



**HAL**  
open science

## $H_\infty$ Robust Control Design for Teleoperation Systems

Bo Zhang, Alexandre Kruszewski, Jean-Pierre Richard

► **To cite this version:**

Bo Zhang, Alexandre Kruszewski, Jean-Pierre Richard.  $H_\infty$  Robust Control Design for Teleoperation Systems. 7th IFAC Symposium on Robust Control Design, Jun 2012, Aalborg, Denmark. hal-00682948

**HAL Id: hal-00682948**

**<https://hal.inria.fr/hal-00682948>**

Submitted on 27 Mar 2012

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# $H_\infty$ Robust Control Design for Teleoperation Systems

B. Zhang A. Kruszewski J.-P. Richard<sup>1</sup>

LAGIS, CNRS UMR 8219, Laboratoire d'Automatique, Génie Informatique et  
Signal Université Lille Nord de France, Ecole Centrale de Lille, 59651  
Villeneuve d'Ascq, France  
(e-mail: {bo.zhang, alexandre.kruszewski, jean-pierre.richard}@ec-lille.fr)

---

**Abstract:** This paper deals with the problem of delay-dependent robust  $H_\infty$  control for time-varying delay teleoperation system with norm-bounded and time-varying model uncertainties. Thanks to our proposed control scheme, Lyapunov-Krasovskii functionals (LKF) and  $H_\infty$  theory, the delay-dependent stability and tracking performance analysis are proposed in terms of Linear Matrix Inequality (LMI) optimization. An illustrative example is given by various simulations to prove that, our proposed solution is efficient to handle time-varying delays and uncertainties under different working conditions, such as abrupt tracking and wall contact motion.

*Keywords:* Teleoperation; Model uncertainties; Time-varying delay;  $H_\infty$  control; Lyapunov-Krasovskii functionals; Linear Matrix Inequality

---

## 1. INTRODUCTION

A typical bilateral teleoperation is a closed-loop structure including the forward (from the master to the slave) and backward (from the slave to the master) transmission, and composed of the human operator, the master haptic-interface robot, the communication medium, the slave robot and the environment. There are two challenges that will be considered in this paper: time-varying delays [Richard (2003)] introduced by long-range or flexible communication links such as the Internet or Wireless 802.11 networks; time-varying model uncertainties that exist in real implementation of bilateral teleoperation, because no real system is 'pure' linear ([Hokayem and Spong (2006)] and the references therein).

Recently, many methods are proposed to address the stability and performance problem of bilateral teleoperation:

- *Passivity-based control* under variable delays: the survey [Nuño et al. (2011)] revisits many passivity-based controllers for bilateral teleoperation system, including the scattering and wave variables. Recently, based on the energy and power considerations, time domain passivity control [Ryu et al. (2005), Ye et al. (2009)] without the transformation of wave variables have been proposed. Overall, the latest passivity-based results can resolve the stabilization problem under time-varying delays, but the system performance is not guaranteed.

- *Non-passive control*: various control strategies have been proposed for a *non-passive environment* under constant or time-varying delays. The readers can refer to [Arcara and Melchiorri (2002), Zhang et al. (2011), Chiasson and Loiseau (2007)] for more details on these methods. However, very few of these methods focus on perturbations and model uncertainties.

Our latest research [Zhang et al. (2011)] presented a force-reflecting proxy control scheme, which guarantees the stabil-

ity and the position/force tracking of the closed-loop system under time-varying delays. This performance is realized by Lyapunov-Krasovskii functionals (LKF) and  $H_\infty$  control [Jiang and Han (2005), Xu et al. (2006), Fridman and Niculescu (2008)], which can be solved by Linear Matrix Inequality (LMI) optimization [Fridman (2006)]. Especially by  $H_\infty$  control, the whole system remains stable despite variations and uncertainties in the dynamics of operator, master robot, communication channels, slave robot, and the environment. Based on the control strategies mentioned above, in this paper, we handle the linear system with time-varying model uncertainties, our design approach can be summarized:

- Control scheme in [Zhang et al. (2011)] is utilized, but the models of master, proxy of master (a remote observer of the master used at the slave side to reduce the impact of the time-varying delays) and slave are combined with time-varying uncertainties.
- Local controllers of master, proxy and slave are designed by Lyapunov functionals and LMI.
- Considering model uncertainties as perturbations, the slave controller is obtained by a less conservative LKF condition. Local and slave controllers will be defined later.

This paper is organized as follows: Section 2 introduces the theorem to be used later. Our main results are given in Section 3. Simulations under the different working conditions are presented in Section 4. Finally we conclude in Section 5.

## 2. PRELIMINARIES

This section is devoted to a general stability theorem with the  $H_\infty$  performance index for uncertain and perturbed systems with time-varying delays. For simplicity reasons, considering one delay in the system,

---

<sup>1</sup> J.-P. Richard is also with INRIA NON-A.

$$\begin{cases} \dot{x}(t) = A_0x(t) + A_1x(t - \tau(t)) + Bw(t), \\ z(t) = Cx(t), \\ x(t_0 + \theta) = \phi(\theta), \dot{x}(t_0 + \theta) = \dot{\phi}(\theta), \theta \in [-h_2, 0], \end{cases} \quad (1)$$

where,  $x(t) \in \mathbf{R}^n$ ,  $w(t) \in \mathbf{R}^l$  is some exogenous disturbance signals, while  $z(t) \in \mathbf{R}^q$  is the objective control output.  $\phi(\theta)$  is the initial state function, and  $\tau(t) \in [h_1, h_2]$ ,  $h_1 \geq 0$  is the time-varying delay.  $A_0, A_1, B$  and  $C$  are constant matrices.

