Achievable Common Rate and Power Allocation in Cooperative Multiple Access Channels

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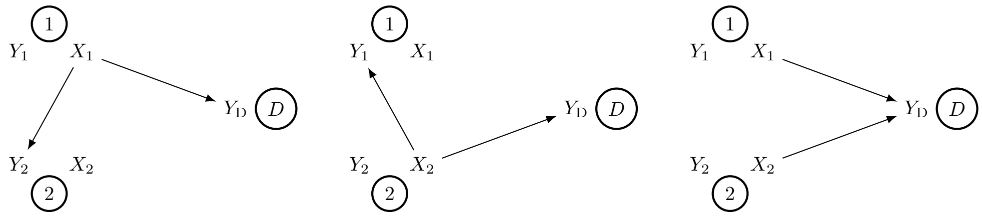
Abstract

In this paper, we aim at obtaining usable bounds on the performance of the cooperative multiple access channel (CMAC) under a Gaussian model. We first show that the problem can be transformed into a convex optimization problem which makes it easily solvable using numerical tools. We propose, as a line of study, to consider the maximal achievable common rate by every node in the network. We then proceed to express closed-form bounds on the capacity region of the CMAC in that common rate scenario. We study simple cooperation schemes based on existing results in relay channels and compare them to other medium sharing approaches. In the end, we show that using the relay-channel based protocols can be efficient for some parameters, but gets less interesting in the Gaussian case if the source-destination links are good enough.

I. Introduction

For more than two decades, the growth of mobile communications led to a renewed interest on the capacity of wireless channels and networks. While the basis of the studies are still the same when compared to classical communication theory, general results have to take into account the strong constraint that nodes can not send and receive information at the same time. Kramer [1] and Khojastepour *et al.* [2] provide straightforward ways to extend the classical capacity theorems to multi-states channels, a general model encompassing half-duplex networks.

Among half-duplex networks, we focus our study in this paper onto 3-nodes networks. The most classical of such models is the relay channel, where a node transmits information to a destination with the help of the other node. In [3], the upper bound on the capacity of the channel is given, along with the now classical decode-and-forward and compress-and-forward lower bounds. Optimizing the capacity of the relay channel under a total power constraint has been the topic of [4], where the authors developed an algorithm akin to waterfilling for the power allocation, for both the half and full duplex relay channel. In [5], the capacity of the coherent full-duplex relay channel is given under different CSI and power allocation schemes. In [6], the authors proposed an adaptative partial decode-and-forward lower bound, and gave results for the optimal power allocation based on fixed-point algorithms.



(a) Node 1 sends (phase 1) (b) Node 2 sends (phase 2) (c) Cooperating phase (phase 3)

Fig. 1. The half-duplex cooperative multiple access channel. The channel is at each time in one of the 3 phases presented here.

Another scheme to consider for the 3-nodes network is the multiple access scheme, where two nodes act as information sources. The capacity of the multiple access channel (MAC) is known in the general case, and can be found in [7]. However, the capacity of the cooperative MAC, where both nodes may help each other in transmitting information to the destination, is still an open problem. This model has been studied by Laneman in his thesis [8], where he gave both an upper-bound and a decode and forward lower-bound on the capacity of the full-duplex CMAC. Sendonaris *et al.* studied this channel extensively in [9], and designed a realistic and usable decode-and-forward scheme, along with its implementation. Their study used full-duplex results but in CDMA orthogonal sub-channels. More recently, Mesbah and Davidson gave an optimal power allocation for the same protocol Sendonaris *et al.* described [10]. They also showed that a more general half-duplex version was able to be solved as a quasiconvex problem, using bisection methods.

In this paper, we show that the upper bound on the capacity of the cooperative MAC may in fact be transformed into a completely convex problem through some variable changes. We also introduce a common rate performance metric, which is the highest common rate both nodes may attain simultaneously. Considering this common rate, we give a lower bound and a loose upper bound in closed form. We then propose a simple protocol based on the superposition of two relay channels as another achievable lower bound. This cooperation scheme allows us to use results on the capacity of relay channels. We show that using a simple time-sharing between the superposed relay channels performs as well as the optimal solution of the full time and energy sharing problem.

