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# Impact of Reputation-Sensitive Users and Competition Between ISPs on the Net Neutrality Debate

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## Abstract

Network neutrality is the topic of a vivid and very sensitive debate, in both the telecommunication and political worlds, because of its potential impact in everyday life. That debate has been raised by Internet Service Providers (ISPs), complaining that content providers (CPs) congest the network with insufficient monetary compensation, and threatening to impose side payments to CPs in order to support their infrastructure costs. While there have been many studies discussing the advantages and drawbacks of neutrality, there is no game-theoretical work dealing with the observable situation of competitive ISPs in front of a (quasi-)monopolistic CP. Though, this is a typical situation that is condemned by ISPs, and, according to them, another reason of the non-neutrality need. We develop and analyze here a model describing the relations between two competitive ISPs and a single CP, played as a three-level game corresponding to three different time scales. At the largest time scale, side payments (if any) are determined. At a smaller time scale, ISPs decide their (flat-rate) subscription fee (toward users), then the CP chooses the (flat-rate) price to charge users. Users finally select their ISP (if any) using a price-based discrete choice model, and decide whether to also subscribe to the CP service. The game is analyzed by backward induction. As a conclusion, we obtain among other things that non-neutrality may be beneficial to the CP, and not necessarily to ISPs, unless the side payments

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are decided by ISPs (through a non-cooperative game).

*Keywords:* Network neutrality, Game theory, Pricing

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## 1. Introduction

There has recently been a strong debate around the so-called *network neutrality*. The debate has been ignited by the increasing traffic asymmetry between Internet Service Providers (ISPs), mainly due to some prominent and resource consuming content providers (CPs) which are usually connected to a single ISP. The typical example is YouTube (owned by Google), accessed by all users while hosted by a single Tier 1 ISP, and whose traffic now constitutes a non-negligible part of the whole Internet traffic. Another example is the subscription-based video service Netflix, that is in the US the most bandwidth-consuming source of traffic, representing 23.3% of the total Internet traffic in late 2011 [1], while having commercial relationships with only one ISP. For those reasons, there has been a surge of protest among ISPs, complaining that the current Internet business model where ISPs charge both end-users and content providers directly connected to them, and have public peering or transit agreements with other ISPs, is not relevant anymore. The main solution proposed is that ISPs should also charge content providers that are associated with other ISPs [2], as first advocated by Ed Whitacre (CEO of AT&T) at the end of 2005 [3].

The underlying concern is that investment is made by ISPs but content providers get an important part of the dividends. The revenue arising from on-line advertising (meaning showing graphical ads on regular web pages) is estimated at approximately a \$24 billion in 2009 [4], while textual ads on search pages has led to a combined revenue of \$8.5 billion in 2007 [5], those figures increasing every year. Meanwhile, transit prices - which constitute the main source of revenues for transit ISPs - are decreasing. ISPs argue that there is no sufficient incentive for them to continue to invest on the network infrastructures if most benefits go to content providers. The threat is to lower the quality of service of CPs that do not pay any fee to them, or even to block their traffic. This possibility has led to protests from CPs and user associations, complaining that this might impact the network development and is an impingement of freedom of speech [3]. The debate was thus launched, essentially at the law and policy makers level, to decide whether the Internet should be *neutral*, i.e., all packets should receive equal treatments in terms of

price and service. In the US, the Federal Trade Commission (FTC) released in 2007 a report not supporting neutrality constraints, increasing the debate at the political level. This debate is also active in Europe and in France, as illustrated by the open consultation on network neutrality launched in 2010. For instance, the French regulation authority, ARCEP, has published in its response a proposal intending to define how net neutrality could be implemented [6, 7].

There has been an increasing attention in the literature on providing a mathematical analysis of the advantages and drawbacks of network neutrality. The idea is to investigate the output of the interactions between selfish actors that are end users, CPs and ISPs, using the framework of non-cooperative game theory [8, 9]. Let us briefly describe here, non exhaustively, some important existing works in that direction. In [10, 11], the authors propose to share the revenue among providers using the Shapley value, the only mechanism that satisfies a set of axioms representing a sense of fairness; in this case CPs participate to the network access cost. The work in [12] analyzes how neutrality or non-neutrality affects provider investment incentives, network quality, and user prices. A similar comparison is made in [13] between a two-sided pricing scheme where ISPs are allowed to charge CPs, and one-sided pricing where such side-payments are not allowed. In each case, at the equilibrium of the game, the levels of investment in content and architecture are determined. The paper gives conditions on the ratio between parameters characterizing advertising rates and end-user price sensitivity, under which a non-neutral network outperforms a neutral one in terms of social welfare. On the other hand, [14] investigates the case where ISPs negotiate joint investment contracts with a CP in order to enhance the quality of service and increase industry profits. It is found that an unregulated regime leads to higher quality investments, but that ISPs have an incentive to degrade content quality. The paper [15] studies the implications of non-neutral behaviors, taking into account advertising revenues and considering both cooperative and non-cooperative scenarios. In [16], we analyze and compare thanks to game-theoretic tools three different situations of interactions *between* ISPs: the case of peering between the ISPs, the case where ISPs do not share their traffic (exclusivity arrangements), and the case where they fix a transfer price per unit of volume. The paper supports the transit price scenario and suggests a limited regulation (enforcing global connectivity) to prevent incumbent ISPs from having a dominant position in the bargaining. Finally, in [17], a game-theoretic model is considered with a single CP, a sin-

gle ISP, a (consumers') demand function that depends on price and quality of service, and involving advertisement and network investment components.

