Bio-inspired paradigms in Network Engineering Games

Eitan Altman

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Eitan Altman
INRIA Sophia-Antipolis,
2004 Route des Lucioles,
06902 Sophia-Antipolis Cedex, France
email: Eitan.Altman@inria.fr

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Abstract

Network Engineering Games (NEGs) is a new emmerging branch of game theory that has been developing in Electrical Engineering Departments. It concerns games that arise in all levels of telecommunication networks. There has been a growing interest among researchers in this community in bio-inspired methodologies in recent years. This is due to two reasons. First, many problems in networking have a lot in common with problems encountered in biology. Some examples are (i) propagation of information in networks, that has often a similar dynamics as the propagation of epidemics within the population. (ii) energy management issues in wireless networks and competition over resources are often similar to issues that have been studied by biologists. Secondly, both equilibria concepts as well as replicator dynamics that arise in evolutionary games are quite relevant to NEGs. In this paper we first present an overview of applications and of tools used in network engineering games, we then describe in more depth bio-inspired tools used in or relevant to network engineering games. We present finally an example of a stochastic epidemic game arising in wireless networks that involves competition over the relaying of information.

1 Introduction

This paper has three goals. First, introduce Network Engineering Games, a relatively new area of application of game theory. After a brief description of the layered structure of telecommunication networks, we present a survey on both applications as well as methodologies related to games that arise in different layers. We then introduce in more details bio-inspired paradigms that are used in network engineering games. These include epidemic, evolutionary games and their extensions. Our third goal is to introduce a detailed example of a competition between epidemics whose solution requires novel tools in Markov games.

1.1 Modeling a telecommunication network: the OSI layer standard

A network may be used to transfer data files, or real time traffic (e.g. voice or video) or both. Transmission may occur over various types of media - cables, wireless radio channesls, optical fibers etc. For each type of media, information is transmitted differently: electrical current is used in cables, electro-magnetic waves in radio channels, and lazer beams are used in optical fibers. The transformation of the original information (data, voice or video) into a form that is suitable for transmission over a given medium is called modulation. There may be various types of modulations that can be used over the same medium, each requiring a different amount resources, and each providing a different quality.

A simple way to represent a network is through nodes, that may stand for equipements such as base-stations, terminals, routers etc, and links which interconnect nodes. In the road traffic context, a link is a binary object: it may exist or not. A capacity may further be associated with it, which describes the maximum amount of traffic that it can carry. In telecommunications, in contrast, the notion of link is not necessarily binary. A radio channel may add noize or interference to the modulated signal resulting in erroneous decoding. Yet by adding some extra redundancy or encoding, errors can be detected and corrected. Encoding and decoding requires resources such as decoding processing that introduces delays.

The transmission of information in a telecommunication involves actions taken at various levels (called layers). Those just described are part of the lowest layer (called the physical layer). There is an "Open Systems Interconnection" (OSI) standard developed already in the 1980s by the International Standards Organization that introduces those different layers, which we briefly describe below.

When one wishes to send or receive a file, or an interactive voice or video call, the information is segmented into packets. Packets are treated at various network layers. Each layer can introduce further errors and delays. Indeed, packets may have to be buffered, and may be lost if there is congestion (buffer overflow).

Layer 1: The Physical Layer

Defines the electrical and physical specifications for devices, their connection to the channel (wired or wireless). It contains aspects such as modulation, power control, some coding and decoding.

Layer 2: The link and Medium Access (MAC) layer

Link and Medium Access layer: take care of communication over a link, (i.e. a local connection between two neighboring network nodes), or of coordinating the access to a common channel. Corrects errors introduced in the physical layer. Takes care of flow control (decisions concerning the rate at which one transmits) and schedulling decisions (which source will have access to transmit over the linkg at a given time) that concern a link.

Layer 3: the Network Layer

The network layer is concerned with routing - deciding how to route each packet. In general, different packets of the same call (data transfer or voice or video call) may follow different routes.

Layer 4: the Transport Layer

This layer takes care of the end-to-end connectivity, and of retransmission if needed. In particular, when transferring data, the destination is able to identify those packets it did not receive well and can request their retransmission.

Layer 5: Session Layer

This layer takes care of opening and closing sessions as well as of initiating dialogues between computers.