II. Model description

A. Network and channel model

Our base model is a half-duplex cooperative multiple access channel (HD-CMAC), composed of two source nodes and a destination node. Each source aims at transmitting its own message, possibly helping the other along the way. The half-duplex constraint implies that the nodes may not send and receive at the same time. We write X_i and Y_i the signal sent and received by the node $i \in \{1, 2\}$, while the message received by the destination is Y_D.

From [2] we can write the upper bound on the capacity of this channel as the capacity of every cut in the network across all the possible states and their associated time-share in the schedule. We consider that the network spends a fraction t_j of its global time in one of the corresponding phase j represented on Fig.1, with $j \in \{1, \dots, 3\}$.

$$R_1 \leq \quad t_1 \log\left(1 + (|h_{1,2}|^2 + |h_{1,D}|^2)P_1^{(1)}\right) + t_3 \log\left(1 + |h_{1,D}|^2(1-\rho^2)P_1^{(3)})\right) \tag{1a}$$

$$R_2 \leq \quad t_2 \log\left(1 + (|h_{2,1}|^2 + |h_{2,D}|^2)P_2^{(2)}\right) + t_3 \log\left(1 + |h_{2,D}|^2(1-\rho^2)P_2^{(3)})\right) \tag{1b}$$

$$\begin{aligned} R_1 + R_2 \leq \quad & t_1 \log\left(1 + |h_{1,D}|^2 P_1^{(1)}\right) + t_2 \log\left(1 + |h_{2,D}|^2 P_1^{(1)}\right) \\ & + t_3 \log\left(1 + |h_{1,D}|^2 P_1^{(3)} + |h_{2,D}|^2 P_2^{(3)} + 2\rho|h_{1,D}||h_{2,D}|\sqrt{P_1^{(3)}P_2^{(3)}}\right) \end{aligned} \tag{1c}$$

Considering the transmission rates of sources 1 and 2 as R_1 and R_2 respectively, we write:

$$R_1 \leq \quad t_1 I(X_1; Y_2, Y_D) + t_3 I(X_1; Y_D | X_2) \tag{2a}$$

$$R_2 \leq \quad t_2 I(X_2; Y_1, Y_D) + t_3 I(X_2; Y_D | X_1) \tag{2b}$$

$$\begin{aligned} R_1 + R_2 \leq \quad & t_1 I(X_1; Y_D) + t_2 I(X_2; Y_D) \\ & + t_3 I(X_1, X_2; Y_D) \end{aligned} \tag{2c}$$

We focus on this paper on an analysis of the Gaussian channel. We can thereby derive the outer bound in a classical manner, as can be seen in [2], [7]. We consider that nodes are subjected to some power allocation, where for each phase j each node i uses $\mathbf{P}_i^{(j)}$ power for transmitting its signal. Furthermore, the channel coefficient $h_{i,k}$ between nodes i and k is stable and symmetrical. Each node is subject to a Gaussian white noise of density N_0 at its receiver. We can thus use the normalized power – w.r.t. the noise density – $P_i^{(j)} = \mathbf{P}_i^{(j)}/N_0$ as the power value in any equation. The signals from the source nodes to the destination have a potential correlation factor ρ. This correlation stems from their cooperation, and requires a coherent transmission between both sources on top of a joint codebook design for the cooperative phase. We consider natural logarithms and our capacity results are thus in nats/s.

The upper bound on the capacity of this channel can be written as the convex closure of all (R_1, R_2) verifying (1). The region changes for different values of the time-sharing vector $\mathbf{t} = (t_1, t_2, t_3)$ and the power-sharing vector $\mathbf{P} = (P_1^{(1)}, P_2^{(2)}, P_1^{(3)}, P_2^{(3)})$, which are the values to be optimized. In order to simplify the expression, we decide to normalize the power further by the value of the channel coefficient of the inter-source link $h_{1,2}$, e.g. changing $P_i^{(j)}$ into $\bar{P}_i^{(j)} = |h_{1,2}|^2 P_i^{(j)}$. This leads us not to consider the 1-D link and 2-D link channel coefficients directly, but rather their relative quality w.r.t. to the inter-source link. We will write $l_1 = h_{1,D}^2/h_{1,2}^2$ and $l_2 = h_{2,D}^2/h_{1,2}^2$ in the remainder of the paper.

