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Abstract: This work is devoted to the design of interval observers for a class of Linear-Parameter-Varying (LPV) systems. Applying High Order Sliding Mode (HOSM) techniques it is possible to decrease the initial level of uncertainty in the system, which leads to improvement of set-membership estimates generated by an interval observer and enlarges the class of the estimated systems. Next, this approach is applied to fault detection by verifying the consistency between the output trajectory and its estimated domain. The efficiency of the proposed approach is demonstrated through computer simulations of an electromechanical system.

Keywords: Sliding-mode, interval estimation, fault detection and isolation.

1. INTRODUCTION

Model-based fault diagnosis is a well established area of research (see Chen and Patton (1999) and Ding (2008) for a recent survey). The main idea of model-based fault detection and diagnosis is to check whether the behavior of the plant is consistent with its fault-free model. Appearance of a significant deviation from the nominal model behavior exposes the faults. Robustness of the diagnosis scheme is determined by its insensitivity to disturbances, measurement noises and model variations. The most model-based approaches use estimation of some relevant internal or observed variables to produce fault-indicating residual signals.

A typical approach to generate an estimate for the internal variable consists in a state observer design. The problem of state estimation for nonlinear systems is very challenging and application important (Meurer *et al.* (2005); Fossen and Nijmeijer (1999); Besanon (2007)). A complete palette of solutions exists for linear systems. In the nonlinear case, the most solutions are based on representation of the estimated system in a canonical form (frequently, partially linear), then particular approaches are available. In general case the LPV equivalent representation of nonlinear systems was found useful (Marcos and Balas (2004); Shamma and Cloutier (1993); Tan (1997)). The basic idea is to replace the nonlinear complexity of the original system by an enlarged parametric variation in the LPV representation that may simplify the observer design. There are several approaches to design observers for LPV systems (Bernard and Gouz (2004); Jaulin (2002); Kieffer and Walter (2004); Moisan *et al.* (2009)). Among them a promising framework for the fault detection consists in synthesis of interval observers (Bernard and Gouz (2004); Moisan *et al.* (2009)). That approach has been recently extended in Rassi *et al.* (2010) to nonlinear systems using LPV representations with known minorant and majorant matrices, and in Rassi *et al.* (2012) for observable nonlinear systems relaxing requirement on cooperativity (monotonicity) of the original system dynamics. The interval observers propagate the parameter uncertainty in the length of interval of the state estimation. The length of interval determines the estimation accuracy of the approach. This is why the uncertainty decreasing is very important for improvement of the interval (set-membership) estimation performance. That has a vital importance for fault detection, since in the set-membership estimation a fault presence is indicated if the measured output leaves its estimated interval domain. Therefore, the interval estimation accuracy directly affects on the minimal detectable amplitude of faults.

The HOSM techniques are very popular for design of observers for linear and nonlinear systems (Barbot *et al.* (2009); Bejarano and Pisano (2011); Moreno and Osorio (2008); Efimov and Fridman (2011); Shtessel *et al.* (2010)). The sliding modes ensure a finite time of the estimation error convergence to zero and complete insensitivity to a matched uncertainty (Edwards and Spurgeon (1998);

Bartolini *et al.* (2008); Perruquetti and Barbot (2002)). Mainly these advances can be achieved under assumption that the systems is strongly observable or strongly detectable (Bejarano and Pisano (2011)). The sliding-mode techniques are also intensively used for fault detection (Efimov *et al.* (2011); Saif and Xiong (2003); Yan and Edwards (2007)).

Following Efimov *et al.* (2012), the objective of this work is to combine both approaches (the interval observers and the HOSM techniques) in order to improve accuracy of estimation achieved by the interval observers. Under a transformation of coordinates, an LPV system has a strongly observable subsystem. Applying HOSM differentiation approach it is possible to estimate the state and the state derivative for this subsystem, which can be further used for improved evaluation of the input and the parameter uncertainty in the rest part of the system. This combination leads to accuracy of the interval estimation improvement, which is a more accurate fault detection. Additionally a relaxation of some applicability constraints usual for interval estimation can be obtained.

The paper is organized as follows. The system of interest, the basic facts from the theories of LPV systems, interval estimation and HOSM techniques are given in Section 2. The interval observer structure and its stability conditions are described in Section 3. The fault detection problem followed by an examples of computer simulation is presented in Section 4.