Layers 6-7 The Presentation and Applicatioin Layers

These layers are concerned with networking aspects that are related to applications, such as the downloading files, the way to connect to the World Wide Web along with the use of HTTP, peer to peer communication etc.

2 Overview of Pioneer Work in Network Engineering Games

In this section we begin with a description of the pioneer research on game theory within the NEG community of telecommunication games. We then provide a broad multidisciplinary overview of the development of networking game theory and of other foundations of game theory that are relevant to telecommunications and to our future research. This state of the art focuses on those areas of networking games that we chose to work on in the proposal. For other networking games that are of interest to NEGs, we refer to the surveys [4, 6, 5] and the books [17, 29, 31, 33, 36, 48, 19, 49, 38, 41, 1].

2.1 Pioneering research related to NEG within the telecommunications community

Whereas the strong developments of game theory and information theory occurred approximately at the same time of history, namely in the middle of the 20th century with the major works by Von Neumann, Morgenstern, Nash, and Shannon, it is only in the last twenty that the community of communication networks has started integrating game theory. Nevertheless, some papers that combine communication theory with game theoretic tools appeared even before 1990: In 1957, point-to-point communications are studied in [16] as a game between the channel encoder (choosing the best input distribution in terms of mutual information) and channel (choosing the worse transition probability in terms of Mutual Information), in 1971, a source coding is seen in [15] as a game between the source encoder and switcher (modifying the source distribution). In [45] the author exploits game theory for the joint signal-and-detector design using game-theoretic techniques to perform multi-parameter optimization, and in [21] a legal encoder-decoder pair fights against a jammer.

The first papers on **power control** using game theory appear on 1998 with the pioneering work [22] and [27]. To get an idea of the impact of game theory on research in communications, I searched on 20/12/2011 on Scholar Google the documents containing wireless networks together with power control. 20500 documents were found. Of these, 3380 appeared in 2011, and 1680 dated from 2000 or earlier. I then repeated the experience restricting further to documents containing game theory. 2600 documents were found. Of these, 20 dated from prior to 2001 and 580 dated from the single year 2011. The share of documents containing game theory thus increased from 1.2

Flow control games appear in the telecommunications community already in 1989 with the dissertation of Douligeris [20]. They were further studied by Mazumdar and by Prof A A Lazar and their coauthors [32, 25].

Routing games were brought into the community of Telecommunication Networks by Orda et al [37] on 1993. The special feature that differentiates this reference from previous work on RGs (in other communities) is that it considers finitely many decision makers, each of which can split its traffic among several paths (we shall present more details later). It is of interest for the telecommunication community since it models well competition between service providers that can control routes for traffic to or from their subscribers. The model studied in [37] restricts to cost of paths that can be expressed as the sum of costs over the links along the path. There are very few papers on RGs with other types of costs: [13] studied a Braess type paradox where the performance measure is blocking probabilities (the Braess paradox comes from road traffic: it consists in a network with a common source and destination, and with two possible paths). As in the original Braess paradox, it is shown that when adding a new link to the network, the delays of all users increase at the (unique) equilibrium. We have further studied such paradoxes in [3] in a network with sources at different nodes. We showed that three equilibria exist and that a Braess paradox occurs there as well. [50] studies numerically a paradox in network of parallel links involving loss of packets due to finite buffers. Beyond these games occurring in specific examples of networks, there has been no theory for RGs with non-additive delays.

3 Paradigms inspired by Road Traffic.

Population (non-atomic) games.
These are games with a continuum (infinite) set of non-atomic players. Non-atomic means that the action of a single player has a negligible impact on the utilities of other players. These games which had an important impact on road traffic engineering are known there as the Traffic Assignment Problem, formalized by Wardrop [47] to model the choice of routes of cars where each driver, modeled as a non-atomic player, minimizes his expected travel delay. This game was solved in [14], by showing that it can be transformed into a global optimization problem. The cost function that appears in the global optimization problem is called a potential of the game. The theory mainly treats additive costs: the cost (delay) over a path is the sum of delays over each of its links. The link cost is assumed to be a function of the total amount of its flow. Some research [38] has been devoted to multimodal traffic where one considers different traffic types (pedestrian, cars, bicycles, trucks etc.). The link cost may differ from one type to another, and may depend explicitly on the flows of each type through the link. In both cases a potential does not exist anymore which renders the problems much harder to solve [38].