B. *Common rate*

Usually, there are tradeoffs to be made between the rates of different nodes in a multi-terminal network. As such, unlike the single link case, there are no unilateraly *best achievable region* in a multi-source channel model. We consider applications in small-scale networks where source nodes have constraints on their effective rates.

$$f_1(\mathbf{t},\alpha,\mathbf{u}) = t_1 \log\left(1 + (1+l_1)\frac{\alpha_1}{t_1}E_{\text{tot}}\right) + t_3 \log\left(1 + l_1 \frac{u_1}{t_3}E_{\text{tot}}\right) \tag{3a}$$

$$f_2(\mathbf{t},\alpha,\mathbf{u}) = t_2 \log\left(1 + (1+l_2)\frac{\alpha_2}{t_2}E_{\text{tot}}\right) + t_3 \log\left(1 + l_1 \frac{u_2}{t_3}E_{\text{tot}}\right) \tag{3b}$$

$$f_3(\mathbf{t},\alpha,\mathbf{u}) = t_1 \log\left(1 + l_1 \frac{\alpha_1}{t_1}E_{\text{tot}}\right) + t_2 \log\left(1 + l_2 \frac{\alpha_2}{t_2}E_{\text{tot}}\right) + t_3 \log\left(1 + \left(l_1\alpha_3 + l_2\alpha_4 + 2\sqrt{l_1 l_2 \alpha_4 u_3}\right)\frac{E_{\text{tot}}}{t_3}\right) \tag{3c}$$

$$f_4(\mathbf{t},\alpha,\mathbf{u}) = t_1 \log\left(1 + l_1 \frac{\alpha_1}{t_1}E_{\text{tot}}\right) + t_2 \log\left(1 + l_2 \frac{\alpha_2}{t_2}E_{\text{tot}}\right) + t_3 \log\left(1 + \left(l_1\alpha_3 + l_2\alpha_4 + 2\sqrt{l_1 l_2 \alpha_3 u_4}\right)\frac{E_{\text{tot}}}{t_3}\right) \tag{3d}$$

Equations (1) define an upper bound on the region for achievable rates for the CMAC, but what we are actually interested in is *which rate is achievable by both nodes*. Going towards this description is straightforward. The common rate semi-line $R_1 = R_2 = R$ will intersect the convex closure of every possible rate regions obtained using (1) at a single point in realistic cases, allowing us to go from treating a region of achievable rates to a single rate variable R. This model is actually readily expanded into different relative demands on the rates of each node. For example, we may well transform the region $\{R_1 \leq \ldots, R_2 \leq \ldots, R_1 + R_2 \leq \ldots\}$ in (1) into the region $\{R \leq \ldots, \alpha R \leq \ldots, (1+\alpha)R \leq \ldots\}$ for some fixed value of $\alpha > 0$. We would then obtain, as a result of our optimization problems, the value of R at the intersection of the convex closure of possible rate regions with the semi-line $R_1 = \alpha R_2$.

III. Power optimization in the CMAC

Our goal in this section is to express outer and inner bounds on the capacity region of the CMAC as solutions to optimization problems, in order to make them solvable numerically and hopefully gain insights on their behaviour. Our work will be heavily reliant on convex optimization techniques. A convex optimization problem in its general form is a minimization of a convex function $f_0(\mathbf{x})$, or equivalently the maximization of a concave function, subject to inequality constraints $f_i(\mathbf{x}) \leq 0$ and equality constraints $h_i(\mathbf{x}) = 0$, further requiring the f_i to be convex functions of $\mathbf{x}$ and the h_i to be linear functions of $\mathbf{x}$ [11]. The equations from (1) do not form convex functions as such, and we thus have to proceed to transformations in order to express the problem correctly. We describe the general problem in the first subsection along with the changes we make, and we analyze it for the remainder of the section.