2. PRELIMINARIES

Euclidean norm for a vector $x \in \mathbb{R}^n$ will be denoted as $|x|$, and for a measurable and locally essentially bounded input $u : \mathbb{R}_+ \to \mathbb{R}$ ($\mathbb{R}_+ = \{\tau \in \mathbb{R} : \tau \geq 0\}$) the symbol $||u||_{[t_0,t_1]}$ denotes its L_∞ norm:

$$||u||_{[t_0,t_1]} = ess\sup\{|u(t)|, t \in [t_0, t_1]\},$$

if $t_1 = +\infty$ then we will simply write $||u||$. Denote by $\mathcal{L}_\infty$ the set of all inputs u satisfying $||u|| < \infty$, and the sequence of integers $1, ..., k$ by $\overline{1, k}$.

In this work we consider the following LPV representation of a nonlinear system:

$$\dot{x} = A(\theta(t))x + B(\theta(t))u(t), \qquad (1)$$
$$y = Cx,\ \psi(t) = y + v(t),$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $y \in \mathbb{R}^p$, $v \in \mathbb{R}^p$ are the state, the input, the output and the measurement noise of the system (1), $\psi(t)$ is the signal available for on-line measurements; $\theta \in \Theta \subset \mathbb{R}^q$ is the scheduling parameter vector, the set Θ is known; the matrix functions $A : \Theta \to \mathbb{R}^{n\times n}$ and $B : \Theta \to \mathbb{R}^{n\times m}$ are given. The instant values of $u(t) \in \mathcal{L}_\infty$, $v(t) \in \mathcal{L}_\infty$ and $\theta(t) \in \mathcal{L}_\infty$ are not known. Almost all existent approaches assume that the vector θ is accessible for measurements, in the following this assumption is relaxed, and only the domain Θ is given.

The fault presence can be modeled by the unknown input $u(t)$ (if the estimates of this input differ from the nominal one, known for the designer, or these estimates exit the predefined nominal set of admissible values for u) or by the vector of parameters $\theta(t)$ (if it also leaves the known domain Θ, which has to lead to a mismatch between the estimated interval for $y(t)$ and the real measurements $\psi(t)$).

Assumption 1. $||x|| \leq X$, $||u|| \leq U$ and $||v|| \leq V$, the bounds $X > 0$, $U > 0$ and $V > 0$ are given.

Boundedness of the state x and the inputs u, v is a standard assumption in the estimation and fault detection theories. Under Assumption 1 the signal $\psi(t)$ is also bounded.

2.1 HOSM differentiation

Taking the s-th time differentiable output $y(t)$ of the system (1), its derivatives can be estimated by the HOSM differentiator from Levant (1998, 2003):

$$\dot{q}_0 = \nu_0, \qquad (2)$$
$$\nu_0 = -\lambda_0|q_0 - \psi(t)|^{s/s+1}\text{sign}[q_0 - \psi(t)] + q_1;$$
$$\dot{q}_i = \nu_i,\ i = \overline{1, s-1}, \qquad (3)$$
$$\nu_i = -\lambda_i|q_i - \nu_{i-1}|^{s-i/s-i+1}\text{sign}[q_i - \nu_{i-1}] + q_{i+1};$$
$$\dot{q}_s = -\lambda_s\text{sign}[q_s - \nu_{s-1}],$$

where λ_k, $k = \overline{0, s}$ are positive parameters to be tuned.

Theorem 1. Levant (2003) Let $y : \mathbb{R}_+ \to \mathbb{R}$ be s-th times continuously differentiable and $v(t) \in \mathcal{L}_\infty$ in (1), then there exist $0 \leq T < +\infty$ and some constants $\mu_k > 0$, $k = \overline{0, s}$ (dependent on λ_k, $k = \overline{0, s}$ only) such that in (3) for all $t \geq T$:

$$|q_k(t) - y^{(k)}(t)| \leq \mu_k||v||^{\frac{s-k+1}{s+1}},\ k = \overline{0, s}.$$

In particular, this result means that if $v(t) \equiv 0$ for all $t \geq 0$, then the differentiator (3) ensures the exact estimation of derivatives in a finite time. In practice, to determine the time T one can analyze the measured error $q_0(t) - y(t)$, which should be identically zero (very close to zero up to the error of integration) for $t \geq T$. Application of HOSM differentiators for unknown input estimation and compensation in linear systems has been studied in Bejarano and Pisano (2011), an extension to nonlinear systems is presented in Efimov *et al.* (2011).

2.2 Interval estimation

For two vectors $x_1, x_2 \in \mathbb{R}^n$ or matrices $A_1, A_2 \in \mathbb{R}^{n\times n}$, the relations $x_1 \leq x_2$ and $A_1 \leq A_2$ are understood elementwise. Given a matrix $A \in \mathbb{R}^{m\times n}$ or a vector $x \in \mathbb{R}^n$, define $A^+ = \max\{0, A\}$, $A^- = A^+ - A$ or $x^+ = \max\{0, x\}$, $x^- = x^+ - x$ respectively.