In those works, there is in general a single ISP, and one or several CPs. Though, in practice, we often have ISPs in competition for customers, while for many services, the CPs are in a quasi monopoly, a characteristic ISPs complain about. (Typical examples are YouTube for non-copyrighted videos, and Netflix for movies and TV shows in the US.) We propose to specifically address this issue in this paper. Remark that in addition to [13], considering competitive ISPs has been proposed in [18], but with competition over consumers, quality and prices for heterogeneous CPs: none of those works consider a monopolistic CP as can be encountered for some applications. In our conference version of the paper [19], whose model was inspired by [15], users were assumed to always go with the cheapest provider. As a consequence, we ended up with a price war (a classical Bertrand competition) such that ISPs always decrease their subscription price in order to attract all demand.

We consider here a more realistic user association model such that users make their choice still based on the price of ISPs, but also on other unknown considerations, hence the use of a classical discrete choice model as in [16]. This requires a derivation of results totally different from [19]. To analyze that situation, we propose a multi-level game where decisions are taken at different time scales. The solutions of the games at the largest time scales, played first, are determined using *backward induction*, meaning that players *anticipate* the impact on, and the resulting solution of, the games played later on at smaller time scales.

The paper is organized as follows. Section 2 presents the basic assumptions of the model we are going to consider, the different levels of game, and the mathematical description of the investigated comparison between the neutral and the non-neutral regimes. It also describes how users select their ISP (if any), and how the aggregated demand at the CP is determined. The next sections present the various game levels for providers' decisions: we describe in Section 3 how the CP, anticipating the decisions of end users, chooses the content price. At a higher level, by backward induction, ISPs play a game on the access charge for end users; this competition is described in Section 4. We then describe the game at the highest level, on the economic relationships between the ISPs and the CP, by determining the side payments of the CP to the ISPs in Section 5. We address the case when those prices are fixed by ISPs, based on a game; we also look at the case

when they are decided by the CP, or by a regulator (maximizing the supply chain value for instance). Section 6 concludes by discussing the impact and relevance of side payments on the providers' revenues, highlighting that it is not always in the interest of the ISPs (but could be), while an appropriate choice of side payments may increase the CP revenue. We also give in that section directions for future research.

## 2. Model

### 2.1. Topology and Pricing Structure

We consider a single CP, whose parameters will be indexed by 1, and two ISPs, named (and indexed by)  $A$  and  $B$ . The access prices charged to users are *flat rate* subscription fees, denoted by  $p_1$ ,  $p_A$  and  $p_B$  for respectively the CP, ISP  $A$  and ISP  $B$ . In order to study non-neutrality, we also introduce side payments  $q_A$  and  $q_B$  representing the per unit of volume prices that the CP has to pay to  $A$  and  $B$ , respectively. All prices are assumed to be positive. Finally, the set of end users is considered continuous and (without loss of generality) of mass one, so that we will indifferently refer to “mass” and “proportion” of users. The charges imposed by actors to other players are summarized in Figure 1, the arrows indicating the cash flows.

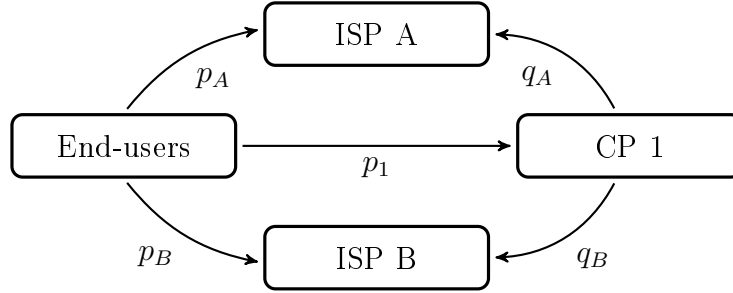


Figure 1: Charging interactions between stakeholders. Prices  $p_1$ ,  $p_A$  and  $p_B$  are positive flat rates, whereas  $q_A$  and  $q_B$  are positive per volume unit prices.

### 2.2. User Demand

Users have to pay both the ISP and the CP to access the content. We assume that without pricing, the global user demand, in terms of volume, for CP data (shared between ISPs  $A$  and  $B$ ) during a unit of time would be

$D_0$ . This corresponds to the case where all users access the content for free, and  $D_0$  can be seen as the average amount of data that a user downloads from the CP. In the general case, the global demand for CP content comes from users subscribing to an ISP. Since users need an Internet access, not only to reach the content of the CP, but also for other purposes (e-mail, web browsing, ...), we de-correlate the ISP choice from the (individual) decision to subscribe to the CP in addition.

Let us focus first on the ISP selection by users. In a previous work [19], we have considered users simply selecting the cheapest ISP (or choosing it randomly if price equality holds). However, this does not take into account the phenomenon of *stickiness* or *loyalty* of the users, proposed in [20]. In the model considered here, user choices are influenced by the ISP subscription prices, but also by other considerations (reputation or preferences) that can be modeled as an additive noise to the main criteria determining the choice. Mathematically, we assume that a user has a valuation of the form  $v_i = \beta \log(1/p_i) + \kappa_i$  for ISP  $i$  (see [16] for details). The case  $p_i = 0$  leads to the maximal possible valuation, independently of the random term, meaning that ISP  $i$  attracts all users, or (say, from symmetry) half of them if the other ISP chooses a null price as well. The parameter  $\beta$  represents the *user sensitivity* to the subscription prices: values of  $\beta$  close to zero lead to a uniform choice over the three alternatives (connect to one of the two ISPs, or not having access to the Internet) regardless of the prices set, whereas large values of  $\beta$  make the users choose the least expensive option. The term  $\log(1/p_i)$  expresses the dissatisfaction for higher prices, the logarithm being used to represent the fact that the same variation of price is felt smaller at high prices than at low prices: users are sensitive to relative price variations rather than absolute ones. Finally,  $\kappa_i$  is an individual-specific random term, taking into account unknown aspects and assumed to follow a Gumbel distribution as in standard discrete choice models [21]. We additionally assume that there is a fictitious price  $p_0$ , assumed to be strictly positive, and representing the cost of the outside option, i.e., the perceived cost of not having access to the Internet. Thus if the (random) valuation associated to that outside option is larger than the ones associated to each ISP, the user prefers not to join the network.

