Speed Based Optimal Power Division in Small Cell Networks

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Abstract

Small cell networks promise to boost the capacity and provide good QoS (Quality of Service) to cell edge users. However they pose serious challenges, especially in the case of high speed unidirectional (e.g., users traveling on highways or in metros) users, in the form of frequent handovers. The extent of the handover losses depends greatly upon the speed of the user and we showed in our previous paper that optimal cell size increases with speed. Thus in scenarios with diverse users (speeds spanning over large ranges), it would be inefficient to serve all of them using common cell radius and it is practically infeasible to design different cell sizes for different speeds. We alternatively propose to allocate power to an user based on its speed. Higher power virtually increases the cell size. We provide an optimal power division, optimal for busy probability (which optimizes an appropriate QoS that considers handover losses), for any given power budget and cell size. We further establish the need for higher power budgets for larger cell sizes, which we called as beta+ power scaling (beta is path-loss factor). We finally obtain optimal cell size, after beta+ scaling and optimal power division.

I. Introduction

Short radius cells appear as a promising solution to boost the capacity of mobile networks and to respond to the increasing demand for video oriented bandwidth demanding services (see [3], [4]), even for cell edge users. One can serve the users at higher data rates (resulting in smaller service times for the same information exchange) while operating at reasonable transmission power. However, operating with short radius cells (Small Cells (SC) or Pico Cells (PC)) raises major issues in terms of mobility management. Indeed, as the cell size decreases, the handover (HO) rate critically increases especially for high or medium speed users which results in un-acceptable rate of call drops: the frequency of handovers (HOs) increases and the risk for insufficient resources/unavailable servers gets higher. In these conditions, the calls drop before completion of the service with higher probabilities.

In [5], we studied the above mentioned trade-offs (associated with mobile users and HOs) and obtained a cell size that is optimal for appropriate metrics, like expected service time, drop or busy probability etc. These optimal cell sizes are derived as a function of various parameters like, user velocity profile, wireless medium properties (in terms of propagation co-efficient), the transmission power etc. One of the important conclusions of the study is, optimal cell size for larger velocities is larger: a) with increase in speed, user crosses a cell faster resulting in a smaller amount of information communicated per cell; b) amount of information required for HO remains the same; c) thus percentage of the time for useful information transfer, decreases with increase in user speed, for the same cell size; d) thus the optimal cell size increases with increase in user speed.

Based on the results in [5], it is suggested in [2] to also boost the power for high or medium speed users. In this paper, we work further on this idea. In [5] we actually show that optimal cell size increases with user speed. Thus one can achieve good performance if cell sizes are varied based on speed. However, it would be a practically infeasible procedure to vary the physical cell size based on user speed. We instead propose to vary the power based on the speed. The higher power virtually increases the cell size: a user at a far away point with higher transmit power can be served at the capacity of a nearby point with smaller power. In this paper, we divide the available power among the various user speed classes according to an optimization criterion (that trades-off between better service rates and frequent HOs), when the same cell size is used for all the users. We obtain a closed form expression for the optimal division. By this division: higher the speed, higher the power allocated. Power allocated to higher speeds increases with path-loss factor and cell size.

If users travel with probability p_i in speed class i and are allocated power P_i, then the average power equals $\sum_i p_i P_i$. The power budget (per cell) specifies the maximum value that this average power can take. To maintain power budget per unit cell dimension constant (which maintains the total power budget constant) one has to scale the power budget linearly with cell size (for one dimensional cells). However this linear scaling will not be sufficient, as will be shown. The distance based attenuation factor scales down by the path loss factor, beta. If the power budget is increased only linearly, the maximum possible rate (capacity) for a cell edge user decreases monotonically with increase in cell size. Thus with constant or linearly increasing (in cell size) power budget, the optimal cell size will always be trivial solution (loss less distance). One needs to scale the power budget by an index that is strictly greater than the path loss factor, beta, to ensure that the rates of the cell edge users either increase or remain the same with cell size, which further avoids the trivial solution. We call this as beta+ power scaling. We illustrate the need for beta+ scaling (see also [1]) and obtain the optimal cell size when the power is boosted according to

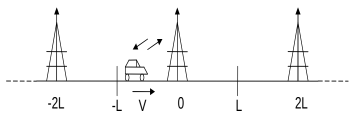

Fig. 1. User moving in a car while deriving service

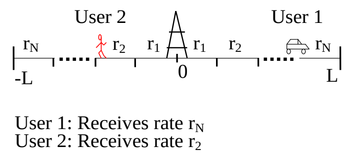

Fig. 2. A one dimensional cell, rate partitioning and user's movement

beta+ scaling and when it is divided among user speed classes according to optimal division.

The method and the tools used for the analysis in this paper could overlap with the ones used in our previous papers ([5] and [1]). But there are significant differences between the three papers. This paper and [1] consider a radically different situation to that in [5], that of finite number of transmission rates. We consider randomly wandering users in [1] while this paper proposes optimal power division among high speed uni-directional users.