Atomic non-splitable games.
The setting is the same as in the previous model except that there are finitely many drivers. The link cost depends in general on the number of drivers using it and are again additive along paths.

Congestion Games.
Rosenthal [39, 40] introduced these games in 1973 and solved them game using a potential similar to [14] assuming that the link cost (or delay) depends only on the sum of flows traversing the link and is the same for all users. The equilibrium is obtained through an integer linear program and can be achieved also in a decentralized way in which players update their decisions (using a best response) in an asynchronous way (one at a time). Again, the potential disappears once we allow costs to depend on the player. Crowding Games, introduced by Milchtaich [34] (and references therein) are a special class of congestion games in which the cost is allowed to be user specific. They too are non-splitable discrete routing games. It extends congestion games in allowing the link cost to depend explicitly on the flow of each player that traverses it rather than on the sum of flows. It is more restrictive than congestion games in that two different paths do not have common links. Under some assumptions on the costs, some properties of congestion games still hold, even when a potential no more exists.

Non-atomic routing games as well as non-splitable atomic games are concerned with decisions of individuals. The theory serves in predicting the congestion as a function of the topology of the network and the capacity of its links. It is thus useful for a network manager or network owners. In contrast, routing games as developed in Electrical Engineering departments are often concerned with the Internet Service Providers (ISPs) each controlling the routes taken by many users (subscribers).

The deregulation of telecom industry in Europe during the 1990-2000 and the opening of national markets to competition between ISPs triggered research on a (relatively) new class of routing games called Competitive Routing (Splitable atomic games). They are concerned with a finite (or more generally discrete) number of decision makers (ISPs) whose decisions concern a continuum (infinite number) of non-atomic individuals. Their systematic study started with the pioneering work by Orda et al. [37]. Yet, some previous work on this framework had already in the context of road traffic earlier. We refer to [24] who showed that the equilibrium in these types of games converges to the Wardrop equilibrium used in road traffic, as the number of players increases to infinity.

For an extensive overview of networking games in road traffic we recommend the book [38]; for the crossover between games in road traffic and in telecommunication, we refer to the whole special issue that we edited on that topic in the journal Networks and Spacial Economics, 4(1), March 2004. The contribution of the community of Algorithmic games to RGs appears in [36].

4 Overview of Bio-inspired paradigms in NEGs

4.1 Evolutionary Games.

In this branch of game theory, pairs of players that play a matrix game are selected at random. These games were introduced in [42] to explain and predict evolution of species i.e. the fraction of the population (or of populations in the case of several classes) that play each possible action at equilibrium. These games introduced various equilibrium notion of equilibrium. The mostly used is the concept of Evolutionary Stationary Strategy, which is a strategy that is immune against mutations. Evolutionary games is concerned not only with equilibrium but also with evolution of competition through the so called replicator dynamics. Through differential equations, they describe how the fraction of the population that uses different actions evolves. They link the utility (called fitness) to reproduction, where a higher fitness means a larger reproduction rate.

There are several reasons that Evolutionary Games and their generalaizations are relevant to NEGs:

- The classical Nash equilibrium concept seems to be too restrictive in many applications in which there are many decision makers. Indeed, a Nash equilibrium describes a stable situation in which no player has an insentive to deviate unilaterally. However, in a situation where many players interact with each other, one would typically prefer to have robustness under simultaneous deviations of some fraction of the decision makers.

- Many systems are developed based on agents that take local decisions regarding the operation of the system. A large effort has been invested in equipping such systems with self organization and optimization features. This brought up the question of dynamics of competition. Many systems are equipped with evolutionary features in which behavior that is considered to be useful or more fit, gets propagated faster in a population. Thus evolution is not only a tool to predict the future and explain the past, but also a tool for designing systems with built in adaptability. Evolution is engineered. Wireless communications is one such system, and the self organisation features are part of the work that is being done in standardisation organizations such as the 3GPP [23].

We next mention some research issues that concern evolutionary games in NEGs.