At a macroscopic level, by discrete choice analysis, the proportion (or equivalently, the mass)  $\sigma_i$  of users selecting ISP  $i$  (with also  $j \in \{A, B\}$ ;

$j \neq i$ ), equals

$$\sigma_i = \begin{cases} \frac{p_i^{-\beta}}{p_A^{-\beta} + p_B^{-\beta} + p_0^{-\beta}} & \text{if } p_A > 0 \text{ and } p_B > 0 \\ 1 & \text{if } p_i = 0 \text{ and } p_j > 0 \\ 1/2 & \text{if } p_A = 0 \text{ and } p_B = 0 \\ 0 & \text{if } p_i > 0 \text{ and } p_j = 0. \end{cases} \quad (1)$$

This repartition function expresses the fact that all users select an ISP if at least one of the subscription prices is null ( $p_A = 0$  or  $p_B = 0$ ).

In this paper, we propose a new aggregated user demand in terms of data volume on ISP  $i$ , where users having selected an ISP then decide whether to use the content offered by the CP, depending on the unit price  $p_1$ . We consider that users' willingness-to-pay for the CP service follows an exponential distribution, independent of the ISP choice. Such a distribution reflects the high heterogeneity of preferences and willingness-to-pay among the population. As a result, the proportion of users subscribing simultaneously to the CP and to ISP  $i$  is  $\sigma_i e^{-\alpha p_1}$ , and the corresponding data volume is given by:

$$D_i = D_0 \sigma_i e^{-\alpha p_1}. \quad (2)$$

The parameter  $\alpha > 0$  represents the sensitivity of users to the CP price: the global demand (sum of demands on each ISP) is a decreasing function of  $\alpha$ .

Notice that demand does not directly depend on the side payments  $q_A$  and  $q_B$ . But the introduction of side payments will induce a reaction on the prices  $p_A$ ,  $p_B$  and  $p_1$  set by ISPs and the CP at equilibrium, which, in turn, indirectly affects demand. Finally, the global volume demand for CP data  $D_A + D_B$  equals  $(\sigma_A + \sigma_B) D_0 e^{-\alpha p_1}$ .

### 2.3. Utility and revenue functions

Among the proportion  $\sigma_A + \sigma_B$  of users having accepted to pay for an access to the network, and then paying a flat-rate price  $p_1$  to the CP, some would have accepted to pay more to benefit from the content of the CP. The surplus of users that would have accepted to pay  $p$  is  $p - p_1$ , while the proportion of users willing to pay more than  $p$  is  $e^{-\alpha p_1}$ .

We can then compute the *user welfare* associated to the existence of the CP, as the sum over all users of the benefit they make accessing the content of the CP. Note that this does not include the benefit that users make by



selecting an ISP, which is associated to other (free) on-line services. The user welfare due to the CP can be computed as:

$$\begin{aligned}
\text{UW}_{\text{CP}} &= (\sigma_A + \sigma_B) \int_{p_1}^{\infty} - \left( \frac{\partial}{\partial x} e^{-\alpha x} \right) (x - p_1) dx \\
&= (\sigma_A + \sigma_B) \frac{e^{-\alpha p_1}}{\alpha} \\
&= \frac{D_A + D_B}{\alpha D_0}.
\end{aligned} \tag{3}$$

The utilities (revenues) of the ISPs come from the end users subscription fee, and from the CP through the possible side payment. The first one depends on the mass of users with the ISP, and the second one on the total amount of volume downloaded by users. Hence, for ISP  $i$  ( $i \in \{A, B\}$ ), the revenue is

$$U_i = p_i \sigma_i + q_i D_i. \tag{4}$$

Remark here that the revenue is always positive since we do not consider the cost of the network.

The utility of the CP in this model is the sum of revenues gained by users subscribing through  $A$  and through  $B$ . It is thus given by

$$U_1 = (p_1/D_0 - q_A)D_A + (p_1/D_0 - q_B)D_B. \tag{5}$$

Since  $p_1$  is decided after  $q_A$  and  $q_B$ , the CP can also ensure a positive revenue by setting  $p_1/D_0 \geq \max(q_A, q_B)$ .

#### 2.4. Multi-stage Decision Problem

The decision variables are the prices  $p_1, p_A, p_B, q_A, q_B$ , impacting end users (demand), as well as revenues of providers. Those variables are decided at different time scales or levels, that can be described as follows.

1. At the largest time scale, the side payments  $q_A$  and  $q_B$  are decided. In the neutral case, they are either fixed to 0, or determined as a common value. They can be different in the non-neutral case, and can be determined either by the ISPs (in a game), the CP, or a regulator. All those options will be investigated. Those determinations will be obtained anticipating the solution of the games below whatever the values of  $q_A$  and  $q_B$  (the so-called *backward induction*).

2. At a smaller time scale, for fixed values of  $q_A$  and  $q_B$ , the ISPs fix their prices  $p_A$  and  $p_B$  during a non-cooperative game to attract customers and maximize their revenues. Here again, the decisions are made anticipating the solutions at lower levels.
3. At an even smaller time scale, the CP sets the price  $p_1$ .