II. System Model

We consider users moving in a fixed direction. The users are moving in one direction (in a one dimensional line $[-D, D]$) and at high speeds, which vary negligibly during the call. One such example (see figure 1) is when a user driving in a car derives its service from portable base stations (BSs) which are installed on street infrastructure (like lamp posts). The users can move in one of the two directions with equal probability, i.e., with half probability. We assume symmetry in both the directions and hence any performance (e.g., busy probability, drop probability etc.), conditioned on the direction of the user, will be equal for both the directions. Thus, unconditional performance would be the same as the performance given a direction, say left to right. *So, without loss of generality we consider only users moving from left to right.* The user moves with speed V, which is uniformly distributed. We assume uniform arrivals. In small cell networks, the (adjacent) BSs are reasonably close and hence one has to design carefully with sufficient reuse factor to avail the advantages of small BS separations. In this work, we indeed assume this is the case, i.e., we assume no interference from the other cells.

Rate Regions: The cell is divided into $2N$ disjoint segments (depending upon distance from BS[1]): users in a segment are served with the same transmission rate. Let $\{\mathbb{A}_n\}_{n\in\mathbb{N}}$ represent these (see figure 2)

$$\mathbb{A}_n := \left[\frac{(n-1)L}{N}, \frac{nL}{N}\right] 1_{\{n>0\}} + \left[\frac{nL}{N}, \frac{(n+1)L}{N}\right] 1_{\{n<0\}} \text{ and } \mathbb{N} \;:=\; \{-N, \cdots, -1, 1, \cdots, N\}. \quad (1)$$

Segments $\mathbb{A}_n, \mathbb{A}_{-n}$ placed at the same distance on either side of BS (figure 2) are served with common rate $r_{|n|}$ and these common rates decrease as the distance from BS increases. Let $\mathbb{R} := \{r_1, \ldots, r_N\}$

[1]In small cell networks (transmission at small distances), distance based propagation losses would be sufficient for deciding the theoretical rate limits as well as the practical transmission rates.

(decreasing set) represent the ensemble of all possible transmission rates. The service rate changes once the user switches from one region to another region.

Arrivals: There are two types of arrivals: 1) arrivals from external world (represented by subscript e and this is done only when there is ambiguity) modeled as Poisson arrivals with parameter λ; 2) handover (HO) arrivals (subscript h) modeled again as Poisson arrivals (see [6], [7]) is the sub-stream of external arrivals whose service is not completed in previous cell. Rate of arrivals into the cell of interest $[-L, L]$ depends upon the cell dimension L and this is shown by either λ_L (for external arrivals) or $\lambda_{h;L}$ (for HO arrivals). For external arrivals, we assume [2] $\lambda_L = \lambda L$ while $\lambda_{h;L}$ will be calculated in later sections. Every arrival, brings along with it Marks (Φ, S), where $\Phi \in \mathbb{N}$ is the position of arrival with distribution $\Pi := \{\pi_n\}$ and S, the number of bytes to be transmitted, is exponentially distributed, i.e., $S \sim \mu exp^{-\mu t} dt$ for some $\mu > 0$.

Resources: A cell can attend K parallel calls. The power per transmission, P, can depend upon L the cell dimension and this dependency will be discussed later.

An example of $\mathbb{R}$: One can *choose the set of possible transmission rates,* $\mathbb{R}$ *and* N *based on the practical channel coding schemes* that would be used in the network design. The analysis presented can be utilized to study a system with any given $\mathbb{R}$ and N. However, in this paper, we consider a specific example. This specific $\mathbb{R}$ is obtained using low SNR approximation of the following theoretical (capacity) rate[3]:

$$r(d) := P\left(1_{\{d \le d_0\}} + r_0 \, |d|^{-\beta} \, 1_{\{d > d_0\}}\right) \text{ with } r_0 = d_0^{\beta}.$$

Here $r(d)$ is the rate at distance d, d_0 is a small lossless distance[4] while β is the propagation co-efficient. We consider a specific system which supports transmission at the maximum possible rate for the entire region. For example in $\mathbb{A}_n$ the farthest user will be at distance $|n|L/N$ and hence maximal transmission rate, that can be allocated, equals

$$r_n = r(|n|L/N) = r_0 P N^{\beta} L^{-\beta} |n|^{-\beta}. \tag{2}$$

Remark: *Alternatively, if the system under consideration can design modulation and or channel coding schemes so as to achieve (almost)* ν *percent of the theoretical rates where* $\nu < 1$ *is a fixed coefficient, then again the above rate structure is applicable (after absorbing* ν *into* r_0 *of (2)).*

[2] If the arrivals in the entire line segment $[-D, D]$ occur at rate λ' those in segment $[-L, L]$ occur at a smaller rate $\lambda_L = \lambda' Prob$(arrival in $[-L, L]$). For the special case of uniform arrivals (i.e., arrivals landing uniformly in $[-D, D]$) $\lambda_L = \lambda L$.