Let us define  $\chi(t) \triangleq \text{col}\{x(t), x(t - \tau(t)), x(t - h_1), x(t - h_2)\}$  (the symbol  $\text{col}\{\}$  represents the column vector, which will also be used in the following) and the corresponding block entry matrices as in [Park et al. (2011)],

$$\begin{aligned} e_1 &= \text{col}\{I, 0, 0, 0\}, & e_2 &= \text{col}\{0, I, 0, 0\}, & e_3 &= \text{col}\{0, 0, I, 0\}, \\ e_4 &= \text{col}\{0, 0, 0, I\}, & e &= e_1A_0^T + e_2A_1^T. \end{aligned} \quad (2)$$

Thus, the system in Eq. 1 can be rewritten as,

$$\begin{cases} \dot{x}(t) = e^T \chi(t) + Bw(t), & x(t) = e_1^T \chi(t), \\ z(t) = Cx(t) = Ce_1^T \chi(t). \end{cases} \quad (3)$$

Considering the Lyapunov-Krasovskii functional in [Fridman (2006), Zhang et al. (2011)],

$$\begin{aligned} V(t, x(t), \dot{x}(t)) &= x(t)^T P x(t) \\ &+ \int_{t-h_1}^t x(s)^T Q_1 x(s) ds + \int_{t-h_2}^t x(s)^T Q_2 x(s) ds \\ &+ h_1 \int_{-h_1}^0 \int_{t+\theta}^t \dot{x}(s)^T R_1 \dot{x}(s) ds d\theta \\ &+ (h_2 - h_1) \int_{-h_2}^{-h_1} \int_{t+\theta}^t \dot{x}(s)^T R_2 \dot{x}(s) ds d\theta. \end{aligned} \quad (4)$$

According to  $H_\infty$  control theory, the performance will be studied by checking  $H_\infty$  performance index  $J(w) < 0$  for a positive scalar  $\gamma$ ,

$$J(w) = \int_0^\infty (z(t)^T z(t) - \gamma^2 w(t)^T w(t)) dt < 0. \quad (5)$$

**Theorem 1.** Suppose there exist matrices of appropriate dimension  $P > 0, Q_i > 0, R_i > 0, S, P_2, P_3, i = 1, 2$ , and a positive scalar  $\gamma$ , such that the condition (6) with notations (7) is feasible, then the system (1) is asymptotically stable and  $J(w) < 0$  for time-varying delay  $\tau_1(t) \in [h_1, h_2]$ .

$$\begin{aligned} \Gamma^1 &= \begin{pmatrix} \Gamma_{11}^1 + e_1 C^T C e_1^T + e P_2 + P_2^T e^T & e_1 P - P_2^T + e P_3 & P_2^T B \\ * & \Gamma_{22}^1 - P_3^T - P_3 & P_3^T B \\ * & * & -\gamma^2 I \end{pmatrix} < 0, \\ \begin{pmatrix} R_2 & S \\ S^T & R_2 \end{pmatrix} &\geq 0, \end{aligned} \quad (6)$$

$$\begin{aligned} \Gamma_{11}^1 &= e_1 Q_1 e_1^T - e_3 Q_1 e_3^T + e_1 Q_2 e_1^T - e_4 Q_2 e_4^T \\ &- (e_1 - e_3) R_1 (e_1 - e_3)^T \\ &- (e_3 - e_2 \ e_2 - e_4) \begin{pmatrix} R_2 & S \\ S^T & R_2 \end{pmatrix} \begin{pmatrix} e_3^T - e_2^T \\ e_2^T - e_4^T \end{pmatrix}, \\ \Gamma_{22}^1 &= h_1^2 R_1 + (h_2 - h_1)^2 R_2. \end{aligned} \quad (7)$$

**Proof.** From  $H_\infty$  stability condition in [Zhang et al. (2011)],  $J(w) < 0$  can be assured if,

$$\dot{V}(t, x(t), \dot{x}(t)) + z(t)^T z(t) - \gamma^2 w(t)^T w(t) < 0. \quad (8)$$

By *Theorem 2* in [Park et al. (2011)], and substituting for  $z(t)$ , we get,

$$\begin{aligned} \dot{V}(t, x(t), \dot{x}(t)) + z(t)^T z(t) - \gamma^2 w(t)^T w(t) \\ \leq \chi(t)^T (\Gamma_{11}^1 + e_1 C^T C e_1^T) \chi(t) + \chi(t)^T (e_1 P + P e_1^T) \dot{x}(t) \\ + \dot{x}(t)^T \Gamma_{22}^1 \dot{x}(t) - \gamma^2 w(t)^T w(t). \end{aligned} \quad (9)$$

Introducing free weighting matrices  $P_2, P_3$  as in [Fridman (2006), He et al. (2002)],

$$0 = 2[\chi(t)^T P_2^T + \dot{x}(t)^T P_3^T][e^T \chi(t) + Bw(t) - \dot{x}(t)]. \quad (10)$$

The expression above is now added into  $\dot{V}(t, x(t), \dot{x}(t)) + z(t)^T z(t) - \gamma^2 w(t)^T w(t)$ , and using notation,

$$\eta(t) = \text{col}\{\chi(t), \dot{x}(t), w(t)\}, \quad (11)$$

leads to,

$$\dot{V}(t, x(t), \dot{x}(t)) + z(t)^T z(t) - \gamma^2 w(t)^T w(t) \leq \eta(t)^T \Gamma^1 \eta(t) < 0, \quad (12)$$

provides that the LMI in Eq. 6 is feasible.

**Remark 2.** There is no particular assumption on  $\dot{\tau}(t)$ . Besides, from *Table 2* in [Park et al. (2011)], we can see that, compared to LKF condition in [Zhang et al. (2011)], our stability criteria can make LMI condition less conservative by decreasing the number of decision variables, this will be illustrated later by the simulation.