A. The general upper bound problem

Functions of the form $t\log(1+x)$ are not, as such, concave functions of $(t,x) \in \mathbb{R}^2$, which is a requirement for our problem. We propose not to consider a power constraint in each slot, but an energy constraint in each slot, where nodes would be limited to use at most $\bar{E}_i^{(j)}$ normalized energy over the fraction of time t_j spent in a slot. This would change the formulations of the logarithm functions in (1) to functions of the form $t\log(1+x/t)$, which is the perspective function of the concave function $\log(1+x)$ over t, and are thus concave [11]. We can also verify this property directly since the Hessian matrix of $t\log(1+x/t)$ is negative semi-definite for all $x \geq 0$ and $t > 0$. This variable change is insightful beyond the fact that it helps changing the problem into a convex one ; it is more natural to express power optimization problems over half duplex channels as total energy allocation problems, where the energy is distributed among different nodes and different slots. We choose to work with an energy-sharing vector represented by α such that $\alpha^T E_{\text{tot}} = (\bar{E}_1^{(1)}, \bar{E}_2^{(2)}, \bar{E}_1^{(3)}, \bar{E}_2^{(3)})$.

The second change we need to make is w.r.t. the ρ parameter appearing in function of the forms $\log(1+(1-\rho^2)x)$ and $\log(1+\rho\sqrt{x})$. Both are non-concave functions of ρ and x. We propose the variable change $\rho_1 = 1-\rho^2$, meaning that we have $\rho = \sqrt{1-\rho_1} = \sqrt{\rho_2}$, along with a new linear equality constraint $\rho_1 + \rho_2 = 1$. While this does not make the functions concave, it allows us to identify the 4 new variables from the products of ρ_1, ρ_2, α_3 and α_4. We note $u_1 = \alpha_3\rho_1$, $u_2 = \alpha_4\rho_1$, $u_3 = \alpha_3\rho_2$ and $u_4 = \alpha_4\rho_2$. The constraint $\rho_1 + \rho_2 = 1$ has to be enforced as $u_1 + u_3 = \alpha_3$ and $u_2 + u_4 = \alpha_4$. The last non-trivial formulation is a function of the form $\log(1+\sqrt{xy})$, which is concave as the composition of a concave function with a concave non-decreasing function[11]. The complete problem can thus be written as follows, with $\mathbf{u}^T = (u_1, u_2, u_3, u_4)$ and the functions f_i described in (3) at the top of the page. Notice that we had to duplicate the last constraint because of our variable change. We use this formulation for the remainder of our study.

$$\begin{aligned} \underset{\mathbf{t},\alpha,\mathbf{u},R}{\text{minimize}} \quad & -R \\ \text{subject to} \quad & R \leq f_1(\mathbf{t},\alpha,\mathbf{u}) \\ & R \leq f_2(\mathbf{t},\alpha,\mathbf{u}) \\ & 2R \leq f_3(\mathbf{t},\alpha,\mathbf{u}) \\ & 2R \leq f_4(\mathbf{t},\alpha,\mathbf{u}) \\ & \mathbb{1}^T\mathbf{t} = 1 \\ & \mathbb{1}^T\alpha = 1 \\ & u_1 + u_3 = \alpha_3 \\ & u_2 + u_4 = \alpha_4 \end{aligned} \tag{4}$$

B. The non-coherent case

This general formula can be simplified if we consider the non-coherent case, which basically means we have $\rho = 0$ in (1). While the problem is still too complex to obtain a closed-form of the maximal rate, we can write an outer bound using Jensen's inequality and the variable changes of the previous section:

$$R \leq (t_1+t_3)\log\left(1 + \frac{(1+l_1)\,\alpha_1 + l_1\alpha_3}{t_1+t_3}E_{\text{tot}}\right) \tag{5a}$$

$$R \leq (t_2+t_3)\log\left(1 + \frac{(\alpha_2 + l_2(\alpha_2+\alpha_4))E_{\text{tot}}}{t_2+t_3}\right) \tag{5b}$$

$$2R \leq \log\left(1 + ((\alpha_1+\alpha_3)l_1 + (\alpha_2+\alpha_4)l_2)\,E_{\text{tot}}\right) \tag{5c}$$