[3] For unit noise variance, capacity equals $log(1 + SNR)$, where signal to noise ratio $SNR = PA$ and attenuation $A = 1_{\{d \le d_0\}} + (d/d_0)^{-\beta} 1_{\{d > d_0\}}$. For low SNRs, $log(1 + SNR) \approx SNR$ and hence capacity equals PA.

[4] Typically d_0 is small and in this paper **we consider optimizing over cell sizes** $L > d_0 N$ so that $r(d) = r_0 P d^{-\beta}$ always.

Handovers: When the user reaches the boundary $\{|\mathbf{x}| = L\}$ the call is handed over to the neighboring cell (if not completed and if free servers are available).

Information to initiate HO: Every new connection requires s_h extra bytes to be exchanged to initiate it. The effect of these bytes (on the system performance) for a new call will be negligible (as it would be once), however one needs to consider their effect on HO calls. These bytes are usually very small in proportion to the actual bytes to be transmitted, i.e., that $s_h \ll S$ with probability one. We also assume that s_h *bytes are exchanged with probability one (w.p.1), while user is in the last region* r_{-N}.

Notations: We denote the transpose by ^{t}. Calligraphic letters represent matrices. Mathbb letters represent sets (e.g., $\mathbb{N}$ - set of segment numbers $\mathbb{R}$ - set of all possible transmission rates, $\mathbb{A}_n$ - rate region n). The contents inside two flower brackets represent either a set or an ordered tuple (as according to convenience): for example $\{r_n\}$ represents the set $\mathbb{R}$ while $\{\pi_n\}$ represents the ordered tuple Π. Lower case letters represent time index (k) or the segment index (n). Lower case bold letters represent the vectors.

Upper case letters either represent system parameters (e.g., D - dimension of Macrocell, L - dimension of small cell, P - Power per transmission, K - Number of servers, N - Total number of possible transmission rates (number of elements in $\mathbb{R}$) , $\Pi = \{\pi_n\}_n = \{Prob($ Arrival in segment $n)\}_n$ - Vector of arrival probabilities etc.) or represent random variables (W - wandering time, S - number of bytes to be transferred, Φ - the segment in which the user is wandering etc.).

When any of the above have to be indexed by n or k and further the dependency on parameter L has to be shown, then we use notation like $\pi_{n;L}$, $W_{n;L}$, P_L, $\Phi_{k;L}$ etc.

This scenario is exactlysimilar to [5], but for one major difference. In [5], it is assumed that the rate of communication can be changed continually. This in some sense gives a "maximal" performance: if one can change rate of communication continually and that too, to the maximum possible one (i.e., capacity) then one obtains the best performance. But in reality this is not possible and we now consider similar situation but with maximum N different possible rates of communication as in the previous sections. This change has significant implications: one requires $\beta+$ power scaling for non trivial optimal cell size (Nd_0) (see also [1]).

III. High Speed users (cars) moving in one direction: Analysis

A. Derivation of appropriate user speed dependent Performance measures

We consider cell 0 $[-L, L]$, without loss of generality and also the users moving from left to right. Let ψ_n represent the probability that a call originated in rate segment n, $\mathbb{A}_n$, is completed before reaching boundary $\{L\}$. The user is serviced at rate $r_{|n|}$ and hence a total of $(L/N)(1/V)r_{|n|}$ bytes are transfered before he crosses $\mathbb{A}_n$ and then at rate $r_{|n+1|}$ and so on till he reaches the boundary point L. Total number of bytes transfered during this transit equal $\sum_{m=n}^{N} r_m L/(NV)$ and hence the user will complete its call (transfer of S bytes of information) before leaving the cell with probability:

$$\psi_n = Prob\left(S < \frac{L}{NV}\sum_{m\geq n}^{N} r_{|m|}\right) = 1 - E\left[e^{-\frac{\mu L \sum_{m=n}^{N} r_{|m|}}{NV}}\right] \text{ as } S \text{ is exponential.}$$

It is difficult to obtain Laplace transform and hence ψ_n, for uniform velocities. However users are moving at high speeds (which are more or less constant)and the speeds are partitioned to different classes. We assume: $V \sim \mathcal{U}([V_{min}, V_{max}])$, with V_{max} close to V_{min} and both away from 0. Then one can approximate,

$$\psi_n \approx \frac{\mu L \sum_{m=n}^{N} r_m}{N} E[1/V] \text{ and } 1 - \psi_n \approx 1$$