Finally, for those fixed values of  $p_1, p_A, p_B, q_A, q_B$ , users choose their ISP (if not too expensive), and decide whether to use the service offered by the CP, as described by formulas (1) and (2).

All those interacting levels are now solved by backward induction, from the smallest time scale to the largest one.

### 3. Content provider price determination

The CP aims at maximizing his revenue  $U_1$ , for fixed values of  $p_A, p_B, q_A, q_B$ , making use of what the total user demand  $D_A + D_B$ , with  $D_i$  given by (2), will be. For convenience, we define  $P_i := p_i^\beta$ .

**Proposition 1.** *Given the side payments  $q_A$  and  $q_B$  and the prices  $p_A$  and  $p_B$  decided by the ISPs, the price of the CP maximizing its revenue (4) is*

$$p_1^* = \begin{cases} \frac{P_A}{P_A + P_B}(D_0 q_B + \frac{1}{\alpha}) + \frac{P_B}{P_A + P_B}(D_0 q_A + \frac{1}{\alpha}) & \text{if } p_A > 0 \text{ or } p_B > 0 \\ D_0 \frac{q_A + q_B}{2} + \frac{1}{\alpha} & \text{if } p_A = 0 \text{ and } p_B = 0 \end{cases} \quad (6)$$

*Proof.* We first consider the case  $p_A > 0$  and  $p_B > 0$ . The derivative of the CP revenue is then

$$\frac{\partial U_1}{\partial p_1} = P_0 e^{-\alpha p_1} \frac{P_A(\alpha q_B + 1/D_0 - \alpha p_1/D_0) + P_B(\alpha q_A + 1/D_0 - \alpha p_1/D_0)}{P_0 P_A + P_0 P_B + P_A P_B}$$

which is strictly positive until  $p_1$  achieves the value given in the first equation of (6), and strictly negative after. Hence the result.

If  $p_A = 0$  (and then  $P_A = 0$ ) and  $p_B > 0$  (the opposite case is symmetric and then omitted), the CP revenue is  $e^{-\alpha p_1}(p_1 - D_0 q_A)$ , whose derivative is  $e^{-\alpha p_1}(1 - \alpha(p_1 - D_0 q_A))$ .

Finally, if  $p_A = p_B = 0$ , then the CP revenue is  $\frac{1}{2}e^{-\alpha p_1}(2p_1 - D_0(q_A + q_B))$ , and its derivative is  $e^{-\alpha p_1}(1 - \alpha p_1 + \frac{\alpha}{2}D_0(q_A + q_B))$ .  $\square$

Notice that the optimal price does not depend on the outside option valuation  $p_0$ . One can also check that it increases with the price  $p_i$  of ISP  $i$  that has the biggest side payment  $q_i$ , and decreases with the other price. In the limit (that can be interpreted as *neutral*) case  $q_A = q_B = q$ , the optimal pricing for the CP is  $qD_0 + 1/\alpha$  whatever the value of  $p_A$  and  $p_B$ . In that case, as an important remark, the CP's revenue is  $\frac{D_A + D_B}{D_0\alpha}$ , which corresponds to the CP-related user welfare: the interest of users and that of the CP coincide here. Finally, remark that the optimal price for the CP is always greater than  $D_0 \min(q_A, q_B) + 1/\alpha$ , because it is a convex combination of  $D_0 q_A + 1/\alpha$  and  $D_0 q_B + 1/\alpha$ . In particular, it is greater than the inverse of the user price sensitivity  $\alpha$ .

#### 4. Pricing game between ISPs

Before the users decide which ISP to join and the CP chooses  $p_1$ , the ISPs play a pricing game, making use of what the CP and users decisions would be. In this section, we determine the Nash equilibrium solutions of this pricing game in an analytical way when there are no side payments, and numerically (because intractable) in the general case. Recall (see [9]) that a Nash equilibrium would be a price profile  $(p_A, p_B)$  such that no ISP can improve (strictly) his utility by unilaterally changing his price. The best-response curves are defined as (by expliciting the dependence of  $U_A$  and  $U_B$  on  $p_A, p_B$ )

$$\begin{aligned} \text{BR}_A(p_B) &= \arg \max_{p_A \geq 0} U_A(p_A, p_B) \text{ and} \\ \text{BR}_B(p_A) &= \arg \max_{p_B \geq 0} U_B(p_A, p_B). \end{aligned}$$

With those notations, a Nash equilibrium is a point  $(p_A^{\text{NE}}, p_B^{\text{NE}})$  for which  $\text{BR}_A(p_B^{\text{NE}}) = p_A^{\text{NE}}$  and  $\text{BR}_B(p_A^{\text{NE}}) = p_B^{\text{NE}}$ . Graphically, if we draw the two best-response curves on the same figure, the set of Nash equilibria is then the (possibly empty) set of intersection points of those curves.

##### 4.1. No side payments

In the case where no side payments are established,  $q_A = q_B = 0$ , we get a simple formulation for the revenue of ISPs. From the previous section, the

optimal CP pricing is  $1/\alpha$ . Using the notation  $P_i = p_i^\beta$ , the revenue of ISP  $A$  is then (the revenue of ISP  $B$  being symmetrical)

$$U_A = \begin{cases} \frac{P_0 P_B p_A}{P_0 P_A + P_0 P_B + P_A P_B} & \text{if } p_A > 0 \text{ and } p_B > 0 \\ 0 & \text{if } p_A = 0 \text{ or } p_B = 0 \end{cases} \quad (7)$$

We first stress that  $p_A = p_B = 0$  is a Nash equilibrium since no player can strictly increase his revenue by unilaterally changing his action: the revenue always remains equal to zero. But setting one's price to zero is a dominated strategy, that is strictly dominated as soon as the adversary price is not zero: it always yields no revenue whereas a strictly positive revenue can be guaranteed with any other choice. Therefore it is not likely to be chosen by ISPs if another equilibrium exists.