**Remark 3.** Our theorem can also be extended to the  $n$  delays case ( $n > 1$ ) and the delay-free case (without  $A_1 x(t - \tau(t))$  in Eq. 1). Note that, *Theorem 1* in our another paper [Zhang et al. (2011)] can well handle the delay-free case.

### 3. MAIN RESULTS FOR FORCE-REFLECTING PROXY CONTROL SCHEME

#### 3.1 System Description and Problem Formulation

The force-reflecting proxy control scheme is presented in Figure 1. Let us give a description of the control scheme:  $F_m(t)$  and  $F_s(t)$  are the actuated inputs of the master and the slave;  $F_h(t)$  and  $F_e(t)$  are the forces of the human operator and environment on the system;  $\hat{F}_h(t)$  and  $\hat{F}_e(t)$  are the estimations of these two forces, which can be obtained by adding the perturbation observers in reality;  $\dot{\theta}_m(t)/\theta_m(t)$ ,  $\dot{\theta}_s(t)/\theta_s(t)$  are the velocities/positions of the master and the slave.

The communication delays  $\tau_1(t), \tau_2(t) \in [h_1, h_2]$ ,  $h_1 \geq 0$ .  $\hat{\tau}_1(t)$  is the estimated network delay, thanks to time-stamped data packet exchanges using a network time protocol as in [Kruszowski et al. (2011)] between the master and slave, the master and slave clocks are synchronized and  $\hat{\tau}_1(t)$  is available at slave's side:  $\hat{\tau}_1(t) = \tau_1(t)$ .

From the master to slave, the information transferred are the velocity/position of the master and the estimated force  $\hat{F}_h(t)$ . However, from the slave to the master, only the estimated force  $\hat{F}_e(t)$  is transferred, so the force tracking,  $F_m(t) = \hat{F}_e(t - \tau_2(t))$ , is realized, if the stability of the whole system is verified.

Note that, in master and slave, there exist norm-bounded and time-varying model uncertainties  $(\Delta A_m(t), \Delta B_m(t), \Delta A_s(t), \Delta B_s(t))$  as follows,

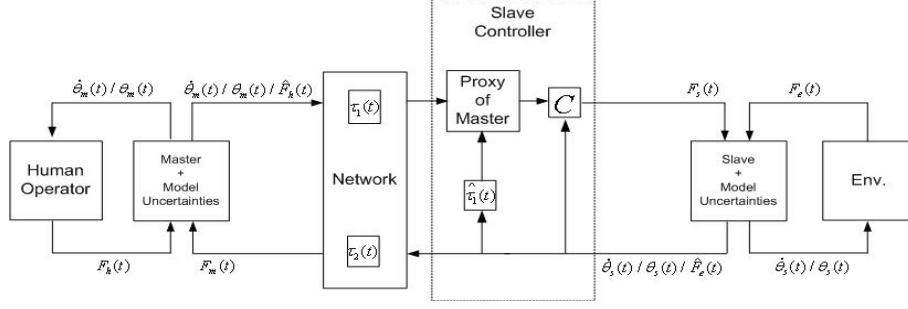


Fig. 1. Force-reflecting proxy control scheme.

$$\begin{aligned}
(\Sigma_m) \quad \dot{x}_m(t) &= ((A_m + \Delta A_m(t)) - (B_m + \Delta B_m(t))K_m^0)x_m(t) \\
&\quad + (B_m + \Delta B_m(t))(F_m(t) + F_h(t)), \\
(\Sigma_s) \quad \dot{x}_s(t) &= ((A_s + \Delta A_s(t)) - (B_s + \Delta B_s(t))K_s^0)x_s(t) \\
&\quad + (B_s + \Delta B_s(t))(F_s(t) + F_e(t)),
\end{aligned} \tag{13}$$

where,  $x_m(t) = \dot{\theta}_m(t)$ ,  $x_s(t) = \dot{\theta}_s(t)$ .  $K_m^0$ ,  $K_s^0$  are the local controllers of the master and slave ensuring the speed stability, they will be designed later. The model uncertainties satisfy,  $i = \{m, s\}$ ,

$$\Delta A_i(t) = G_i \Delta(t) D_i, \quad \Delta B_i(t) = H_i \Delta(t) E_i, \tag{14}$$

where,  $G_i$ ,  $D_i$ ,  $H_i$ ,  $E_i$  are constant matrices of appropriate dimension and  $\Delta(t)$  is a time-varying matrix,  $\Delta(t)^T \Delta(t) \leq I$ .

In the slave controller, the proxy of master is like a remote observer of the master, which is used at the slave side to reduce the impact of the time-varying delays. Thus, the model of proxy, including the local controller, is same with the master, but the model uncertainties are different ( $\hat{\theta}_p(t)/\theta_p(t)$  is the velocity/position of the proxy of master),

$$\begin{aligned}
(\Sigma_p) \quad \dot{x}_p(t) &= ((A_m + \Delta A_p(t)) - (B_m + \Delta B_p(t))K_m^0)x_p(t) \\
&\quad - (B_m + \Delta B_p(t))F_p(t - \tau_1(t)) \\
&\quad + (B_m + \Delta B_p(t))(\hat{F}_e(t) + \hat{F}_h(t - \tau_1(t))),
\end{aligned} \tag{15}$$

where,  $x_p(t) = \dot{\theta}_p(t)$ ,  $\Delta A_p(t) = G_p \Delta(t) D_p$ ,  $\Delta B_p(t) = H_p \Delta(t) E_p$ .  $G_p$ ,  $D_p$ ,  $H_p$ ,  $E_p$  are constant matrices. The gain  $L = (L_1 \ L_2 \ L_3)$  is used to synchronize the position between the master and the proxy of master,

$$F_p(t - \tau_1(t)) = L \begin{pmatrix} \dot{\theta}_p(t - \hat{\tau}_1(t)) \\ \dot{\theta}_m(t - \tau_1(t)) \\ \theta_p(t - \hat{\tau}_1(t)) - \theta_m(t - \tau_1(t)) \end{pmatrix}. \tag{16}$$