Let $P_{e,ho}$ ($P_{h,ho}$) represent the probability that a new (HO) call is handed over (again) to the neighboring cell. Recall here that we are modeling the HOs also as Poisson arrivals. A new call can arrive in any $\mathbb{A}_n$ with probability π_n while an handed over call always occurs at $-L$, i.e., in rate region $\mathbb{A}_{-N}$ (note we consider only calls from left to right without loss of generality). Then by unconditioning (w.r.t. the event of arrival being in $\mathbb{A}_n$),

$$\begin{aligned} P_{e,ho} &= 1 - \sum_{n=-N}^{N} \pi_n \psi_n = 1 - PL^{1-\beta}E[1/V]C_{e,ho} \text{ with } C_{e,ho} := \frac{\mu}{N} r_0 N^{\beta} \sum_{n=-N}^{N} \pi_n \sum_{m=n}^{N} |m|^{-\beta} \\ P_{h,ho} &= 1 - \psi_{-N} = 1 - PL^{1-\beta}E[1/V]C_{h,ho} \text{ with } C_{h,ho} := \frac{\mu}{N} r_0 N^{\beta} \sum_{n=-N}^{N} |n|^{-\beta}. \end{aligned} \quad (3)$$

Extra HO bytes s_h: So far, in the analysis we did not consider the effect of the extra bytes required for HO s_h, which only impact a HO call. In the following, we will modify the expressions derived so far (only for the ones specific to HO calls), after considering the effect of these s_h HO bytes. The time taken for exchange of HO bytes s_h has to be included in the time of the cell utilized by the user and hence the (HO) service time b_h (which by approximation is the time taken to traverse the entire cell) does not change with s_h. However $P_{h,ho}$, the probability of a HO call completing the call before reaching the (opposite) boundary, changes to the following

$$\psi_{-N,h} = Prob\left(S + s_h < \frac{L}{NV}\sum_{m=-N}^{N} r_m\right) \approx (C_{h,ho}PL^{1-\beta}E[1/V] - \mu s_h) = 1 - P_{h,ho}. \quad (4)$$

Limit Velocity: Before we proceed further, we would like to note down an important result regarding the limit of the speeds that can be supported, which is a counter part of the [5, Theorem 2] for a system supporting finite number (N) of transmission rates. A user entering at $-L$ when moving with speed V, can transfer in a cell, at maximum

$$g^N(L) := \frac{L}{NV}\sum_{m=-N}^{N} r_m = C_{h,ho}\frac{PL^{1-\beta}}{\mu V}, \quad (5)$$

bytes of information, out of which s_h are used for HO purposes. So, useful communication is possible (and $\psi_{-N,h} > 0$, note $g_L^N = C_{h,ho} P L^{1-\beta}/(\mu V)$) only when $g^N(L) > s_h$ with probability 1. With $\beta > 1$ (the practical range of path loss factors), $g^N(L)$ reduces with L and so useful communication is not possible for any cell size if $g^N(Nd_0)$ itself is less than s_h and hence we have:

Theorem 1: When $\beta > 1$, there exists a limit on the maximum velocity that can be supported by the system for a given power P(substituting r_m from (2)):

$$V_{lim}(P) := \frac{1}{s_h} r_0 \sum_{n=-N}^{N} |n|^{-\beta} P d_0^{1-\beta}. \ \square$$

Comparison between systems with finite and infinite $\mathbb{R}$: Theorem 1 is equivalent of the limit result derived for a system supporting infinite number (continuum) of 'maximal' transmission rates, [5, Theorem 2]. In [5], we considered a system which can change the transmission rate instantaneously (with the distance between the user and the serving BS) to the maximum possible one (capacity). We derived the total number of bytes that can be communicated while the user moving at speed V traverses one entire cell (see expression for $g(L)$ as given by equation (3) in [5] with $\beta > 1$). This quantity determines the system performance to a great extent and is reproduced here ([5, equation (3)]):

$$g(L) = \frac{2Pd_0}{V(\beta-1)} \left(\beta - \left(\frac{L}{2d_0} \right)^{1-\beta} \right).$$

One can see that $g(L)$ increases with L and finally saturates to $2Pd_0\beta/(V(\beta-1))$. On the other hand, in contrast, for the system considered in this paper with finite set of transmission rates ($\mathbb{R}$, with $|\mathbb{R}| = N$), the corresponding number (of bytes transfered in a cell) is given by $g^N(L)$ defined in (5). For this finite system $g^N(L)$ actually reduces with L. This is because when the number of possible transmission rates is fixed at N, even with the 'best'[5] rate allocations given by (2), the serving (transmission) rate at any point in the cell reduces as L increases. *Thus with limited resources (limited number of feasible transmission rates) the system behavior changes drastically* . One can achieve a system close to that in [5] if N increases appropriately with L.

Because of these effects resulting from the limitation of resources, one would need the transmit power P to the scaled (with L) by a proportionality factor greater than β (the beta+ power scaling mentioned in the Introduction). The beta+ scaling is discussed at the end of this subsection after deriving the appropriate system metrics.