4.1.1 Delayed replicator dynamics

Decision making occurs in wireless communications in different time scales. For example, in the WCDMA technology, power control decisions are taken 1500 times per second. Routing decisions that concern which base station should a mobile be associated with occur at a much slower time scale. A wireless telephone terminal has a typical life time of 2-3 years. This means that although many decisions are taken at a very fast time scale, the fitness related to these decisions suffers a huge delay. Indeed, in evolutionary games, we interpret the fitness of a given protocole as a measure of the rate of growth in its use. Now, most protocoles (such as power control) are performed without the user's intervention. This does not mean that the user is not a player. Indeed, when deciding which cellular phone to buy, a technology that has better performance and lower costs will be prefered. This huge delay between control decisions taken and the fitness received for using them may cause instabilities in replicator dynamics that could be used for controlling the evolution of the system. This motivated research on delayed replicator dynamics in NEG applications [44].

4.1.2 Complex interactions between players

Evolutionary game is usually concerned with pairwize interactions between randommly selected pairs of players. In applications in NEGs, the number of players that are involved in an interaction may also be less or more than 2. It may be a random number. This number may be known or not known to a player when it takes a decision. Another source of difficulties is that there need not be reciprocity in who plays with who: player A may be affected by decisions of a player B without B being affected by A. Indeed, this can happen whenever the receiver of B is close to A but the receeiver of A is far from B. These features of evolutionary games in network engineering are summarized in [43].

4.1.3 Markov Decision Evolutionary Games (MDEG)

We introduced in [8] Markov Decision Evolutionary Games (MDEGs) that extended EVGs: each player has an internal state. The fitness received in a local interaction with another player depends then not only on the actions chosen but also on the internal states of the players. Moreover, the internal states of individuals that interact change with probabilities that depend on both the actions and the internal states of the individuals. Finally, an individual's objective is not to maximize its fitness but rather to maximize its sum of expected fitness during its lifetime, or its time-average fitness. We have been able so far to use this model in several problems in wireless communications and computed the equilibrium.

4.1.4 Sequential Anonymous Games (SAG)

This class of games introduced in [28] has a structure similar to MDEG. The difference is that in MDEGs, strategies interact through pairwise interactions between players, whereas in SAGs, the interactions involves a cumulative effect of a whole infinite class of players. As in MDEGs, each player has its own Markov chain whose transition probabilities depend on the state and action of that player as well as the global state and the policy used by other players. Only one special utility functions (the discounted cost) has been studied in [28].

4.2 Control of Epidemics

Computer viruses have been reported to cause a damage of 17 billion US$ on 2000. Already in 1998, the relation between computer viruses and epidemiology are suggested [35]. Since then, viruses and tools to fight them have become more sophisticated. In the biology literature, there has been almost no research using game models in epidemics. We found no references to games in epidemics before 2000. Some isolated research on the topic has appeared in the last years [9, 10, 12, 11]. These papers use simple game theoretic tools to model some decisions related to fighting against viruses (namely vaccination decisions and decisions concerning observations of epidemics).

We have been using optimal control theory within the classical epidemic models so as to (i) identify the worst possible computer-viruses attacks in a given network (ii) to fight viruses whose behavior is described by classical epidemic models. These results were also novel with respect to epidemiology (and not only computer viruses). The challenge is to model (i) and (ii) together, i.e. to consider the strategic interaction between security software and a malicious malware, in which each side is aware of the other. We shall model such interactions through zero sum dynamic game formulation which we shall use to get new insight on the vulnerability of the networks to attacks and to efficient defense strategies. Some first steps in that direction are reported in

Epidemics and their control have also been used to model diffusion of files in a peer to peer (P2P) network, and to study energy efficient propagation of information in wireless networks based on smartphones which communicate with each other using direct links (Bluetooth or WiFi) rather than using the cellular network telephony.

5 Challenges in Bio-Inspired Games

We specify below three challenges in developing bio-inspired game foundations for NEGs.

(1) Creating an Atomic Evolutionary Game framework Already in routing games we mentioned that the community of NEGs is faced with situations in which a decision maker (e.g. an Internet Service Provider) controls the decision of a large number of individuals (e.g. the subscribers routes). In existing evolutionary games (EVGs), a player is related to one interaction in time and space. EVG theory does not consider situations in which several individuals that are involved in different (possibly simultaneous) interactions correspond to some common decision maker. We are thus faced with creating a new class of EVGs. The need for creating such framework arises also in other communities, as we learnt from the discussion in [46], pages 73-74 that tries to identify who should be considered as a player in the biological context: an individual? A group of individuals? A species? (Or equivalently a gene?) A strategy? The reference mentions bees as an example. Pairwise interactions between bees from different beehives have an impact on the utility of the whole beehive (or perhaps of the whole corresponding species of bees).