Here, we can completely drop the time variables by noting that functions of the form $t\log(1+x/t)$ are increasing in t, and that any solution where $t_3<1$ can thus not be better that one where $t_3=1$. In a similar fashion, we have that solutions where $\alpha_3>0$ or $\alpha_4>0$ can not be optimal either because it is easy to construct a better solution where $\alpha_3=\alpha_4=0$:

$$R \leq \log\left(1+(1+l_1)\alpha_1 E_{\text{tot}}\right) \tag{6a}$$

$$R \leq \log\left(1+(1+l_2)\alpha_2 E_{\text{tot}}\right) \tag{6b}$$

$$2R \leq \log\left(1+\alpha_1 l_1 E_{\text{tot}}+\alpha_2 l_2 E_{\text{tot}}\right) \tag{6c}$$

From this point, we see that when considered in this optimization problem, the Jensen's outer bound reduces to the full-duplex CMAC, where nodes can send and receive at the same time, all the time. Solving for the maximum common rate under the constraint $\alpha_1+\alpha_2=1$ gives a problem that has a closed form solution, although the proof is too long to fit in this paper. We give the optimal value of the global power-sharing variable $\alpha_1=\alpha_J^*$, where the J stands for Jensen:

$$\alpha_J^* = \begin{cases} 1-\frac{\sqrt{(2+l_1+l_2)^2+4l_1(1+l_2)^2E_{\text{tot}}}-(2+l_1+l_2)}{2(1+l_2)^2E_{\text{tot}}} & \text{if } l_1>l_2 \\ 1/2 & \text{if } l_1=l_2 \\ \frac{\sqrt{(2+l_1+l_2)^2+4l_2(1+l_1)^2E_{\text{tot}}}-(2+l_1+l_2)}{2(1+l_1)^2E_{\text{tot}}} & \text{if } l_1<l_2 \end{cases} \tag{7}$$

It is interesting to make a comparison with a classical multiple access scheme, with both nodes using the channel at the same time for their own transmission without trying to listen to the other node's information. The capacity of the multiple access scheme is known to be attainable through time-sharing between different superposition coding/successive cancellation decoding schemes [7]. The expression of the achievable region for this scheme is very close to the one in (6). Since there is no cooperation, we only have to replace the terms $(1+l_1)$ and $(1+l_2)$ with l_1 and l_2 respectively, because the nodes do not listen to each other. From this point, we see that when considered in this optimization problem, the Jensen's outer bound reduces to the full-duplex CMAC, where nodes can send and receive at the same time, all the time.the inter-source link. Using the same method as before, we can obtain the optimal power sharing parameter α_N^* where the N stands for *non-cooperative*:

$$\alpha_N^* = \begin{cases} 1-\frac{\sqrt{(l_1+l_2)^2+4l_1l_2^2E_{\text{tot}}}-(l_1+l_2)}{2l_2^2E_{\text{tot}}} & \text{if } l_1>l_2 \\ 1/2 & \text{if } l_1=l_2 \\ \frac{\sqrt{(l_1+l_2)^2+4l_2l_1^2E_{\text{tot}}}-(l_1+l_2)}{2l_1^2E_{\text{tot}}} & \text{if } l_1<l_2 \end{cases} \tag{8}$$

C. Relay superposition

We propose in this section to design a cooperation scheme by directly using results from achievable rates in relay channels. Consider the model from Fig.1. By separating phase 3 into 2 sub-phases, where each source node act as a relay for the other and do not send its own information. The general model transforms into a parallel relay channel model, with each node alternating in the role of source or relay node. While this scheme is probably suboptimal, it has the merit of being easy to implement since we can just take any existing cooperation protocol on relay channels and compute a time-sharing parameter.

The upper-bound on the performance of such an approach would be derived from the upper-bound on the capacity of the half-duplex relay channel, as given in [4], [2]. In the remainder of the paper, although we limit our analysis to this upper bound on the capacity, it would be possible to use the achievable rate of any scheme for the half-duplex relay channel, such as decode-and-forward or compress-and-forward [3], [8]. The simplest of all relay superposition schemes is the *separate relay optimization*, where we blindly use a time sharing value t_r to split the time between the phases where node 1 is the source and the ones where it is a relay. If $t_r = 1$, then the whole time is attributed to node 1 being the source, which means $R_2 = 0$ and $R_1 = R_1^*$, the maximum rate node 1 may attain with the help of node 2. On the other hand, we have similarly $R_1 = 0$ and $R_2 = R_2^*$ when $t_r = 0$.