Next,  $K = (K_1 \ K_2 \ K_3)$  is the gain of the controller  $C$ ,

$$F_s(t) = K \begin{pmatrix} \dot{\theta}_s(t) \\ \dot{\theta}_p(t) \\ \theta_s(t) - \theta_p(t) \end{pmatrix}. \tag{17}$$

Thus, our following works are to solve:

**Problem 1:** the local controllers of master, proxy and slave,  $K_m^0$  and  $K_s^0$ , are designed to make the master, proxy and slave robustly stable with respect to the model uncertainties.

**Problem 2:** the proxy of master and the controller  $C$  (together defined as the slave controller) are designed to make the whole system stable and achieve the position/force tracking.

### 3.2 Problem 1: Local Controller Design

The local controllers are designed by a Lyapunov functional and LMI, taking the master as an example, and considering the Lyapunov functional  $V(t, x_m(t)) = x_m(t)^T P x_m(t)$ ,  $P = P^T > 0$ , we verify  $\dot{V}(t, x_m(t)) < 0$  by using *Lemma 1* in [Xu and Liu (2003)] and introducing  $\rho_A > 0$ ,  $\rho_B > 0$ , then applying Schur formula leads to,

$$\begin{aligned}
\Gamma^2 &= \begin{pmatrix} \Gamma_{11}^2 & \rho_A^{-1} P G_m & \rho_B^{-1} P H_m & K_m^0 T E_m^T & D_m^T \\ * & -I & 0 & 0 & 0 \\ * & * & -I & 0 & 0 \\ * & * & * & -\rho_B^{-1} I & 0 \\ * & * & * & * & -\rho_A^{-1} I \end{pmatrix} < 0, \\
\Gamma_{11}^2 &= A_m^T P + P A_m - K_m^0 T B_m^T P - P B_m K_m^0.
\end{aligned} \tag{18}$$

Multiplying  $\Gamma^2$  by  $\text{diag}\{P^{-T}, \dots, P^{-T}, I\}$  at the left side,  $\text{diag}\{P^{-1}, \dots, P^{-1}, I\}$  at the right side, defining  $N_m = K_m^0 P$ , then the result  $K_m^0 = N_m P^{-1}$  follows,

$$\begin{aligned}
\Gamma^3 &= \begin{pmatrix} \Gamma_{11}^3 & \rho_A^{-1} G_m P & \rho_B^{-1} H_m P & N_m^T E_m^T & P D_m^T \\ * & -I & 0 & 0 & 0 \\ * & * & -I & 0 & 0 \\ * & * & * & -\rho_B^{-1} I & 0 \\ * & * & * & * & -\rho_A^{-1} I \end{pmatrix} < 0, \\
\Gamma_{11}^3 &= A_m P + P A_m^T - N_m^T B_m^T - B_m N_m.
\end{aligned} \tag{19}$$

**Remark 4.** The proxy has the same local controller with master ( $K_m^0$ ), and for the slave, local controller ( $K_s^0$ ) can be obtained by the same procedure.

### 3.3 Problem 2: Global Stability and Performance Analysis

The objective of this subsection is to provide the stability and performance analysis by designing the proxy of master and the controller  $C$  under time-varying delays and uncertainties. Firstly, we design the proxy of master,  $L$ , by Lyapunov-Krasovskii functional,  $H_\infty$  control and LMI to synchronize the position between the master and the proxy. Considering the models of the master and proxy as the following system,

$$\begin{cases} \dot{x}_{mp}(t) = (A_{mp}^0 + \Delta A_{mp}^0(t))x_{mp}(t) \\ \quad + (A_{mp}^1 + \Delta A_{mp}^1(t))x_{mp}(t - \tau_1(t)) \\ \quad + (B_{mp} + \Delta B_{mp}(t))w_{mp}(t), \\ z_{mp}(t) = C_{mp}x_{mp}(t), \end{cases} \tag{20}$$

where,

$$\begin{aligned}
x_{mp}(t) &= \begin{pmatrix} \dot{\theta}_p(t) \\ \dot{\theta}_m(t) \\ \theta_p(t) - \theta_m(t) \end{pmatrix}, \quad w_{mp}(t) = \begin{pmatrix} \hat{F}_e(t) + \hat{F}_h(t - \tau_1(t)) \\ F_m(t) + F_h(t) \end{pmatrix}, \\
z_{mp}(t) &= \begin{pmatrix} \theta_p(t) - \theta_m(t) \end{pmatrix},
\end{aligned} \tag{21}$$

$$\begin{aligned}
A_{mp}^0 &= \begin{pmatrix} A_m - B_m K_m^0 & 0 & 0 \\ 0 & A_m - B_m K_m^0 & 0 \\ 1 & -1 & 0 \end{pmatrix}, \\
\Delta A_{mp}^0(t) &= \begin{pmatrix} \Delta A_p(t) - \Delta B_p(t) K_m^0 & 0 & 0 \\ 0 & \Delta A_m(t) - \Delta B_m(t) K_m^0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \\
A_{mp}^1 &= \begin{pmatrix} -B_m L_1 & -B_m L_2 & -B_m L_3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \\
\Delta A_{mp}^1(t) &= \begin{pmatrix} -\Delta B_p(t) L_1 & -\Delta B_p(t) L_2 & -\Delta B_p(t) L_3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \\
B_{mp} &= \begin{pmatrix} B_m & 0 \\ 0 & B_m \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} B_{mp}^1 & B_{mp}^2 \end{pmatrix}, \\
\Delta B_{mp}(t) &= \begin{pmatrix} \Delta B_p(t) & 0 \\ 0 & \Delta B_m(t) \\ 0 & 0 \end{pmatrix}, \quad C_{mp} = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}.
\end{aligned} \tag{22}$$