Lemma 2. Let $x \in \mathbb{R}^n$ be a vector variable, $\underline{x} \leq x \leq \overline{x}$ for some $\underline{x}$, $\overline{x} \in \mathbb{R}^n$.

1) If $A \in \mathbb{R}^{m\times n}$ is a constant matrix, then

$$A^+\underline{x} - A^-\overline{x} \leq Ax \leq A^+\overline{x} - A^-\underline{x}. \qquad (4)$$

2) If $A \in \mathbb{R}^{m\times n}$ is a matrix variable,$\underline{A} \leq A \leq \overline{A}$ for some $\underline{A}$, $\overline{A} \in \mathbb{R}^{m\times n}$, then

$$\underline{A}^+\underline{x}^+ - \overline{A}^+\underline{x}^- - \underline{A}^-\overline{x}^+ + \overline{A}^-\overline{x}^- \qquad (5)$$
$$\leq Ax \leq \overline{A}^+\overline{x}^+ - \underline{A}^+\overline{x}^- - \overline{A}^-\underline{x}^+ + \underline{A}^-\underline{x}^-.$$

A matrix $A \in \mathbb{R}^{n\times n}$ is called Hurwitz if all its eigenvalues have negative real parts, it is called Metzler if all its elements outside the main diagonal are not negative. Any solution of the linear system

$$\dot{x} = Ax + \omega(t),\ \omega : \mathbb{R}_+ \to \mathbb{R}^n_+,$$

with $x \in \mathbb{R}^n$ and a Metzler matrix A, is elementwise nonnegative for all $t \geq 0$ provided that $x(0) \geq 0$ (see Smith (1995)). Such dynamical systems are called cooperative (monotone).

3. INTERVAL OBSERVER DESIGN

For brevity of presentation the case $p = 1$ is considered only. We will need the following assumptions.

Assumption 2. For all $\theta \in \Theta$, there is an invertible matrix $S(\theta) \in \mathbb{R}^{n\times n}$ such that the system (1) can be represented as follows:

$$x = S(\theta) \begin{bmatrix} z_1 \\ z_2 \end{bmatrix},\ y = c^{\mathrm{T}} z_1,$$

$$\dim\{z_1\} = n_1,\ \dim\{z_2\} = n_2,\ n_1 + n_2 = n,$$

$$\dot{z}_1 = A_0 z_1 + b_0[a_{11}(\theta)^{\mathrm{T}} z_1 + a_{12}(\theta)^{\mathrm{T}} z_2 + b_1(\theta)^{\mathrm{T}} u], \tag{6}$$

$$\dot{z}_2 = A_{21}(\theta) z_1 + A_{22}(\theta) z_2 + B_2(\theta) u, \tag{7}$$

where

$$c = [1\ 0 ... 0]^{\mathrm{T}},\ b_0 = [0 ... 0\ 1]^{\mathrm{T}},$$

$$A_0 = \begin{bmatrix} 0 & 1 ... 0 & 0 \\ \vdots & \ddots & \vdots \\ 0 & 0 ... 0 & 1 \\ 0 & 0 ... 0 & 0 \end{bmatrix}$$

is a canonical representation, the vector functions $a_{11}(\theta)$, $a_{12}(\theta)$, $b_1(\theta)$ and the matrix functions $A_{21}(\theta)$, $A_{22}(\theta)$, $B_2(\theta)$ have corresponding dimensions.

It is worth to stress that for $n_1 = 1$ this assumption is always true (at least the output coordinate can be chosen in the vector z_1).

Assumption 3. Let there exist a vector function $f(\theta) \in \mathbb{R}^{n_2}$ such that

$$[A_{22}(\theta) z_2 + B_2(\theta) u] - f(\theta)[a_{12}(\theta)^{\mathrm{T}} z_2 + b_1(\theta)^{\mathrm{T}} u] = \Delta_1 z_2 + \Delta_2(\theta) u$$

for some Hurwitz matrix $\Delta_1 \in \mathbb{R}^{n_2\times n_2}$ and $\Delta_2 : \Theta \to \mathbb{R}^{n_2\times m}$.

Assumption 4. There exists a matrix $P \in \mathbb{R}^{n_2\times n_2}$ such that the matrix $D = P^{-1}\Delta_1 P$ is Hurwitz and Metzler (H&M).