Expected Service time: the amount of time for which a user is served in a cell. Let b_n represent the average amount of time for which service is received in a cell, given that the call is originated in region

[5]In (2), we divided the cell into equal area regions and assigned each region with the best the possible transmission rate, the 'maximal' rate (capacity) of the farthest user of the region.

n. When high speed users travel in small cells, with high probability, a call is not completed in one cell and hence one can approximate b_n with the time taken to reach the boundary:

$$b_n = \frac{L}{N}(N-n)\, E\left[\frac{1}{V}\right].$$

The service time of a new call (b_e) and that of a handed over call (b_h) on average equals (by unconditioning)[6]:

$$b_e = \sum_{i=-N}^{N} b_n \pi_n = C_{b,e} L E[1/V] \text{ and } b_h = b_{-N} = C_{b,h} L E[1/V] \text{ with } C_{b,e} := \sum_{n=-N}^{N} \pi_n \frac{N-n}{N}, \; C_{b,h} := 2.$$

Non elastic traffic and relevant performance metrics: We now continue with the derivation of the performance metrics that capture the trade off between HOs losses and the better service rates. In this work, we consider non elastic traffic. Non elastic traffic comprises of users demanding immediate service. These users (e.g., voice calls) drop the call if it is not picked up immediately, i.e., if all the K servers are busy (see [1], [5]). The probability that a call is not picked up immediately is called the Busy probability and the probability that a call that was picked up is ever dropped before completing its service is called the Drop probability. Both these performance metrics depend upon HOs as well the average service times and hence captures the required trade-off. In this paper we work with metrics proportional to P_{Busy}. One can obtain P_{Drop} as in ([1], [5]) and we omit its discussion here. Performance metrics in this section are derived in a way similar to that in [5] and [1]. We briefly explain the derivations and more details can be found in the two papers.

HO Arrival rate: As a first step we obtain the rate, $\lambda_{h,L}$, at which HOs occur. Let λ_L represent the fractions of the calls that arrive in the cell of interest $[-L, L]$ which equals λL for uniform arrivals (for appropriate $\lambda > 0$). A fraction of the new arrivals as well as HO calls get converted (again) to HO calls. The calls that have not finished their service before reaching the boundary are exactly this fraction, whose value is given by $P_{e,ho}$ and $P_{h,ho}$ respectively for new arrivals and the calls that resulted from a HO. By memory less nature of S (the bytes to be transferred) we have a stochastic equivalence between the calls entering and leaving the cell (see [5], [1]). Using this, $\lambda_{h;L}$ satisfies the fixed point equation, $\lambda_{h;L} = \lambda_L P_{e,ho} + \lambda_{h;L} P_{h,ho}$. Hence, from equations (3), (4)

$$\lambda_{h;L} = \frac{\lambda_L P_{e,ho}}{1 - P_{h,ho}} = \lambda L \frac{1 - P L^{1-\beta} E[1/V] C_{e,ho}}{P L^{1-\beta} E[1/V] C_{h,ho} - \mu s_h}.$$

[6]The higher speed users are handed over more frequently than the lower speed ones. So, the distribution of the HO user velocities need not be uniform even if the new users arrive with uniform speeds. However as in [5, Appendix B], one can show that the HO user speed distribution converges to uniform distribution as the cell size reduces to zero.

Overall expected service time (considering HO and new arrivals) equals (see (6)),

$$\bar{b} = \frac{1}{\lambda_L + \lambda_{h,L}} \left(\lambda_L b_e + \lambda_{h,L} b_h \right).$$

Load factorThe product of the average service times and the arrival rate gives the rate at which the overall load is arriving into the system and the ratio of this product with the number of servers is called load factor, which is given by:

$$\rho = \frac{1}{K}(\lambda_L + \lambda_{h,L})\bar{b}.$$

Simplifying

$$\rho = \frac{1}{K}\left(\lambda_L b_e + \lambda_{h,L} b_h\right) = \frac{1}{K}\lambda L^2 E[1/V] \left(C_{b,e} + C_{b,h} \frac{1 - PL^{1-\beta} E[1/V] C_{e,ho}}{PL^{1-\beta} E[1/V] C_{h,ho} - \mu s_h} \right). \quad (6)$$

Busy probability: A small cell catering to non elastic traffic can be modeled by an M/G/K/K queue (as in [5]). Then using Erlang's loss formula the busy probability can be calculated as,

$$P_{Busy} = \frac{\rho^K / K!}{\sum_{k=0}^{K} \rho^k / k!}.$$

Busy probability, depends upon L only via ρ and both are differentiable in L (see [5] for similar details) and by differentiating twice one can immediately obtain the following:

Lemma 1: Optimizers of ρ and P_{Busy} are same, i.e.,

$$L^*_\rho := \min_L \rho = \min_L P_{Busy}(L) =: L^*_{P_{Busy}}. \quad \square$$

Because of Lemma 1, we hence forth work with the load factor ρ.

