) Sensitivity of bending stiffness I to the skin thickness of the wing of the XRF1 configuration

FIGURE 14:

6 Conclusion and near-future work

This paper presents the achieved developments in an ongoing PhD program at ONERA which aims at extending the aeroelastic adjoint towards an aerostructural adjoint using the

CFD code *elsA* in order to take in account the wing flexibility changes during the optimization process. This approach lays on an Euler-Bernoulli beam model which constitutes a reasonable alternative to FE model for preliminary design optimization processes of high aspect ratio wings.

Aerodynamic-only and aeroelastic optimizations have been conducted and a specific tool, InAirSsi, for structural modeling has been developed. This tool is an input-output system that provides the structural model for a given cfd mesh, analyses material stresses, estimates the structural weight, and supplies the aerostructural adjoint solver with the necessary sensitivities, so that at each iteration of the optimizer, the flexibility of the wing is taken into account in objective function gradient computation, the structure is tested to resist limit loads and the weight is estimated. A preliminary design of the primary structure has been completed with InAirSsi.

The near-future work consists of performing an aerostructural optimization of the weighted sum of drag coefficient and structural weight in order to analyse the effect of wing stiffness on both structures and aerodynamic performance.

```
#####
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##### * * (o) * * #####

#structural design parameters
CS 7
input CONTROL_SECTIONS 3.5 6.02 8.39 10. 15.02 21.66 28.49

input SKIN_EXTRADOS_THICKNESS 0.005 0.004 0.006 0.005 0.005 0.005 0.005 0.005
input SKIN_INTRADOS_THICKNESS 0.004 0.004 0.006 0.005 0.005 0.005 0.005 0.005
input CAPS_INTRADOS_THICKNESS 0.03 0.03 0.035 0.025 0.025 0.03 0.03
input CAPS_EXTRADOS_THICKNESS 0.03 0.03 0.035 0.025 0.025 0.03 0.03
input FRONT_SPAR_THICKNESS 0.02 0.02 0.02 0.02 0.02 0.02 0.02
input REAR_SPAR_THICKNESS 0.02 0.02 0.02 0.02 0.02 0.02 0.02

#####
##Case data##

parameter CARD_PARAM 12 # alpha_geom+alpha_struct
parameter SKIN_MESH SKIN-XRF1-INI.dat
parameter BEAM_NODES 21

#####
## Aircraft Case ##
parameter MIN_SPAN 3.4
parameter MAX_SPAN 28.8
parameter FRONT_SPAR_PLACEMENT 0.25 #in chord ratio
parameter REAR_SPAR_PLACEMENT 0.75 #in chord ratio
parameter REF-SURF 184.41
parameter MTWOW 250000
#parameter 2nd_STRUCT law

#####
##Material characteristics##
parameter YOUNG_MODULUS 75 #expressed in GPa
parameter SHEAR_MODULUS 27 #expressed in GPa
parameter DENSITY 2800 #expressed in kg/m3
parameter SIGMA_LIMIT 500 #expressed in MPa
parameter TAU_LIMIT 2800 #expressed in MPa
parameter SECU 1.64
```

FIGURE 15: Input sytem of InAirSsi

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