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# Parametric Dictionary Learning in Diffusion MRI

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## I. INTRODUCTION

In this work, we propose an approach to exploit the ability of compressive sensing to recover diffusion MRI signal and its characteristics from a limited number of samples. Our approach is threefold. First, we learn and design a parametric dictionary from a set of training diffusion data. This provides a highly sparse representation of the diffusion signal. The use of a parametric method presents several advantages: we design a continuous representation of the signal, from which we can analytically recover some features such as the ODF; besides, the dictionary we train is acquisition-independent. Next, we use this sparse representation to reconstruct the signal of interest, using cross-validation to assess the optimal regularization parameter for each signal reconstruction. The use of cross-validation is critical in the  $\ell_1$  minimization problem, as the choice of the parameter is sensitive to the noise level, the number of samples, and the data sparsity. Third, we use a polynomial approach to accurately extract ODF maxima. In the last section, we motivate and describe the choice of experimental parameters for the HARDI contest.

## II. PARAMETRIC DICTIONARY LEARNING

### A. Sparse reconstruction

We reconstruct the signal in an over-complete dictionary of continuous functions. Given a signal  $\mathbf{y} \in \mathbb{R}^N$ , this is expressed from the coefficient vector  $\mathbf{x} \in \mathbb{R}^M$  as

$$\mathbf{y} = \mathbf{H}\mathbf{D}\mathbf{x} + \epsilon. \quad (1)$$

The matrix  $\mathbf{H}$  is the standard  $N \times R$  design matrix of the underlying family of continuous functions. The dictionary  $\mathbf{D} \in \mathbb{R}^{R \times M}$ , is a linear sparsifying transform, such that the signal  $\mathbf{y}$  can be reconstructed from a limited number of atoms. This matrix does not depend on the choice of a specific acquisition protocol. The reconstruction is done by minimizing  $\|\mathbf{y} - \mathbf{H}\mathbf{D}\mathbf{x}\|_2 + \lambda\|\mathbf{x}\|_1$ . Here we propose to learn the dictionary  $\mathbf{D}$  from a training dataset.

### B. Parametric dictionary learning

We can rewrite Eq. 1 as  $\mathbf{y} = \mathbf{H}\mathbf{c} + \epsilon$ , and  $\mathbf{c} = \mathbf{D}\mathbf{x}$ . Given a training set of signals  $\mathbf{Y}$  of size  $N \times S$ , we first estimate the coefficients  $\mathbf{C} = (\mathbf{H}^T\mathbf{H})^{-1}\mathbf{H}^T\mathbf{Y}$ . Then we look for a dictionary  $\mathbf{D}$  of size  $R \times M$  and a code  $\mathbf{X}$  of size  $M \times S$  verifying

$$\arg \min_{\mathbf{D}, \mathbf{X}} \|\mathbf{C} - \mathbf{D}\mathbf{X}\|_2^2 + \alpha\|\mathbf{X}\|_1, \text{ s.t. } \forall m \leq M, \|\mathbf{d}_m\|_2 = 1 \quad (2)$$

with  $\mathbf{d}_m$  the  $m^{\text{th}}$  column of  $\mathbf{D}$ . We use the python library scikit-learn [5] to solve this problem.

## III. CROSS-VALIDATION

We use cross validation to assess  $\lambda$  [6]. In particular, we use a  $K$ -fold Cross Validation, which consists in splitting the entire data set in  $K$  subsets.  $K - 1$  subsets are used to reconstruct the signal whereas the  $K^{\text{th}}$  subset enables a tight estimation of regularization parameter  $\lambda_K$  via the evaluation of a cross validation distance. This operation is repeated  $K$  times by considering another subset. Then, we keep an average value,  $\lambda = \frac{1}{K} \sum_{i=1}^K \lambda_i$ . To avoid a drastic increase of the

computational effort, we split our data set in 5 partitions, i.e.  $K = 5$ . Our previous experiments showed that it is sufficient to obtain a close approximation of the optimal  $\lambda$ .

## IV. A POLYNOMIAL APPROACH FOR MAXIMA EXTRACTION

The HARDI contest is based on fiber orientation estimation via the extraction of ODF maxima. After computing the ODF, we extract its maxima using a polynomial approach that can analytically bracket and numerically refine with high precision *all* maxima, ensuring that none are missed [4]. First, leveraging the linear bijection between the spherical harmonic basis and the homogeneous polynomial basis of the same order and degree, we rewrite the ODF as a polynomial,  $P(\mathbf{x} = [x_1, x_2, x_3])$ , with  $\|\mathbf{x}\|_2 = 1$ . Second we formulate the optimization problem for computing the ODF extrema:

$$\partial F / \partial x_1 = \partial F / \partial x_2 = \partial F / \partial x_3 = \|\mathbf{x}\|_2^2 - 1 = 0, \quad (3)$$

where using Lagrange multipliers  $F(\mathbf{x}, \Lambda) = P(\mathbf{x}) - \Lambda(\|\mathbf{x}\|_2^2 - 1)$ . However, instead of optimizing, which is inherently a local approach dependent on initializations, we solve the polynomial system Eq. 3 using a polynomial system solver that can analytically bracket all roots – missing none and refine them numerically to high precision. Finally, we identify the maxima from the extrema using the Bordered Hessian, which is the generalization of the Hessian.

## V. EXPERIMENTAL PARAMETERS

For the HARDI contest, we choose  $\mathbf{H}$  to be constituted of the continuous SPF basis functions [1], [2] and constrain our dictionary  $\mathbf{D}$  to represent a combination of the SPF functions. We choose the SPF basis because it facilitates the ODF estimation [3]. We learn our dictionary on the training data set given for the HARDI contest. Then we probe the testing data set on 15 measurements uniformly spread on a single shell at  $b$  value  $b = 2000\text{s} \cdot \text{mm}^{-2}$ .

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