Because  $H_\infty$  control theory is devoted to minimize the modeling imperfections, uncertainties and expected disturbances, we consider the time-varying uncertainties in master and proxy system as perturbations,

$$\begin{aligned}
\varphi_p(t) &= (\Delta A_p(t) - \Delta B_p(t) K_m^0) \dot{\theta}_p(t) \\
&\quad + \Delta B_p(t) (\hat{F}_e(t) + \hat{F}_h(t - \tau_1(t))), \\
\varphi_m(t) &= (\Delta A_m(t) - \Delta B_m(t) K_m^0) \dot{\theta}_m(t) + \Delta B_m(t) (F_m(t) + F_h(t)),
\end{aligned} \tag{23}$$

and at the proxy side,

$$\mu_p(t) = -\Delta B_p(t) L x_{mp}(t - \tau_1(t)). \tag{24}$$

$L$  will be fixed later, we add uncertainties in Eq. 23 and Eq. 24 into  $w_{mp}(t)$ . Thus, the system in Eq. 20 is rewritten as,

$$\begin{cases} \dot{x}_{mp}(t) = A_{mp}^0 x_{mp}(t) + A_{mp}^1 x_{mp}(t - \tau_1(t)) + \tilde{B}_{mp} \tilde{w}_{mp}(t), \\ z_{mp}(t) = C_{mp} x_{mp}(t), \end{cases} \tag{25}$$

where,

$$\begin{aligned}
\tilde{w}(t) &= \begin{pmatrix} B_m \hat{F}_e(t) + B_m \hat{F}_h(t - \tau_1(t)) + \varphi_p(t) + \mu_p(t) \\ B_m F_m(t) + B_m F_h(t) + \varphi_m(t) \end{pmatrix}, \\
\tilde{B}_{mp} &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}.
\end{aligned} \tag{26}$$

Then, the following theorem is obtained.

**Theorem 5.** Suppose there exist matrices of appropriate dimension  $P > 0$ ,  $Q_i > 0$ ,  $R_i > 0$ ,  $S$ ,  $\bar{P}_2$ ,  $P_3$ ,  $i = 1, 2$ , and positive scalars  $\gamma$ ,  $\xi$ ,  $\xi_i$ ,  $i = 1, 2, 3$ , such that the condition (28) with notations (29) is feasible, then the system (20) is asymptotically stable and  $J(w) < 0$  for time-varying delays  $\tau_1(t) \in [h_1, h_2]$ , and with the following proxy control gain,

$$L = M \bar{P}_2^{-1}. \tag{27}$$

$$\begin{aligned}
\Gamma^4 &= \begin{pmatrix} \Gamma_{11}^1 + \Gamma_{11}^4 + \Gamma_{11}^4{}^T & \Gamma_{12}^4 & \Gamma_{13}^4 & e_1 \bar{P}_2^T C_{mp}^T \\ * & \Gamma_{22}^4 - \xi \bar{P}_2 - \xi \bar{P}_2^T & \xi \tilde{B}_{mp} & 0 \\ * & * & -\gamma^2 I & 0 \\ * & * & * & -I \end{pmatrix} < 0, \\
\begin{pmatrix} R_2 & S \\ S^T & R_2 \end{pmatrix} &\geq 0,
\end{aligned} \tag{28}$$

$$\begin{aligned}
\Gamma_{11}^4 &= \begin{pmatrix} \bar{P}_2^T A_{mp}^0{}^T & \xi_1 \bar{P}_2^T A_{mp}^0{}^T & \xi_2 \bar{P}_2^T A_{mp}^0{}^T & \xi_3 \bar{P}_2^T A_{mp}^0{}^T \\ -M^T B_{mp}^1{}^T & -\xi_1 M^T B_{mp}^1{}^T & -\xi_2 M^T B_{mp}^1{}^T & -\xi_3 M^T B_{mp}^1{}^T \\ 0 & 0 & 0 & 0 \end{pmatrix}, \\
\Gamma_{12}^4 &= e_1 P + \xi \begin{pmatrix} \bar{P}_2^T A_{mp}^0{}^T \\ -M^T B_{mp}^1{}^T \\ 0 \end{pmatrix} - \begin{pmatrix} \bar{P}_2 \\ \xi_1 \bar{P}_2 \\ \xi_2 \bar{P}_2 \\ \xi_3 \bar{P}_2 \end{pmatrix}, \quad \Gamma_{13}^4 = \begin{pmatrix} \tilde{B}_{mp} \\ \xi_1 \tilde{B}_{mp} \\ \xi_2 \tilde{B}_{mp} \\ \xi_3 \tilde{B}_{mp} \end{pmatrix}.
\end{aligned} \tag{29}$$

**Proof.** We use *Theorem 1* on system (25), a series of steps is made to deal with nonlinear matrix terms [Fridman and Shaked (2001)]:

- supposing  $P_2 = (\bar{P}_2 \ \xi_1 \bar{P}_2 \ \xi_2 \bar{P}_2 \ \xi_3 \bar{P}_2)$  and  $P_3 = \xi \bar{P}_2$  (the definition of  $P_2$  is for getting  $L$  by LMI, but it introduces the conservatism, till now, this is still an open problem),
- multiplying  $\Gamma^1$  by  $\text{diag}\{\bar{P}_2^{-T}, \dots, \bar{P}_2^{-T}, I\}$  at the left side,  $\text{diag}\{\bar{P}_2^{-1}, \dots, \bar{P}_2^{-1}, I\}$  at the right side,
- making the transformation  $A_{mp}^1 = -B_{mp}^1 L$  and defining  $M = L \bar{P}_2$ , applying Schur formula, then the result follows.