(2) Engineering Evolution EVGs have been used in biology to explain evolution of species and to predict future evolution. The challenge in introducing EVGs into engineering communities is to use EVGs to engineer evolution and not just to predict or explain evolution. As an example, replicator dynamics have been used as models of evolution dynamics as observed in biology. In engineering we have the option of designing these dynamics by optimizing its parameters, or even to propose new types of dynamics.

(3) Epidemic vectors and acceleration: In the context of diffusion of contents in a network, epidemics compete with each other on popularity. There are several measures of popularity:

- (i) the total number of destinations that were interested by the content (either downloaded it, or read its description)
- (ii) the same number but restricted to a recent period (e.g. number of downloads in the past week).

Two types of actors wish to accelerate the epidemics:

A source of information (e.g. movie producers) is interested to maximize the number of people who will get information (who would eventually watch the film). Within the game context of competition between such epidemics, each source of information can use several complementary social networks to spread information: e.g. twitter, facebook and youtube. This can be viewed as a vector of epidemies. For example, short advertisements on twitter would be one epidemy, they would provide with links to youtube which would diffuse a trailer for a film (2nd epidemics). In facebook or in other social networks, more details as well as cinema critics and ranking would consist of a 3rd epidemics. The challenge is to model the coupling between these three related epidemics as well as the resulting epidemic which consists on going to watch the film, and to provide guidelines to engineer them.

A second actor is the content provider (CP). It may be the owner of caches and of social networks. CPs are interested to use their resources so as to maximize the rate of downloads. By identifying sufficiently early popular content, the CP can target that content when assigning resources. It can provide it with larger visibility, put it in caches etc. This is also a new topic within the context of games between contents.

6 An example of an epidemic game

We present in some details an example of a game in wireless networks. It illustrates the need in novel tools in stochastic games, and on the same time it is concerned with diffusion of information and has several features that places it within the class of bio-inspired games.

Consider a network with N mobile users each having some content that it wishes to disseminate in some town. These mobiles are the N players in a stochastic game that we describe below. The competition will be over the access to some resources (relays) and the actions available will correspond to transmissioin power management. This will include transmission power control and control of the recharging of the baterry.

6.1 The description of the problem and its model

We call the content of the ith source a type i content. The content of the players is disseminated through a fleet of M mobile relays. We assume that a mobile relay can store no more than one content. The transmission between the source and a relay may occur when they are within some range of each other. This range is determined by the trasmission power which is controlled by the sources. When source n transmits at its lowest available power, then it meats each of the relays at times that form a Poisson process with rate λ_i. We call the process that describes the times in which a mobile source and a mobile relay are within range of each other a "contact process". Each source has several available transmission power levels. When transmitting at the jth power level, then the contact process with each relay is a Poisson point process with rate $\lambda_i a_i^j$, where $a_i^j \geq 1$ are some contsants.

System state. We shall describe the system as the combined state of each player. The state of player i is given by a couple (x_i, k_i).

- The first component is an integer corresponds to the number of relays that have content type i. We call this the "dissemination state". It is non-negative. The sum of x_i over all players i is bounded by M, the total number of relays. We shall denote by $\mathcal{M}$ the set of states in which the bound is actually achieved. $\mathcal{M}$ corresonds to the set of states in which each destiniation has already some content.

- The second component corresponds to the energy state of the battery. We consider $K+1$ possible energy levels, where $k=0$ corresponds to an empty battery, and $k=K$ corresponds to a full battery.

We shall denote the state space of a player i by $(\mathbf{X}_i, \mathbf{K}_i)$.

Actions. Each player controls the transmission power. The larger the transmission power, the larger is the range of the transmission and therefore the rate of the contact process between the source and each one of the relays. The actions (of choosing transmission powers) are described as the real numbers a_i^j, so that when using action a_i^j, player i increases the rate of the Poisson contact process by a multiplicative factor of a_i^j.