**Proposition 2.** *Assuming that there are no side payments, i.e.  $q_A = q_B = 0$ , then*

- *if  $\beta \leq 1$ , there is a unique Nash equilibrium different from  $(0, 0)$  with  $P_A^{NE} = P_B^{NE} = \infty$ ,*
- *if  $1 < \beta < 2$ , there is a unique Nash equilibrium different from  $(0, 0)$  with  $P_A^{NE} = P_B^{NE} = \frac{2 - \beta}{\beta - 1} P_0$ ,*
- *if  $\beta \geq 2$ ,  $(0, 0)$  is the unique Nash equilibrium, yielding no revenue for the ISPs.*

The proof relies on the following general result about symmetric games.

**Lemma 1.** *If the best response function  $BR_A$  and  $BR_B$  are*

- *equal:  $BR_A = BR_B = BR$ ,*
- *single-valued,*
- *strictly increasing,*

*then  $(p_A, p_B)$  is a Nash equilibrium if and only if  $p_A = p_B = p$  with  $p$  a fixed point of the best-response function:  $p = BR(p)$ .*

*Proof.* (Lemma) The pair of prices  $(p_A, p_B)$  is a Nash equilibrium if and only if  $p_A = \text{BR}(p_B)$  and  $p_B = \text{BR}(p_A)$ . Let us suppose that  $p_A \neq p_B$ , for instance  $p_A > p_B$  (should the indexes be permuted). Then:

$$p_B = \text{BR}(p_A) > \text{BR}(p_B) = p_A,$$

where the inequality comes from the strict increasingness of  $\text{BR}$ , hence a contradiction. At Nash equilibrium,  $p_A = p_B$  is then a necessary condition, and from the definition of such an equilibrium, it is necessary and sufficient to have  $p = \text{BR}(p)$ .  $\square$

*Proof.* (Proposition) Assuming that  $p_A > 0$  and  $p_B > 0$ , the derivative of ISP  $A$  revenue (7) is

$$\frac{P_B P_0}{(P_A P_B + P_B P_0 + P_A P_0)^2} (P_A(1 - \beta)(P_B + P_0) + P_B P_0).$$

Hence the derivative has the same sign as the affine function of  $P_A$ :  $P_A(1 - \beta)(P_B + P_0) + P_B P_0$  which is strictly positive while  $P_A$  is smaller than the unique root, and negative afterwards. Hence, given  $p_B > 0$ , the best-response of ISP  $A$  is

$$\text{BR}_A(P_B) = \begin{cases} +\infty & \text{if } \beta \leq 1 \\ \frac{P_B P_0}{(\beta - 1)(P_B + P_0)} & \text{otherwise.} \end{cases}$$

The case  $\beta \leq 1$  is then solved.

For the case  $\beta > 1$ , notice that the best response is the same function for ISP  $B$  due to symmetry, that equals  $\text{BR}(P) = \frac{1}{(\beta-1)(1/P_0+1/P)}$ , and is a strictly increasing function of  $P$ . Hence it follows from the previous lemma that every Nash equilibrium is symmetric, which results here in the necessary and sufficient condition at Nash equilibrium  $P_B^{\text{NE}} = P_A^{\text{NE}} = \text{BR}_A(P_A^{\text{NE}})$ . The last equation has a unique strictly positive solution in the case  $1 < \beta < 2$ , the one given in the proposition, and no solution otherwise.  $\square$

This proposition shows in particular that, when the price sensitivity of users is high ( $\beta \geq 2$ ), we are led to the same price war as in the model of Bertrand competition studied in [19]. But for smaller levels of price sensitivity, this does no longer happen: the price set by ISPs at equilibrium is strictly positive, hence providing some revenue from users to both ISPs.

#### 4.2. Positive side payments

In the general case, the computation of the Nash equilibrium or even the best response function is not analytically tractable. We are then led to study numerically the price competition between ISPs. From here, we take  $\alpha = 1$ ,  $p_0 = 1$ ,  $D_0 = 1$ , and  $\beta = 1.5$ .

Let us first remark that if the price set by an ISP (say ISP  $A$ ) is equal to zero, then the other ISP (ISP  $B$ ), at a Nash equilibrium, sets his price to zero as well. This is because ISP  $B$  does not attract any users if his price is not zero, and then his revenue is null, whereas the revenue is  $\frac{1}{2}q_B D_0 e^{-\frac{\alpha}{2} D_0 (q_A + q_B) - 1}$  otherwise, hence strictly positive.

Numerical computations show that there is a set of side payments  $q_A$  and  $q_B$  for which the price war phenomenon between ISP happens. That set is shown in Figure 2. In the following, we will denote by  $\mathcal{P}_w$  the set of side payments for which a price war between ISPs takes place, leading to null subscription prices.

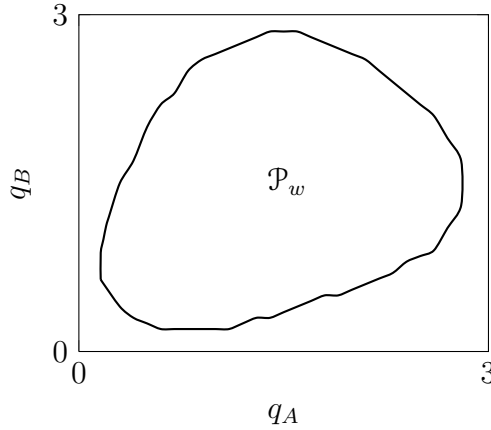


Figure 2: Set  $\mathcal{P}_w$  of side payments  $(q_A, q_B)$  for which the Nash equilibrium is such that  $p_A^{\text{NE}} = p_B^{\text{NE}} = 0$ , *i.e.* price war holds.