Note that, the main difference of proxy with the Luenberger observer is that, the correction term acts as an input of the system (e.g.  $F_p(t - \tau_1(t))$ ).

The position tracking between the master and the proxy of master has been achieved. And then, the position tracking between the proxy of master and the slave is assured by the controller  $C$ . The model of the system containing the proxy of master, the controller  $C$  and the slave, is given as follow,

$$\begin{cases} \dot{x}_{ps}(t) = (A_{ps} + \Delta A_{ps}(t)) x_{ps}(t) + (B_{ps} + \Delta B_{ps}(t)) w_{ps}(t), \\ z_{ps}(t) = C_{ps} x_{ps}(t). \end{cases} \tag{30}$$

Note that, the input of the proxy,  $F_p(t - \tau_1(t))$ , is also considered as a perturbation,

$$\begin{aligned}
x_{ps}(t) &= \begin{pmatrix} \dot{\theta}_s(t) \\ \dot{\theta}_p(t) \\ \theta_s(t) - \theta_p(t) \end{pmatrix}, \quad z_{ps}(t) = \begin{pmatrix} \theta_s(t) - \theta_p(t) \end{pmatrix}, \\
w_{ps}(t) &= \begin{pmatrix} F_e(t) \\ \hat{F}_e(t) + \hat{F}_h(t - \tau_1(t)) - F_p(t - \tau_1(t)) \end{pmatrix},
\end{aligned} \tag{31}$$

and,

$$\begin{aligned}
A_{ps} &= \begin{pmatrix} A_s - B_s K_s^0 - B_s K_1 & -B_s K_2 & -B_s K_3 \\ 0 & A_m - B_m K_m^0 & 0 \\ 1 & -1 & 0 \end{pmatrix}, \\
\Delta A_{ps}(t) &= \begin{pmatrix} \Delta A_s(t) - \Delta B_s(t) K_s^0 - \Delta B_s(t) K_1 & -\Delta B_s(t) K_2 & -\Delta B_s(t) K_3 \\ 0 & \Delta A_p(t) - \Delta B_p(t) K_m^0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \\
B_{ps} &= \begin{pmatrix} B_s & 0 \\ 0 & B_m \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} B_{ps}^1 & B_{ps}^2 \end{pmatrix}, \\
\Delta B_{ps}(t) &= \begin{pmatrix} \Delta B_s(t) & 0 \\ 0 & \Delta B_p(t) \\ 0 & 0 \end{pmatrix}, \quad C_{ps} = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}.
\end{aligned} \tag{32}$$

The following terms are considered as perturbations at the slave side,

$$\begin{aligned}
\varphi_s(t) &= (\Delta A_s(t) - \Delta B_s(t) K_s^0) \dot{\theta}_s(t) + \Delta B_s(t) F_e(t), \\
\mu_s(t) &= -\Delta B_{ps}^1(t) K x_{ps}(t).
\end{aligned} \tag{33}$$

So,

$$\begin{aligned}\tilde{w}_{ps}(t) &= \begin{pmatrix} B_s F_e(t) + \varphi_s(t) + \mu_s(t) \\ B_m \hat{F}_e(t) + B_m \hat{F}_h(t - \tau_1(t)) - B_m F_p(t - \tau_1(t)) + \varphi_p(t) + \mu_p(t) \end{pmatrix}, \\ \tilde{B}_{ps} &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}.\end{aligned}\quad (34)$$

The system Eq. 30 is rewritten as,

$$\begin{cases} \dot{x}_{ps}(t) = A_{ps}x_{ps}(t) + \tilde{B}_{ps}\tilde{w}_{ps}(t), \\ z_{ps}(t) = C_{ps}x_{ps}(t). \end{cases}\quad (35)$$

The controller gain  $K$  can be obtained by *Theorem 4* in [Zhang et al. (2011)] which handles  $H_\infty$  controller design for the perturbed system without delays. Now, the position tracking between the master, the proxy and slave are ensured. Finally, the objective is to ensure the global stability of the whole system described by,

$$\begin{cases} \dot{x}_{mps}(t) = (A_{mps}^0 + \Delta A_{mps}^0(t))x_{mps}(t) \\ \quad + (A_{mps}^1 + \Delta A_{mps}^1(t))x_{mps}(t - \tau_1(t)) \\ \quad + (B_{mps} + \Delta B_{mps}(t))w_{mps}(t), \\ z_{mps}(t) = C_{mps}x_{mps}(t), \end{cases}\quad (36)$$

where,

$$\begin{aligned}x_{mps}(t) &= \begin{pmatrix} \dot{\theta}_s(t) \\ \dot{\theta}_p(t) \\ \dot{\theta}_m(t) \\ \theta_s(t) - \theta_p(t) \\ \theta_p(t) - \theta_m(t) \end{pmatrix}, \quad w_{mps}(t) = \begin{pmatrix} F_e(t) \\ \hat{F}_e(t) + \hat{F}_h(t - \tau_1(t)) \\ F_m(t) + F_h(t) \end{pmatrix}, \\ z_{mps}(t) &= \begin{pmatrix} \theta_s(t) - \theta_p(t) \\ \theta_p(t) - \theta_m(t) \end{pmatrix}.\end{aligned}\quad (37)$$