For each player i, the set $\mathbf{A}_i$ actions available to player i includes $a_i^1 = 1$, which means to transmit at the lowest available power. It includes the action $\overline{a}_i$ which denotes the one corresponding to the largest transmission power. It also contains the action $a_i^0 = 0$ in which there is no transmission but in which the battery may charge and may thus move result to higher energy states. We assume that player i can only charge the battery when it is not full, i.e. when $k_i < K$. On the other hand, when the battery is empty then $a_i = 0$ is the only available action. The time it takes to increase the battery energy level by one unit is exponentially distributed with parameter μ. Let $\mathbf{A}$ be the product action space of $\mathbf{A}_i$, $i = 1, ...N$.

Transition probabilities. Transmission at a larger power has two outcomes: a larger rate of the contact process and a larger rate of depletion of the battery. As mentioned, the action a_i corresponds to accelerating the the rate of the contact process (between source i and any one of the relays) by a factor of a_i. Let $Q(k,a)$ be the rate at which the energy level decreases when at level k and when a is used. Let $\rho = \max_{k,a} Q(k,a)$. We use the standard uniformization approach [30] to describe the game as a discrete time Markov game.

The uniformization is based on adding fictitious transitions at each state so that the total rate of transitions (which we call λ below) is the same in all states and under all actions. (Adding fictitious transitions at a given state means to add transitions from that state to itself in a way that does not change the probability distribution of the stochastic processes describing the states and actions.) Then one considers the state and action processes right after a real or a fictitious transition occurs. We denote by σ_n, $n = 1, 2, 3, ...$ these instants. The process observed at these times is a discrete time Markov game. The adaptation of the uniformization to the Risk sensitive criterion criterion can be found in [18].

The transition probabilities are defined as follows. Define $\lambda = M \sum_{i=1}^{N} \lambda_i \overline{a}_i + \mu + \rho$. Then we have

$$P_{(\mathbf{x},\mathbf{k}),\mathbf{a},(\mathbf{z},\mathbf{m})} = \tag{1}$$

$$
\begin{cases}
(M-|\mathbf{x}|)\frac{a_i\lambda_i}{\lambda} & for \quad \mathbf{z}=\mathbf{x}+e_i, \mathbf{m}=\mathbf{k}, \\
 & \qquad \mathbf{x}\in\mathbf{X}\setminus\mathcal{M}, a_i>0, \\
\frac{Q(k_i,a_i)}{\lambda} & for \quad \mathbf{z}=\mathbf{x}, \mathbf{m}=\mathbf{k}-e_i, \\
 & \qquad a_i>0, \\
\frac{\mu}{\lambda} & for \quad \mathbf{z}=\mathbf{x}, \mathbf{m}=\mathbf{k}+e_i, \\
 & \qquad m_i<K, a_i=0, \\
1-\frac{1}{\lambda}\left[\sum_{i=1}^{N}(M-|\mathbf{x}|)a_i\lambda_i+\mu 1\{a_i=0\}+Q(k_i,a_i)\right] & for \quad \mathbf{z}=\mathbf{x}, \mathbf{x}\in\mathbf{X}, \mathbf{m}=\mathbf{k}.
\end{cases}
$$

The cost. Each relay meets each end user according to a Poisson process with rate η. We assume that when such an end user seeks for content type i then it needs it within some time T_i. We define the ith type faliure probability as the probability that an end user mobile searching for type i content does not reach it within time T_i. Player i's goal is to minimize this probability.

If there were a fixed number $\overline{x}_i$ of relays that had a content type i, then a mobile user that arrives to town and wanted to receive content i would have to wait an exponentially distributed time with mean $1/(\overline{x}_i\eta)$. Thus, the probability that it would have to wait longer than T_i would be given by

$$P_f(i) = E\left[\exp(-\eta\overline{x}_iT_i)\right] \tag{2}$$

For any content type i, $i=1,...,N$, the number $X_i(t)$ of relays that have a content type i is non-decreasing. Furthermore, it is bounded since $\sum_{i=1}^{N} X_i(t) \le M$. Therefore after some finite time τ, $X_i(t)$ reaches a limit denoted by $\overline{x}_i$ for all i. For all $t>\tau$, the type i faliure probability for a mobile arriving at t is given by (2).