Figures 3 to 6 show the prices and revenues at Nash equilibrium (recall that this is for  $q_A$  and  $q_B$  fixed). Numerical computations point out the fact that the revenue of the CP, and the user welfare he creates, are always equal at equilibrium, the reason why we do not plot user welfare here. While this equality is clear when side payments are the same, our numerical results suggest that it remains true in the general case.

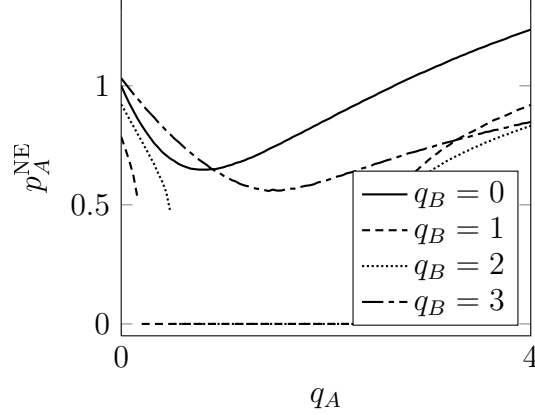


Figure 3: ISP  $A$  user price  $p_A^{\text{NE}}$  at equilibrium as a function of the side payments  $q_A$  with  $q_B \in \{0, 1, 2, 3\}$ .

Figure 3 represents the price  $p_A^{\text{NE}}$  set by ISP  $A$  at Nash equilibrium, when the side payment  $q_A$  varies, and for several ISP  $B$  side payments  $q_B$ . This reveals that the price at equilibrium first decreases with the side payment set by the ISP, and then increases. For some value of the opponent ISP side payment, it goes to zero when a threshold is reached. This threshold corresponds to the case where the side payment revenue that ISPs get by setting their prices to zero, and then attracting the whole set of users, becomes larger than the one they get on a limited market share with both the subscription fees and the side payments. Finally, there is no monotonicity in the opponent side payment.

Figure 4 displays the optimal price  $p_1^*$  of the CP in terms of  $q_A$  for different values of  $q_B$ . We can notice the discontinuity due to the price war thresholds (for the cases  $q_B = 1$  and  $q_B = 2$ , since there are no such thresholds for the two other cases from Figure 2). It can be remarked that the optimal price increases with  $q_A$  (and  $q_B$ ) both before and after the thresholds, but not in general. One can also check here the general property that  $p_1^* \geq D_0 \min(q_A, q_B) + 1/\alpha$ , which, in particular, ensures a strictly positive revenue to the CP.

Figure 5 shows that the revenue of ISPs is not monotonous with the side payment. Moreover, depending on the ISP  $B$  side payment, the maximal revenue of ISP  $A$  is reached either for a null side payment (e.g., when  $q_B = 3.0$ ) or for a strictly positive value (e.g., when  $q_B = 0.0$ ). On the other hand, the CP revenue, plotted in Figure 6, has a tendency to decrease with

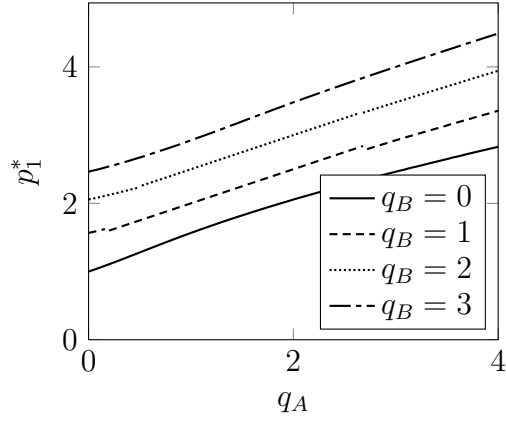


Figure 4: CP optimal price at equilibrium as a function of the side payments  $q_A$  with  $q_B \in \{0, 1, 2, 3\}$ .

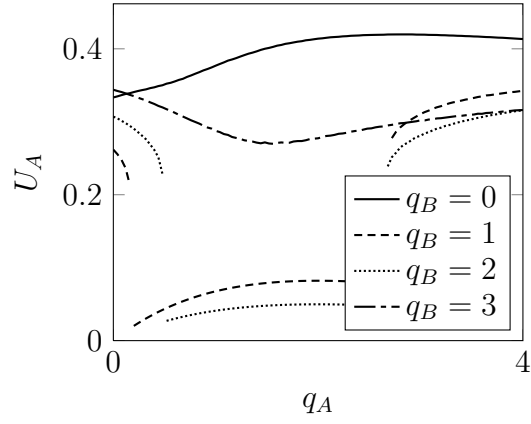


Figure 5: ISP A revenue at equilibrium as a function of the side payments  $q_A$  with  $q_B \in \{0, 1, 2, 3\}$ .



side payments, even if it is not strictly the case, as can be seen for  $q_B = 1$  and small values of  $q_A$ . When a discontinuity occurs, the CP revenue is

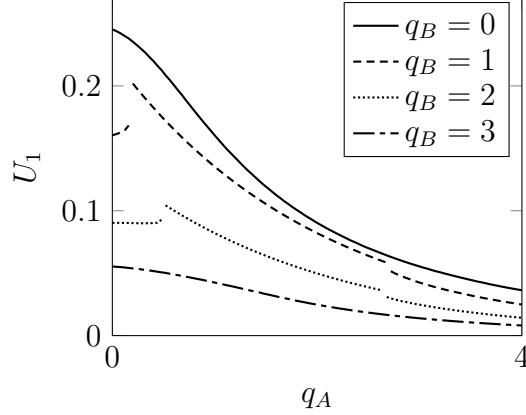


Figure 6: CP revenue at equilibrium as a function of the side payment  $q_A$ , with  $q_B \in \{0, 1, 2, 3\}$ .

maximized for the smallest side payment leading to the price war.