So, we can get,

$$\begin{aligned}A_{mps}^0 &= \begin{pmatrix} A_s - B_s K_s^0 - B_s K_1 & -B_s K_2 & 0 & -B_s K_3 & 0 \\ 0 & A_m - B_m K_m^0 & 0 & 0 & 0 \\ 0 & 0 & A_m - B_m K_m^0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \end{pmatrix}, \\ \Delta A_{mps}^0(t) &= \begin{pmatrix} (1,1) & -\Delta B_s(t)K_2 & 0 & -\Delta B_s(t)K_3 & 0 \\ 0 & (2,2) & 0 & 0 & 0 \\ 0 & 0 & (3,3) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \\ (1,1) &= \Delta A_s(t) - \Delta B_s(t)K_s^0 - \Delta B_s(t)K_1, \\ (2,2) &= \Delta A_p(t) - \Delta B_p(t)K_m^0, \quad (3,3) = \Delta A_m(t) - \Delta B_m(t)K_m^0, \\ A_{mps}^1 &= \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & -B_m L_1 & -B_m L_2 & 0 & -B_m L_3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \\ \Delta A_{mps}^1(t) &= \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & -\Delta B_p(t)L_1 & -\Delta B_p(t)L_2 & 0 & -\Delta B_p(t)L_3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \\ B_{mps} &= \begin{pmatrix} B_s & 0 & 0 \\ 0 & B_m & 0 \\ 0 & 0 & B_m \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \Delta B_{mps}(t) = \begin{pmatrix} \Delta B_s(t) & 0 & 0 \\ 0 & \Delta B_p(t) & 0 \\ 0 & 0 & \Delta B_m(t) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \\ C_{mps} &= \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}.\end{aligned}\quad (38)$$

Considering the model uncertainties as perturbations, the whole system is described in the form as follow with novel definition of perturbation  $\tilde{w}_{mps}(t)$ ,

$$\begin{cases} \dot{x}_{mps}(t) = A_{mps}^0 x_{mps}(t) + A_{mps}^1 x_{mps}(t - \tau_1(t)) + \tilde{B}_{mps} \tilde{w}_{mps}(t), \\ z_{mps}(t) = C_{mps} x_{mps}(t), \end{cases}\quad (39)$$

$$\tilde{w}_{mps}(t) = \begin{pmatrix} B_s F_e(t) + \varphi_s(t) \\ B_m \hat{F}_e(t) + B_m \hat{F}_h(t - \tau_1(t)) + \varphi_p(t) \\ B_m F_m(t) + B_m F_h(t) + \varphi_m(t) \end{pmatrix}, \quad \tilde{B}_{mps} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.\quad (40)$$

By *Theorem 1*, we can verify the global stability of the system. Then the force tracking,  $F_m(t) = \hat{F}_e(t - \tau_2(t))$ , is achieved on the basis of the control scheme.

#### 4. ILLUSTRATIVE EXAMPLE

To evaluate the performance of the proposed approach, different working conditions have been simulated. The maximum amplitude and sampling time of time-varying delays are  $0.2s$  (greater amplitude of delays can also be handled) and  $0.001s$ , which satisfy most network-based applications of teleoperation system. Note that, the time-varying delays in two channels are asymmetric.

The master, the proxy of master and the slave models are described as simple integrators,  $1/s$ ,  $1/s$  and  $2/s$ . The model uncertainties are all modeled as  $\Delta A_i(t) = \Delta B_i(t) = \sin(t)$ , thus,  $G_i = D_i = H_i = E_i = 1$ ,  $i = \{m, p, s\}$ . By our local controller design procedure,  $K_m^0 = 49.6361$ ,  $K_s^0 = 7.5208$ .

The gains of the proxy of master and the controller  $C$ , the corresponding  $\gamma_{min}^L$ ,  $\gamma_{min}^K$ , and the global stability with  $\gamma_{min}^g$  are presented as follows,

$$\begin{aligned}L &= (0.8926 \quad -2.1504 \quad 158.2891), \quad \gamma_{min}^L = 0.0122, \\ K &= (9.0646 \quad -3.7807 \quad 445.6875), \quad \gamma_{min}^K = 0.0058, \\ \gamma_{min}^g &= 0.0120.\end{aligned}\quad (41)$$

For comparison reasons, the Lyapunov-Krasovskii functional in [Fridman (2006), Zhang et al. (2011)] is also applied in the design of slave controller under the same design procedure, and we get,

$$\gamma_{min}^L = 0.0156, \quad \gamma_{min}^K = 0.0058, \quad \gamma_{min}^g = 0.0126.\quad (42)$$

We can see that, Lyapunov-Krasovskii functional proposed in this paper improves the results and reduces the conservatism.

Note that, the  $\rho_A$ ,  $\rho_B$ ,  $\xi$ ,  $\xi_i$ ,  $i = 1, 2, 3$  proposed above are important variables when verifying the LMI conditions. A query loop algorithm is considered to search for their best values.

##### 4.1 Tracking in Abrupt Changing Motion

Figure 2 shows the position tracking between the master and slave under time-varying delays and uncertainties, where the human operator ( $F_h(t)$ ) is modeled as the pulse generator.

From the Figure, we can see that, our method achieves the position tracking, especially at the mutation point (amplified part in Figure 2), good position convergence between the master and slave has been presented.

##### 4.2 Tracking in Wall Contact Motion

Similarly, the position tracking in wall contact motion is presented in Figure 3. Here, the slave is driven to the hard wall with a stiffness of  $K_e = 30kN/m$  located at the position  $x = 1.0m$ .