Beta+ power scaling: Let $C_{cell} := g^N(L)/L$ represent the 'capacity per cell', the maximum information that can be transfered in a cell normalized by cell size (see also [1]). From (5), $C_{cell} \propto PL^{-\beta}$ and reduces monotonically with L for constant P. The fundamental quantity C_{cell} will not decrease with L only if the power P_L (power P_L now on includes L in notations) scales by a proportionality factor greater than β, i.e., if $P_L \propto L^{\beta+\gamma}$ for some $\gamma > 0$, as then, $C_{cell} \propto P_L L^{-\beta} \propto L^\gamma$. *We call this as beta+ scaling.* To maintain the total power constant, one should scale[7] $P_L \propto L$. Practical values of β are above 2 and so *systems with larger cell sizes can perform better only if their total power budget is boosted (via beta+ scaling).* Performance metrics like drop or busy probability (or ρ) further degrade due to HO losses. In fact, with constant total power ($P_L \propto L$) from (6), $L^*_\rho = N d_0$. From (6) ρ depends on L via two terms

[7] There exists approximately D/L cells in the system and if one uses transmit power P_L for cell size L, then the total power used in the system is proportional to P_L/L.

L^2 and $-P_L L^{1-\beta}$, $-P_L L^{1-\beta}$ decreases with L for beta+ scaling and there would be a threshold γ_0[8] such that $L^*_\rho > N d_0$ (for all $\gamma > \gamma_0$).

B. Optimal power law for a given cell size L and power constraint $\bar{P}$:

As explained in the Introduction, based on the results of [5], we are mainly interested in dividing the users into different categories based on their speeds and then dividing the power across various user classes. From (6), load factor ρ depends upon the user speed profile only via $E[1/V]$. We consider I disjoint velocity classes, each of which is identified with its $E[1/V]$[9] in one of the I values $\{1/V_1, \cdots, 1/V_I\}$. Let p_i represent the probability that $E[1/V]$ equals $1/V_i$ and let P_i represent the transmit power allocated to users of speed class i.

The HO rates of the different user classes can be calculated as before and they equal:

$$\lambda_{h,L,i} = \lambda_{L,i} \frac{P_{e,ho,i}}{1 - P_{h,ho,i}} = \lambda L p_i \frac{1 - P_i L^{1-\beta} E[1/V_i] C_{e,ho}}{P_i L^{1-\beta} E[1/V_i] C_{h,ho} - \mu s_h}.$$

Similarly the expected service time for different user classes can be calculated and then the overall load factor ρ simplifies:

$$\rho(L, \mathbf{P}) = \lambda_L b_e + \sum_i \lambda_{h,L,i} b_{h,i} = \lambda L^2 \left(C_{b,e} \sum_i p_i E[1/V_i] + C_{b,h} \sum_i p_i E[1/V_i] \frac{1 - \delta_i C_{e,ho}}{\delta_i C_{h,ho} - \mu s_h} \right), \quad (7)$$

where $\delta_i := P_i E[1/V_i] L^{1-\beta}$ We are interested in power division $\mathbf{P} := \{P_1, \cdots, P_I\}$ which optimizes ρ while satisfying: $\sum_i p_i P_i \leq \bar{P}$. The average power constraint can be satisfied by substituting $P_I = \frac{1}{p_I}\left(\bar{P} - \sum_{i<I} p_i P_i\right)$ and then (with $\mathbf{P}_{-I} := (P_1, \cdots, P_{I-1})$)

$$\rho(L; \bar{P}) := \rho\left(L, \left(\mathbf{P}_{-I}, \frac{\bar{P} - \sum_{i<I} p_i P_i}{p_I}\right)\right) = \lambda L^2 \left(C_{b,e} \sum_i p_i E[1/V_i] + C_{b,h} \sum_{i<I} p_i E[1/V_i] \frac{1 - \delta_i C_{e,ho}}{\delta_i C_{h,ho} - \mu s_h} \right)$$
$$+ \lambda L^2 C_{b,h} p_I E[1/V_I] \frac{p_I - \left(\bar{P} - \sum_{i<I} p_i P_i\right) E[1/V_I] L^{1-\beta} C_{e,ho}}{\left(\bar{P} - \sum_{i<I} p_i P_i\right) E[1/V_I] L^{1-\beta} C_{h,ho} - p_I \mu s_h}.$$

We are now interested in the optimal power division (constraint $g^N_{L,i} = \delta_i C_{h,ho} > \mu s_h$ is needed for useful communication):

$$\mathbf{P}^*_{-I}(L; \bar{P}) = \arg \max_{\{\mathbf{P}_{-I} \in \mathbb{P}\}} \rho(L; \bar{P}) \text{ with } \mathbb{P}(\bar{P}, L) := \left\{ \mathbf{P}_{-I} : \min_{i \leq I} \delta_i > \mu s_h / C_{h,ho} \text{ with } P_I := \frac{\bar{P} - \sum_{i<I} p_i P_i}{p_I} \right\}.$$

[8]Load factor ρ increases with L via term L^2 at a slope 2 while it decreases with L via term $-P_L L^{1-\beta}$ at slope $1+\gamma$. As γ increases, the decrease slope $(1+\gamma)$ increases and eventually ρ would decrease with L.