Finally, over the price war set  $\mathcal{P}_w$ , the CP subscription price is  $p_1 = D_0 \frac{q_A + q_B}{2} + \frac{1}{\alpha}$  and the revenues are

$$U_i = \frac{1}{2} q_i D_0 e^{\frac{\alpha D_0}{2} (q_A + q_B) - 1} \quad (8)$$

$$U_1 = \text{UW}_{\text{CP}} = \frac{1}{\alpha} e^{\frac{\alpha D_0}{2} (q_A + q_B) - 1}. \quad (9)$$

## 5. Side payments determination

We consider at the highest level three possibilities for the choice of the side payments  $q_A$  and  $q_B$ . We first look at the case when they are determined by the CP (even if unlikely in practice), then the case when they result from a game played between ISPs, and finally the case when they are determined by a regulator (e.g., to maximize social welfare).

Since we don't have the analytical expression for the ISPs price at Nash equilibrium, we provide numerical results, where we take  $\alpha = 1$ ,  $\beta = 1.5$ ,  $p_0 = 1$  and  $D_0 = 1$ .

### 5.1. Determined by the CP

The revenue of the CP is maximized when the side payments are  $q_A = q_B = 0.3$  as illustrated in Figure 7 (instead of plotting a hard-to-read 3D-curve of CP revenue in terms of  $q_A$  and  $q_B$ , we have preferred to draw 2D-curves in terms of one of the parameters for values various of the second parameter close to optimal). It is interesting to notice that, for such a set of side payments, there is a price war on the user prices, i.e.,  $p_A^{\text{NE}} = p_B^{\text{NE}} = 0$  here. In fact it corresponds to the symmetric ( $q_A = q_B$ ) point of the price war set  $\mathcal{P}_w$  described in Figure 2 for which the sum of side payments is minimized. Indeed, if  $p_A^{\text{NE}} = p_B^{\text{NE}} = 0$ , then the revenue of the CP can be rewritten as  $U_1 = \frac{1}{\alpha} e^{-\frac{\alpha D_0}{2}(q_A + q_B) - 1}$ , and then is maximized when  $q_A + q_B$  is minimal.

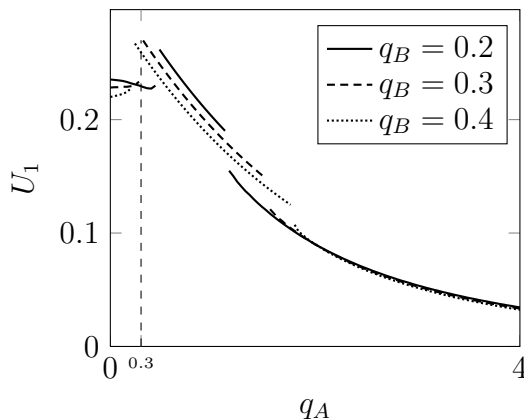


Figure 7: CP revenue at equilibrium as a function of the side payment  $q_A$ , with  $q_B \in \{0.2, 0.3, 0.4\}$ . The maximal revenue is reached when  $q_A = q_B = 0.3$ .

At this point the revenues of the stakeholders are  $U_A = U_B \approx 0.04$ , and  $UW_{\text{CP}} = U_1 \approx 0.27$ . Hence the revenues of ISPs are much smaller than the one of the CP. Note that the situation is quite counter-intuitive, since the CP gains to introduce side payments. This is because those payments exacerbate the competition between ISPs, which is beneficial to end users, and finally to the CP who can reach more customers.

### 5.2. Determined by the ISPs, through a game

If we instead assume that the side payments are non-cooperatively determined by the ISPs, we are led to study the best response of each ISP to the other ISP side payment. As shown in Figure 8, the best response is

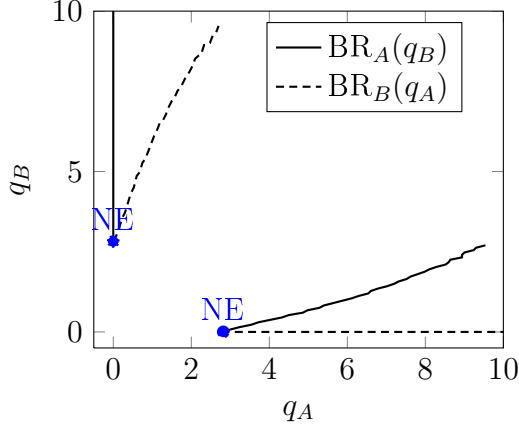


Figure 8: The optimal side payment of each ISP as a function of the opponent ISP side payment. There is a threshold  $q_i \approx 2.80$  beyond which the best response falls to zero. There are two Nash equilibria (NE) where one ISP sets its side payment to zero.

first increasing in the other ISP side payments, and then falls to zero above a threshold, which is approximately 2.80. Since the best-response to a null price is 2.80, it follows that  $(0, 2.80)$  and the symmetric point  $(2.80, 0)$  are Nash equilibria, and they are the only ones. It is interesting to notice that the resulting side payments are not symmetric at Nash equilibrium, so are the revenues equalling 0.42 for the ISP with side payment 2.80 and 0.34 for the other ISP.