Solving $t_r R_1^* = (1 - t_r) R_2^*$ gives us an interesting result for the common achievable rate. In that case, we can express R, the common rate, and $t_r^{(c)}$ the time-sharing value that attains it as:

$$t_r^{(c)} = \frac{R_2^*}{R_1^* + R_2^*} \qquad R = \frac{R_1^* R_2^*}{R_1^* + R_2^*} \tag{9}$$

The common rate R is in fact half the harmonic mean of the maximum achievable rates of each source if the whole time is devoted to their transmission. This result has an intuitive explanation since the common rate is indeed an average of rates, obtained by inverting the sum of the inverse rate of each source. It is to be noted that this result is readily extended to higher dimensions, although constrained by the paper size this global result is relegated to a future contribution.

At this point, we infer that this scheme may in fact be suboptimal to the achievable rate with superposed relays. By trading transmission time for higher transmission power in the *global relay optimization* problem, a less constrained formulation could attain a higher common rate. From [4] we know that the upper-bound on the capacity of the relay channel is written as the minimum of two functions, with a time-sharing and an energy-sharing vectors as parameters. Since we consider a half-duplex model where both nodes may send information simultaneously, we have to consider in total, for each instance of relay channel in the network, 2 time-sharing variables and 3 energy-sharing variables – the source node may use a different energy value between the phase where it transmits alone and the cooperating phase. This means that this global problem would have 4 time variables and 6 energy variables, both separately summing to one. To finalize the problem, we note that we can express the common rate as the minimum of 4 functions, 2 for each superposed relay channel.

Once again, this problem is too complex to treat analytically, but we do gain valuable insights from it, as will be shown shortly. We compare this relay superposition scheme with a direct transmission scheme with time sharing, where nodes do not cooperate and use the channel one at a time. Although we do not provide analyses of coherent lower bounds on the CMAC, we do plot out the outer bound for the coherent case of our study. This bound is readily computed from (4) and can serve as a basis for comparison of the achievable rates for the non-coherent case. Coherent lower bounds are given in [8], [9], [10], and we plan to study them in future works.

Fig.2 presents the analysis of the schemes described in this paper. We can readily see that the separate optimization performs as well as the global optimization problem for the superposed relays. This means that there is little point in treating the full problem, and that optimizing each superposed relay separately before applying the optimal time-sharing from (9) is sufficient. The problem is thus simpler to manage, since it is possible to show that the optimization of the time-sharing and power-sharing inside the relay channel can be solved using a fixed-point algorithm, as is done in [6] for decode-and-forward protocols. Furthermore, we see that in high-SNR scenarios, the concurrent approach is actually more efficient than the relay superposition, while being a lot less complicated to manage provided that the destination can perform successive cancellation decoding. The coherent upper-bound is also arguably quite close to the non-coherent one.

Fig.3 confirms the previous results. When using the same values for l_1 and l_2 as in Fig.2, the relay superposition