[9]We in fact assume users in speed class i, travel with probability one at speed V_i, for simplifying explanations. One can easily generalize.

Clearly, $\mathbb{P}$ is non-empty[10] and the optimization will have a solution if the power constraint $\bar{P}$ is sufficiently large and we assume the same in this paper. The partial derivatives are (for all $i < I$):

$$\begin{aligned}\frac{d\rho}{dP_i} &= \lambda C_{b,h} p_i (E[1/V_i])^2 L^{3-\beta} \frac{C_{e,ho}\mu s_h - C_{h,ho}}{(P_i E[1/V_i] L^{1-\beta} C_{h,ho} - \mu s_h)^2} \\ &\qquad -\lambda C_{b,h} p_i (E[1/V_I])^2 L^{3-\beta} \frac{\mu s_h C_{e,ho} - C_{h,ho}}{(P_I E[1/V_I] L^{1-\beta} C_{h,ho} - \mu s_h)^2} \\ &= \lambda C_{b,h} p_i L^{3-\beta} \left(C_{e,ho}\mu s_h - C_{h,ho}\right) \left(\frac{(E[1/V_i])^2}{(P_i E[1/V_i] L^{1-\beta} C_{h,ho} - \mu s_h)^2} - \frac{(E[1/V_I])^2}{(P_I E[1/V_I] L^{1-\beta} C_{h,ho} - \mu s_h)^2} \right).\end{aligned}$$

Calculating the partial derivatives and equating them to zero, $\partial\rho/\partial P_i = 0$ for all u, the optimal power division $\mathbf{P}(L, \bar{P})$, for a given cell size and power constraint $\bar{P}$ equals:

$$\mathbf{P}^*(L) = \begin{bmatrix} -1 & 0 & 0 & \cdots & 0 & 1 \\ 0 & -1 & 0 & \cdots & 0 & 1 \\ & & \vdots & & & \\ 0 & 0 & 0 & \cdots & -1 & 1 \\ p_1 & p_2 & p_3 & \cdots & p_{I-1} & p_I \end{bmatrix}^{-1} \begin{bmatrix} \mu s_h \frac{E[1/V_1]-E[1/V_I]}{E[1/V_1]E[1/V_I]L^{1-\beta}Chho} \\ \mu s_h \frac{E[1/V_2]-E[1/V_I]}{E[1/V_2]E[1/V_I]L^{1-\beta}Chho} \\ \vdots \\ \mu s_h \frac{E[1/V_{I-1}]-E[1/V_I]}{E[1/V_{I-1}]E[1/V_I]L^{1-\beta}Chho} \\ \bar{P} \end{bmatrix} \quad (8)$$

$$= \begin{bmatrix} p_1 - 1 & p_2 & p_3 & \cdots & p_{I-1} & 1 \\ p_1 & p_2 - 1 & p_3 & \cdots & p_{I-1} & 1 \\ & & \vdots & & & \\ p_1 & p_2 & p_3 & \cdots & p_{I-1} - 1 & 1 \\ p_1 & p_2 & p_3 & \cdots & p_{I-1} & 1 \end{bmatrix} \begin{bmatrix} \mu s_h \frac{E[1/V_1]-E[1/V_I]}{E[1/V_1]E[1/V_I]L^{1-\beta}Chho} \\ \mu s_h \frac{E[1/V_2]-E[1/V_I]}{E[1/V_2]E[1/V_I]L^{1-\beta}Chho} \\ \vdots \\ \mu s_h \frac{E[1/V_{I-1}]-E[1/V_I]}{E[1/V_{I-1}]E[1/V_I]L^{1-\beta}Chho} \\ \bar{P} \end{bmatrix} \quad (9)$$

This can be solved easily and

$$P_i^*(L; \bar{P}) = \bar{P} + \frac{\mu s_h}{L^{1-\beta} C_{h,ho}} \sum_{j<I} p_j \left(\frac{1}{E[1/V_I]} - \frac{1}{E[1/V_j]} \right) - \frac{\mu s_h}{L^{1-\beta} C_{h,ho}} \left(\frac{1}{E[1/V_I]} - \frac{1}{E[1/V_i]} \right). \quad (10)$$

[10]The set $\mathbb{P}$ is inverse image of a continuous mapping on a connected set and it grows with increase in $\bar{P}$.