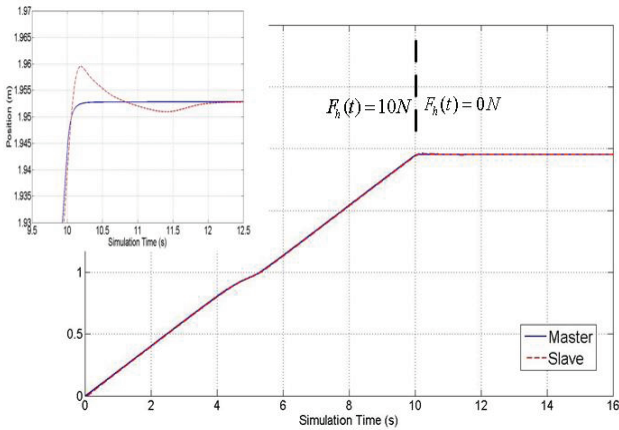


Fig. 2. Position response in abrupt tracking motion.

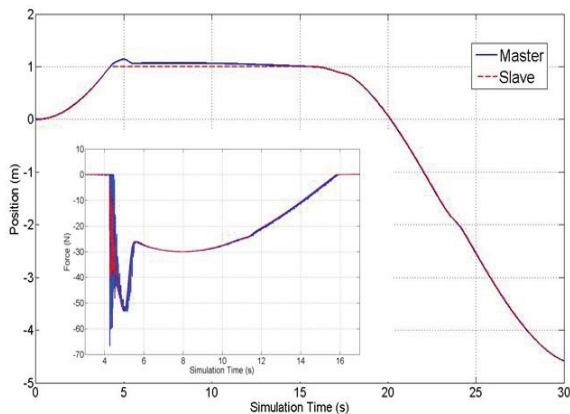


Fig. 3. Position/force response in wall contact motion.

Based on  $H_\infty$  control, the time-varying model uncertainties are handled by our proxy of master and controller  $C$ . The force tracking,  $F_m(t) = \hat{F}_e(t - \tau_2(t))$  can be seen in the smaller Figure of Figure 3.

## 5. CONCLUSIONS

In this paper, we extend our control approach in [Zhang et al. (2011)] to address the stability and performance problem under model uncertainties. Thanks to Lyapunov-Krasovskii functionals,  $H_\infty$  control theory and LMI, firstly, local controllers are designed, and then, the slave controller is achieved.

The simulations achieved by YALMIP and SIMULINK demonstrate that such a teleoperation system can eliminate the influence of perturbations introduced by model uncertainties, and can run in different working conditions.

## REFERENCES

- Arcara, P. and Melchiorri, C. (2002). Control schemes for teleoperation with time delay: a comparative study. *Robotics and Autonomous Systems*, 38(1), 49–64.
- Chiasson, J. and Loiseau, J.J. (2007). *Applications of Time Delay Systems*. Springer.
- Fridman, E. (2006). A new Lyapunov technique for robust control of systems with uncertain non-small delays. *IMA J. Math. Contr. Infor.*, 23(2), 165–179.

- Fridman, E. and Niculescu, S.I. (2008). On complete Lyapunov-Krasovskii functional techniques for uncertain systems with fast-varying delays. *Int. J. Robust Nonlinear Control*, 18(3), 364–374.
- Fridman, E. and Shaked, U. (2001). New bounded real lemma representations for time-delay systems and their applications. *IEEE Trans. Automat. Contr.*, 46(12), 1973–1979.
- He, Y., Wu, M., She, J.H., and Liu, G.P. (2002). Parameter-dependent Lyapunov functional for stability of time-delay systems with polytopic-type uncertainties. *IEEE Trans. Automat. Contr.*, 49(5), 828–832.
- Hokayem, P.F. and Spong, M.W. (2006). Bilateral teleoperation: an historical survey. *Automatica*, 42(12), 2035–2057.
- Jiang, X. and Han, Q.L. (2005). On  $H_\infty$  control for linear systems with interval time-varying delay. *Automatica*, 41(12), 2099–2106.
- Kruszewski, A., Jiang, W.J., Fridman, E., Richard, J.P., and A.Toguyeni (2011). A switched system approach to exponential stabilization through communication network. *IEEE Trans. Contr. Syst. Technol.*, to appear.
- Nuño, E., Basañez, L., and Ortega, R. (2011). Passivity-based control for bilateral teleoperation: a tutorial. *Automatica*, 47(3), 485–495.
- Park, P., Ko, J.W., and Jeong, C. (2011). Reciprocally convex approach to stability of systems with time-varying delays. *Automatica*, 47(1), 235–238.
- Richard, J.P. (2003). Time delay systems: an overview of some recent advances and open problems. *Automatica*, 39(10), 1667–1694.
- Ryu, J.H., Preusche, C., Hannaford, B., and Hirzinger, G. (2005). Time domain passivity control with reference energy following. *IEEE Trans. Contr. Syst. Technol.*, 13(5), 737–742.
- Xu, B. and Liu, Y.H. (2003). Delay-dependent/delay-independent stability of linear systems with multiple time-varying delays. *IEEE Trans. Automat. Contr.*, 48(4), 697–701.
- Xu, S., Lam, J., and Zou, Y. (2006). New results on delay-dependent robust  $H_\infty$  control for systems with time-varying delays. *Automatica*, 42(2), 343–348.
- Ye, Y.Q., Pan, Y.J., and Gupta, Y. (2009). Time domain passivity control of teleoperation systems with random asymmetric time delays. In *Proceedings of IEEE Conf. on Decision and Control, and Chinese Control Conference (CDC'09, CCC'09)*, 7533–7538. Shanghai, China.
- Zhang, B., Kruszewski, A., and Richard, J.P. (2011). Tracking improvement based on the proxy control scheme for bilateral teleoperation system under time-varying delays. In *2011 IEEE Int. Conf. on Emerging Technologies and Factory Automation (ETFA'11)*. Toulouse, France.