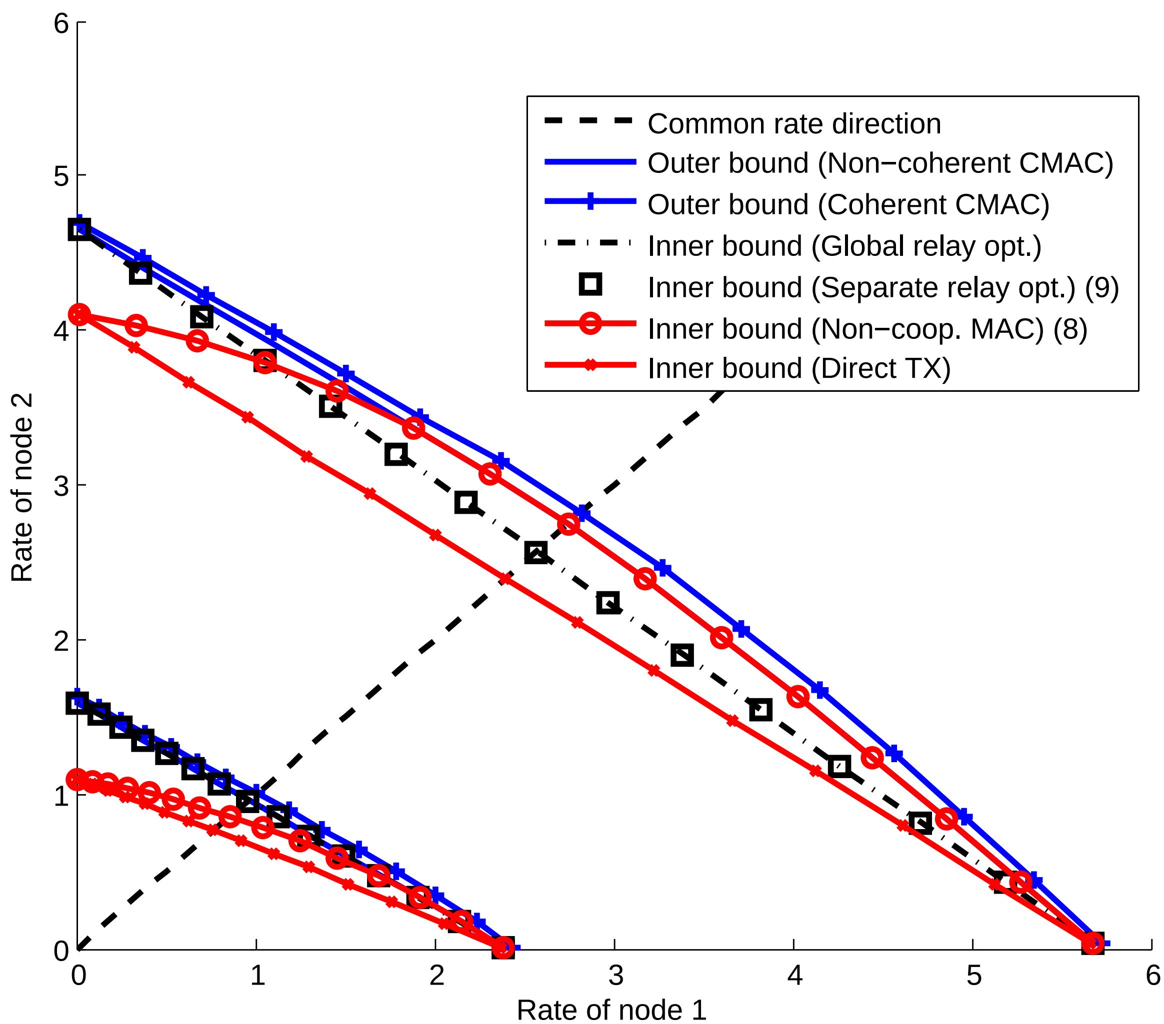


Fig. 2. Comparison of the Pareto front of achievable regions for the different schemes discussed in the paper, with $l_1 = 1$ and $l_2 = 0.2$, meaning that one link is of poor quality.

protocol performs better at low SNR, virtually attaining the non-coherent upper bound in that region. It loses its lead at high SNR, where the non-cooperative concurrent access scheme meets it. We can also see on Fig.3 that if the intersource link is relatively equal or worse than the source-destination links, cooperation is actually counter-productive in the non-coherent case. It is expected that in the coherent case, lower-bounds and cooperation schemes that exploit the coherency would perform better and close the substantial gap with the coherent upper bound – with a price on the protocol's complexity.