With this interpretation of $\overline{x}_i$, we find it convenient to rewrite the cost (2) as

$$P_f(i) = E\Big[\exp(-\eta T_i\int_0^\infty dX_i(t))\Big].$$

After uniformization, this becomes:

$$P_f(i) = E\left[\exp\left(-\eta T_i\left(X_i(0)+\sum_{n=0}^{\infty}1\{X_i(\sigma_{n+1})-X_i(\sigma_n)=1\}\right)\right)\right] \tag{3}$$

Define

$$\delta_j^1(v,\mathbf{x},\mathbf{k}) = \frac{v(\mathbf{x}+e_j,\mathbf{k})}{v(\mathbf{x},\mathbf{k})}, \qquad \delta_j^2(v,\mathbf{x},\mathbf{k}) = \frac{v(\mathbf{x},\mathbf{k}+e_j)}{v(\mathbf{x},\mathbf{k})}.$$

Define for each player i, $\mathbf{k}\in\mathbf{K}$, $\mathbf{x}\in\mathbf{X}\setminus\mathcal{M}$, $\mathbf{a}\in\mathbf{A}$ and $v\in R^{|\mathbf{X}|}$:

$$
\begin{aligned}
J^i(v,\mathbf{x},\mathbf{k},\mathbf{a}) &= \frac{1}{v(\mathbf{x},\mathbf{k})}\sum_{\mathbf{z},\mathbf{m}} P_{(\mathbf{x},\mathbf{k}),\mathbf{a},(\mathbf{z},\mathbf{m})}\exp(-\eta T_i 1\{z_i=x_i+1\})v(\mathbf{z},\mathbf{m}) \\
&= \frac{1}{\lambda}\sum_{j=1}^{N}\Big[(M-|\mathbf{x}|)a_j\lambda_j(\exp(-\eta T_i)1\{j=i\})\delta_j^1(v,\mathbf{x},\mathbf{k})) \\
&\quad +1\{a_j\neq 0\}Q(k_j,a_j)\delta_j^2(v,\mathbf{x},\mathbf{k})-\mu\delta_j^2(v,\mathbf{x},\mathbf{k}-e_i)1\{a_i=0\}\Big]
\end{aligned}
$$

and set $J^i(v, \mathbf{x}, \mathbf{k}, \mathbf{a}) = 1$ for $x \in \mathcal{M}$. $v(\mathbf{x}, \mathbf{k})$ times $J^i(v, \mathbf{x}, \mathbf{k}, \mathbf{a})$ is the total cost for player i if at time 0 the system is at state $\mathbf{x}$, player j takes action a_j (where a_j is the jth component of the action vector $\mathbf{a}$) and the utility to go for player i from the next transition onwords is $v(\mathbf{y})$ if the state after the next transition is $\mathbf{y}$.

Let $\mathbf{u}$ be a mixed stationary multi-policy. With some abuse of notation we define for each player i and for each $\mathbf{x} \in \mathbf{X} \setminus \mathcal{M}$ and $\mathbf{k}$

$$J^i(v, \mathbf{x}, \mathbf{k}, \mathbf{u}) = \frac{1}{\lambda} \sum_{j=1}^{N} \Big[(M - |\mathbf{x}|) \left[\sum_{a \in \mathbf{A}_j} u_j(a|\mathbf{x}, \mathbf{k}) a \right] \lambda_j \Big(\exp(-\eta T_i 1\{j = i\}) \delta_j^1(v, \mathbf{x}, \mathbf{k}) \Big)$$

$$+ \sum_{a \in \mathbf{A}_x, a \neq 0} Q(k_j, a) u_j(a|\mathbf{x}, \mathbf{k}) \delta_j^2(v, \mathbf{x}, \mathbf{k}) - \mu \delta_j^2(v, \mathbf{x}, \mathbf{k} - e_i) u_j(0|\mathbf{x}, \mathbf{k}).$$

and set $J^i(v, \mathbf{x}, \mathbf{u}) = 1$ for $x \in \mathcal{M}$.

$$1 = \min_{u \in \Delta(\mathbf{A}_i)} J^i(v_i, \mathbf{x}, \mathbf{u}) \tag{4}$$

Theorem 1. *(i) The fixed point equation (4) has a solution v^*.*
(ii) Let v^* be such a fixed point. Any mixed stationary multi-policy $\mathbf{u}$ that achieves the argmin of (4) for all i is a mixed stationary Nash equilibrium.