Now let us compare that outcome to the case without any side payments. From Subsection 4.1, with  $\beta = 1.5$ , the revenue of ISPs is 0.33. Hence the ISPs global revenue increases by about 15%, which goes in the direction of ISPs arguments about side payments improving their revenue. On the other side, the CP revenue decreases from 0.25 to 0.06, hence losing nearly 75% of its value. The benefit of ISPs is then at the expense of the CP and consequently of the user welfare.

### 5.3. Determined by a regulator

A regulator can either decide to maximize the revenue of the supply chain (sum of utilities of the ISPs plus the CP), the user welfare (end-users surplus), or the social welfare (including user welfare and all providers utilities).

The total value of the *supply chain* is the total revenue got from the users, i.e.,  $U_1 + U_A + U_B$ .

*User welfare* can be decomposed into two components: the user welfare due to the existence of the CP -that is computed in (3)-, and the user welfare due to the presence of the ISPs. Let us focus on the latter part: we have assumed that users not connected to the Internet perceive a cost  $p_0$  (thus  $p_0$  can be seen as the value of the connectivity service). When a user decides to subscribe to ISP  $i$  and pay the corresponding price  $p_i$ , its benefit is then  $p_0 - p_i$  with respect to the no-ISP situation: the user does not bear anymore the cost  $p_0$  of not having Internet access, and instead perceives the monetary cost  $p_i$ . Aggregating over the whole population, the user welfare that is due to the presence of the ISPs (with their prices  $p_A$  and  $p_B$ ) equals

$$UW_{\text{ISPs}} = \sigma_A(p_0 - p_A) + \sigma_B(p_0 - p_B).$$

The global user welfare generated by the system (ISPs and CP) is therefore

$$UW = UW_{\text{CP}} + UW_{\text{ISPs}} \quad (10)$$

Finally, *social welfare* is defined as the overall value of the service for the society. It therefore includes the surpluses of all actors, and equals  $SW = U_1 + U_A + U_B + UW$ . Note that Social Welfare also corresponds to the total value that the service has for subscribers, without considering any monetary exchanges because they stay within the society. We indeed obtain, simplifying the sum of the terms in SW:

$$SW = (\sigma_A + \sigma_B) \left( p_0 + \left( p_1 + \frac{1}{\alpha} \right) e^{-\alpha p_1} \right),$$

where the term  $(\sigma_A + \sigma_B)p_0$  is the value of the connectivity service for ISPs' subscribers, and the other term is the value of the CP service for CP subscribers, computed as  $(\sigma_A + \sigma_B) \int_{p_1}^{\infty} \alpha e^{-\alpha x} x \, dx$ .

### 5.3.1. Side payments to maximize User Welfare

Since CP revenue and user welfare are equal at Nash equilibrium, it follows that user welfare is maximized when the CP revenue is maximized. This case has already been treated in Subsection 5.1.

### 5.3.2. Side payments to maximize Social Welfare

We have obtained numerically that the social welfare is maximized for the same side payments than the ones maximizing the CP revenue and the user welfare.

### 5.3.3. Side payments to maximize the supply chain value

The supply chain value is maximized when the side payments are both null, which has been studied in Subsection 4.1. In this neutral case the revenue of ISPs is approximately 0.33 whereas that of the CP (and the induced user welfare) is 0.25. Remark that among the three alternatives considered in this subsection, this one leads to the fairest revenue sharing between stakeholders.

## 6. Discussion, conclusions and future work

We have provided in this paper a model describing the interactions between two ISPs in competition, a CP, and end users connecting to the network. With respect to the literature, we believe that considering competitive ISPs and a single CP is a more realistic representation of the current network where we have a quasi-monopole for some applications (for instance YouTube or Netflix), while several ISPs are in competition (an argument of ISPs). The goal is to study the impact of side payments on providers' revenues, and conclude whether they can help reduce the unfairness of the current revenue sharing among all actors, as claimed by ISPs in the current network neutrality debate.

In this paper, we have presented a three-level game where (from the largest to the shortest time scale) the side payments are first determined, then a pricing game is played between ISPs, followed by the content provider price, and finally, knowing all those prices, end users choose their ISP (or none if too expensive) and possibly decide to subscribe also to the CP service. All those levels are played by backward induction, meaning that players anticipate the solutions of the later games when choosing their strategies.

Our results have highlighted the fact that side payments, *unless decided by ISPs*, have little chance to address the concern from ISPs regarding the fairness of the revenue sharing associated to users accessing content through their infrastructures. This is due, to a great extent, to the competition played among ISPs on the access prices that drives their revenues to low values. On the other hand, the CP being in a monopolist situation, always obtains significant revenues. An interesting paradox we have highlighted is that side payments may be beneficial to the CP. Nevertheless, when side payments are decided by the ISPs (non-cooperatively), it can be beneficial to them, but at the expense of both the CP and the users. Remarkably, in terms of social and user welfare, the optimal side payments are the same than

those maximizing the CP revenue. But looking at the whole supply chain, in order to avoid too big disparities between revenues of providers, the neutral case is the most suitable. If the side payments are decided non-cooperatively by ISPs, in our experiment, one (only) is a big winner, while the other ISP gains a bit more than in the neutral case. This asymmetry may be a problem and can create complicated tensions and negotiations.

As future research, we would like to go into several directions: first to include several CPs with different contents, but such that some end users are targeting only a subset of them, for all possible subsets. ISPs may also charge each other to let the CPs not connected to them reach their own customers (transit pricing). Other extensions to our model could include architecture investment and content innovation characteristics, for the ISPs and the CP respectively.

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