IV. Conclusion

In this paper, we studied the half-duplex cooperative multiple access channel. We transformed the general formulation of the upper bound region into a convex problem, and proposed to study the maximum rate achievable by both nodes. We obtained both numerical and exact bounds on the capacity of this channel. We further proposed

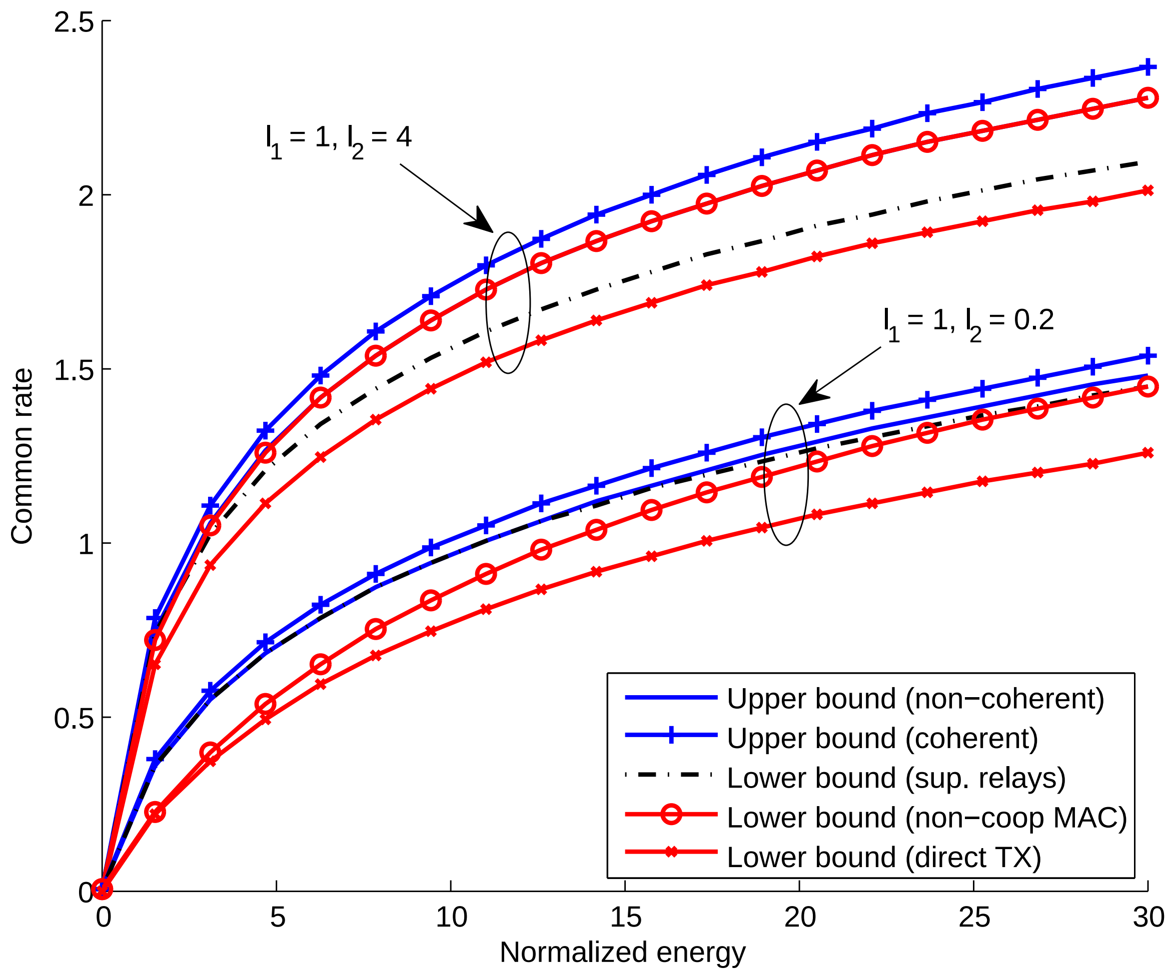


Fig. 3. Example of the bounds on the achievable common rate with respect to the total available (normalized) energy.

a simple relay time-sharing scheme, and compared it to the concurrent access approach in multi-user channels. We showed that the relay superposition scheme is performing well when the channels between a source and the destination is bad. When the available energy is high, or when the inter-source channel is relatively bad, the concurrent access approach is better.

Further studies should most importantly include fading channels, where cooperation should shine because the spatial diversity helps countering the loss in capacity due to channel variations. Furthermore, the focus could be turned towards coherent approaches, and work on more complex cooperative schemes. Finally, it would be interesting to find the turnover point between the different schemes and design a protocol switching between different approaches.

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