Proof. (4) are describes a set of N dynamic programming equations, where the ith one corresponds to the best response for player i when the other use the stationary policies $u_j, j \neq i$. (The dynamic programming formulation for the total cost risk sensitive criterion can be found in [26].) The rest of the proof follows the same steps as those of the standard total cost criterion for absorbing stochastic games, see [7]. ∎

6.2 Aggregation

We show that one can transform the stochastic game into an equivalent one which has the same action space but a much simpler state space: the dissemination state space $\mathbf{X}$ can be replaced by a one dimensional set given by $\overline{\mathbf{X}} := \{0, 1, ..., M\}$. We call the latter the aggregated dissemination state space, and we call $\overline{\mathbf{X}} \times \mathbf{K}$ the aggragated state space. An aggregated dissemination state $i \in \overline{\mathbf{X}}$ corresponds to the set of dissemination states $\mathbf{x} \in \mathbf{X}$ such that $|\mathbf{x}| = i$. It thus counts the total number of relays that have some content. Taking the summation in (1) we get the following transition probabilities for the aggregated Markov game:

$$P_{(x,\mathbf{k}),\mathbf{a},(z,\mathbf{m})} = \tag{5}$$

$$\begin{cases} (M - |\mathbf{x}|) \frac{\sum_{i=1}^{N} a_i \lambda_i}{\lambda} & for \quad z = x + 1, \mathbf{m} = \mathbf{k}, \\ & \quad x \in \{0, ..., M-1\}, a_i > 0, \\ \frac{Q(k_i, a_i)}{\lambda} & for \quad z = x, \mathbf{m} = \mathbf{k} - e_i, \\ & \quad a_i > 0, \\ \frac{\mu}{\lambda} & for \quad z = x, \mathbf{m} = \mathbf{k} + e_i, \\ & \quad m_i < K, a_i = 0, \\ 1 - \frac{1}{\lambda} \left[\sum_{i=1}^{N} (M - |\mathbf{x}|) a_i \lambda_i + \mu 1\{a_i = 0\} + Q(k_i, a_i) \right] & for \quad \mathbf{z} = \mathbf{x}, \mathbf{x} \in \mathbf{X}, \mathbf{m} = \mathbf{k}. \end{cases}$$

The aggregated state process has the Markov property: the dependence of the next aggregated state on the history is only through the current aggregated state and actions.

Next we show that the cost also has this feature. Indeed, for a given initial state $x_i(0)$, we can rewrite (3) as

$$P_f(i) = \exp(-\eta T_i x_i(0)) \prod_{n=1}^{\infty} E\left[\frac{\lambda_i A_i(n)}{\sum_{j=1}^{N} \lambda_j A_j(n)} \exp(-\eta T_i) + \left(1 - \frac{\lambda_i A_i(n)}{\sum_{j=1}^{N} \lambda_j A_j(n)}\right)\right] \quad (6)$$

Hence we see that there is no dependence on the individual dissemination state. Note however that there is a dependence on the individual battery state since the actions $A_j(n)$ at the nth time slot depend on whether individual battery state. In particular, if the battery is empty then only recharging is possible.

We obtain the following main result.

Theorem 2. *Any equilibrium in the game with aggregated state is also an equilibrium in the original game.*

Indeed, let $\mathbf{u}$ be an equilibrium in this game. i.e. the actions of each player depend on the dissemination states at a time n only through their sum.

Let it be used in the original game, and assume that some player i deviates to a policy is not in that class. By doing so, the player cannot do better. This is due to a general result on aggregation in Markov Decision Processes, see Theorem 6.3 in [2]. This ends the proof.

7 Conclusion

We presented in this paper the area of Network Engineereing Games that has attracted considerable attention in last years. We presented a survey of applications and tools used in it and focused in particular on bio-inspired games. We presented in some details an example of a game in wireless networks. It illustrated the usefulness of the risk sensitive criteria in stochastic games formulations. This gives rise to a multiplicative type of dynamic programming instead of an additive one. We note that in this example we did not include a cost per energy. If we had included one then we would have obtained a problem that combines additive and multiplicative type criteria which is a new area that needs exploration.

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