Application of timestepping schemes based on time discontinuous Galerkin methods to multi-dimensional examples

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Thorsten Schindler[1], Vincent Acary[2]

[1] *Institute of Applied Mechanics, Technische Universität München, Boltzmannstraße 15, 85748 Garching, Germany, thorsten.schindler@mytum.de*
[2] *INRIA Grenoble, 655 avenue de l'Europe, Montbonnot, 38334 Saint Ismier Cedex, France, vincent.acary@inrialpes.fr*

Summary: Classic timestepping schemes for nonsmooth dynamical systems are locally of integration order one both in smooth and nonsmooth periods. A consistent enhancement based on time discontinuous Galerkin methods and splitting of smooth and nonsmooth force propagation is presented. Convergence and general behaviour are discussed with different multi-dimensional examples with and without considering friction.

Introduction

In nonsmooth mechanical systems, the velocity function contains smooth and nonsmooth propagation episodes. With classic function derivatives, one has to distinguish these two ranges. Measure theory is the correct mathematical tool which yields a uniform description. The interpretation in the sense of distributions permits the following problem formulation.

Problem 1 (Distribution differential inclusion) *Solve*

$$q(0) := q_0 \ , \tag{1}$$

$$v(0) := v_0 \ , \tag{2}$$

$$< \dot{q}, \varphi_q >_{\mathcal{D}^*,\mathcal{D}} =< v, \varphi_q >_{\mathcal{D}^*,\mathcal{D}} \ , \quad \forall \varphi_q \in \mathcal{D}(I) \ , \tag{3}$$

$$< D^1 v, \varphi_v >_{\mathcal{D}^*,\mathcal{D}} =< m^{-1} f, \varphi_v >_{\mathcal{D}^*,\mathcal{D}} + < m^{-1} \mathrm{d}i, \varphi_v >_{\mathcal{D}^*,\mathcal{D}} \ , \quad \forall \varphi_v \in \mathcal{D}(I) \ , \tag{4}$$

$$\textit{contact and impact relations}\ (q, v, \mathrm{d}i, t) \in \mathcal{N} \ . \tag{5}$$

The initial values of position q and velocity v are given by q_0 and v_0, respectively. The position offers a classical time derivative almost everywhere. On the other hand, non-regular reaction measures $\mathrm{d}i$ require to use the distributional derivative $D^1 v$ of v. The space of all real-valued $\mathcal{C}^\infty$-functions with compact support contains the test functions for position φ_q and velocity φ_v. It is denoted by $\mathcal{D}(I)$. Its dual space $\mathcal{D}^*(I)$ contains distributions. Inverse mass m^{-1}, external force f as well as time t can be interpreted in this general setting using the notion of the primal-dual pairing $< *, * >_{\mathcal{D}^*,\mathcal{D}}$.

Classical timestepping schemes discretise the equations of motion of Problem 1 including the constraints with integration order one. This avoids impact detections: a large number of contact transitions can be handled efficiently. Event-driven schemes resolve the exact contact transition times. Between the events, the motion of the system is computed by a classic integration method for differential algebraic equations. This is very accurate but the detection of events can be time consuming and is not possible for Zeno phenomena.

Time Discontinuous Galerkin Methods

To consistently improve the behaviour during smooth episodes, classical timestepping schemes are embedded in time discontinuous Galerkin methods. In Problem 1, one assumes that ansatz and test functions might have jumps across discretisation intervals, and that they are of higher order inside discretisation intervals. The second assumption states that there is not an instantaneous influence of the analytic nonsmooth dynamics on the numerical solution: the exact time of discontinuity is not resolved. Figure 1 shows that one can define the location of discontinuities of velocity and interaction impulses at the left or right interval border. Piecewise constant

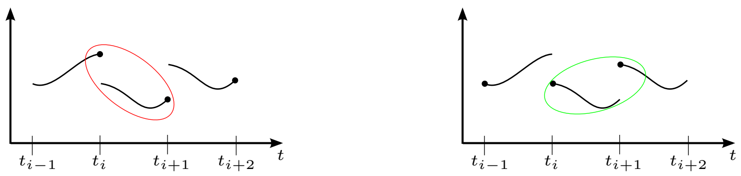

Figure 1: Piecewise smooth propagation with left and right discontinuity.

functions and right interaction impulse discontinuity yield classic explicit (right velocity discontinuity) and implicit (left velocity discontinuity) timestepping schemes.

For each interval $[t_i, t_{i+1}]$, we use splitting techniques

$$\mathrm{d}i := r\mathrm{d}t + p_{i+1}\delta_{t_{i+1}}$$

to take care of contact reactions r separated from the single impact reaction p_{i+1} always evaluated at t_{i+1} with the Dirac measure $\delta_{t_{i+1}}$ because of stability reasons. Hence, contacts benefit from higher order ansatz functions and impacts yield local integration order one.

Numerical Experiments

We analyse convergence and general behaviour of a D^{1-} timestepping scheme (right velocity discontinuity, linear velocity ansatz function) and the Moreau-Jean timestepping scheme (constant velocity ansatz function) with different examples with and without considering friction (e.g. bouncing ball example, slider-crank mechanism [2], circuit breaker, cam follower).

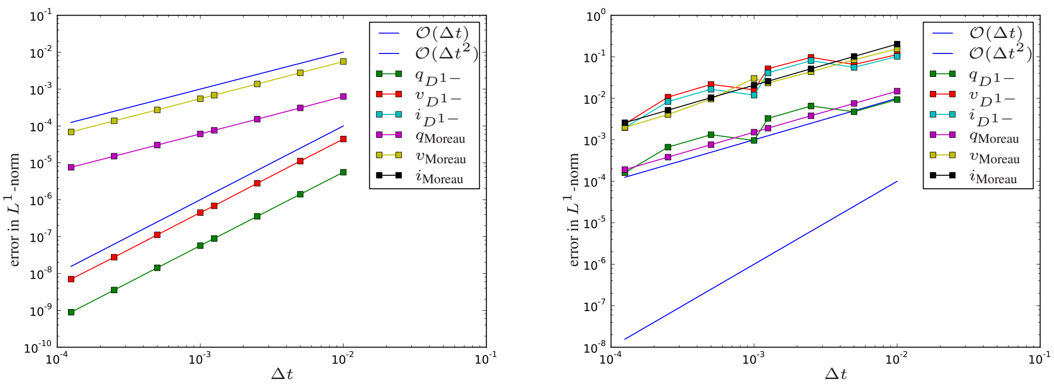

Figure 2: Convergence diagram (bouncing ball): free flight and finite accumulation of impacts.

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