Assume the set $\{1/V_1, \cdots, 1/V_I\}$ is arranged in decreasing order and that I is the highest velocity class. Then one can easily obtain the following properties of the power division.From (10), allocated power increases with the speed (which in turn implies an increased virtual cell). Also, the disparity in allocated powers increases with cell size, the disparity in speeds and the path-loss factor.

C. Optimal cell size after using the optimal power law (10) and beta+ scaling

As already mentioned one needs a sufficiently large power constraint for a non empty domain of optimization, $\mathbb{P}$. Further, by beta+ scaling, we have a L dependent power constraint i.e., $\bar{P} = \tilde{P}L^{\beta+\gamma}$ for some $\tilde{P}, \gamma > 0$ and we assume $\tilde{P}$ is sufficiently large. The final goal of this paper is to obtain the optimal cell size when the users are allocated (optimal) transmission powers based on their speeds as in (10) and with beta+ scaling:

$$L^* = \arg \max_{\left\{L:\mathbb{P}(\tilde{P}L^{\beta+\gamma}, L)\neq\emptyset\right\}} \rho\left(L, \mathbf{P}^*(L; \tilde{P}L^{\beta+\gamma})\right).$$

However $\beta+$ scaling implies increasing total power used by the system (P_L/L) and hence we consider the following joint cost to obtain the optimal cell size (ω_p is the weight given to power):

$$L_g^* = \arg\max g(L) \quad \text{with} \quad g(L) := \rho\left(L, \mathbf{P}^*(L; \tilde{P}L^{\beta+\gamma})\right) + w_P\tilde{P}L^{\beta+\gamma-1}. \tag{11}$$

This is a complicated optimization and we obtain the optimal cell size via some numerical examples.

IV. Numerical examples

In Figure 3 we plot optimal power division as a function of β with settings as mentioned in the figure. Additionally, we have two speeds: $E[1/V] \in [.067 \quad .011]$ with $(p_1, p_2) = (0.2, 0.8)$. As evident from (10), we notice: the more the losses (path-loss factor) are, the more diverse the two allocated powers are.

We next consider another example in Figure 4 where we obtain optimal cell size after $\beta+$ scaling and optimal power division. We consider two user speeds ($I = 2$) and fix the lower speed at $E[1/V_1] = 1/15$ while vary the second one. The settings are same as before (except for $\omega_p = 0.005$ and $\tilde{P} = 0.0005$) and we plot the optimal cell size for both total cost (L_g^* given in (11)) as well as ρ (L_ρ^* when $\omega_p = 0$ in g) as a function of $V_2 := 1/E[1/V_2]$. We also plot the power budget per cell size ($\tilde{P}L^{\beta+\gamma}$), ρ at the optimizers. We observe that: 1) optimal cell size (and hence power budget) increases with velocity; 2) performance (ρ) degrades with increase in velocity.

Conclusions

High speed users pose serious design problems for small cell networks in the form of frequent handovers. Further, the optimal cell (dimension) design is sensitive to the speed of the user. Alternately,

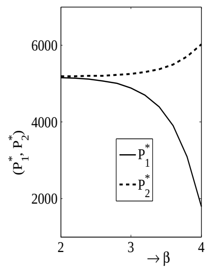

Fig. 3. Optimal power division ($\bar{P} = 0.0001$, $L = 160$)

Common Settings

$N = 3$ $d_0 = 5$

$r_0 = .4d_0^{\beta}$ $\gamma = 0.5$

$K = 500$

$\lambda = 10^{-7}$ $s_h = 0.2$

$\mu = 0.000001$

$\omega_p = 0.0002$

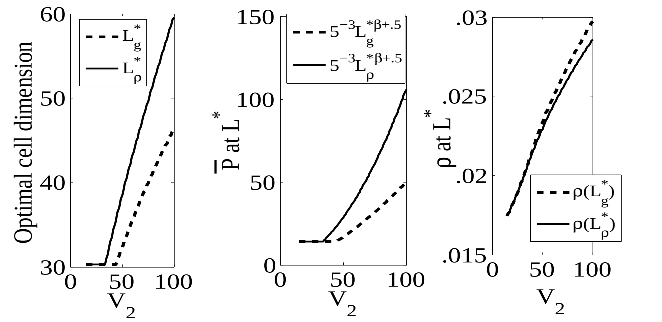

Fig. 4. Optimal cell size, performance and power budget versus V_2

we propose to reflect the speed based (optimal) design variations in the allocated powers. We obtain an optimal power division (closed form expression) among various user speed classes, that optimally trades-off between the good service rates and the frequent handovers, for any given average power constraint and the cell size. The optimal division ensures larger power to higher speeds and the differences in the powers allocated increase with path loss factor and the disparities in the speeds. We also establish that, for larger cells to work more efficiently, one has to boost the total transmit power used in the system. We showed that this boosting depends upon the path loss factor beta and called it as beta+ scaling. We finally propose a joint cost to obtain optimal cell size, when the network employs optimal power division as well as beta+ scaling.

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