Assumption 2 states that there exists a transformation of coordinates, which represents the system (1) as a pair of interconnected subsystems (6) and (7). The subsystem (6) is strongly observable since it has the canonical representation c, A_0, b_0 (the conditions of existence of such a transformation for linear time-invariant systems are analyzed in Bejarano and Pisano (2011)). However, the system is not necessarily detectable (the dynamics of (1) could be non-minimum phase as in Shtessel *et al.* (2010)) since there is no requirement on stability of the matrix function $A_{22}(\theta)$. This relaxation may be important for application of interval observer design method for estimation in uncertain non-minimum phase systems. Instead, Assumption 3 states that the matrix $\Delta_1 = A_{22}(\theta) - f(\theta)a_{12}(\theta)^{\mathrm{T}}$ is Hurwitz (the matrix $A_{22}(\theta)$ can be stabilized by an output feedback, or the pair of matrices $(A_{22}(\theta), a_{12}(\theta)^{\mathrm{T}})$ is observable for all $\theta \in \Theta$) and independent in θ. Under mild conditions of the main result in Rassi *et al.* (2012), in this case there is a matrix $P \in \mathbb{R}^{n_2\times n_2}$ such that D is H&M, as it is stated in Assumption 4.

Under these assumptions it is proposed to use the differentiator (3) to estimate the state z_1 and its derivative $\dot{z}_1$, then from (6) we get an improved estimate on the signal $a_{12}(\theta)^{\mathrm{T}} z_2 + b_1(\theta)^{\mathrm{T}} u$, which can be applied for design of an interval observer for the system (7) in the new coordinates $r = P^{-1} z_2$. Let us consider these steps consequently.

Under Assumption 2 the output y of the system (6) has n_1 derivatives. Therefore according to Theorem 1 and Assumption 1, there exist parameters λ_k, $k = \overline{0, n_1}$ in (3) with $s = n_1$ and $T > 0$ such that for all $t \geq T$:

$$|q_k(t) - y^{(k)}(t)| \leq \mu_k V^{\frac{n_1 - k + 1}{n_1 + 1}},\ k = \overline{0, n_1}$$

for some constant μ_k, $k = \overline{0, n_1}$. Thus $z_1(t) = \hat{z}_1(t) + e_1(t)$ and $\dot{z}_{1,n_1}(t) = q_{n_1}(t) + e_2(t)$ for all $t \geq T$, where $\hat{z}_{1,i}(t) = q_{i-1}(t)$ and $|e_{1,i}(t)| \leq \mu_{i-1} V^{\frac{n_1 - i + 2}{n_1 + 1}}$ for $i = \overline{1, n_1}$, $|e_2(t)| \leq \mu_{n_1} V^{\frac{1}{n_1 + 1}}$. The variables $\hat{z}_1$ and q_{n_1} are available for a designer, the errors e_1 and e_2 are upper bounded by some functions of V. Substitution of these variables into the last equation of (6) gives:

$$q_{n_1} + e_2 = a_{11}(\theta)^{\mathrm{T}}[\hat{z}_1 + e_1] + a_{12}(\theta)^{\mathrm{T}} z_2 + b_1(\theta)^{\mathrm{T}} u,$$

or equivalently

$$a_{12}(\theta)^{\mathrm{T}} z_2 + b_1(\theta)^{\mathrm{T}} u = q_{n_1} + e_2 - a_{11}(\theta)^{\mathrm{T}}[\hat{z}_1 + e_1].$$

Substituting this equality in the differential equation (7) we obtain

$$\dot{z}_2 = \Delta_1 z_2 + [A_{21}(\theta) - f(\theta)a_{11}(\theta)^{\mathrm{T}}](\hat{z}_1 + e_1) + f(\theta)(q_{n_1} + e_2) + \Delta_2(\theta) u, \tag{8}$$

which is a stable system according to Assumption 3.

Applying the transformation of coordinates $r = P^{-1} z_2$, the system (8) can be rewritten as follows

$$\dot{r} = Dr + G_1(\theta)(\hat{z}_1 + e_1) + G_2(\theta)(q_{n_1} + e_2) + G_3(\theta) u, \tag{9}$$

where $G_1(\theta) = P^{-1}[A_{21}(\theta) - f(\theta)a_{11}(\theta)^{\mathrm{T}}]$, $G_2(\theta) = P^{-1} f(\theta)$ and $G_3(\theta) = P^{-1}\Delta_2(\theta)$. The dynamics of (9) is cooperative and stable, and all uncertain functions or variables in the right hand side of (9) belong to an interval for $\theta \in \Theta$:

$$\underline{G_j} \leq G_j(\theta) \leq \overline{G_j},\ j = \overline{1, 3};\ |u(t)| \leq U;$$

$$|e_{1,i}(t)| \leq \bar{e}_{1,i} = \mu_{i-1} V^{\frac{n_1 - i + 2}{n_1 + 1}},\ i = \overline{1, n_1};$$

$$|e_2(t)| \leq \bar{e}_2 = \mu_{n_1} V^{\frac{1}{n_1 + 1}}$$

for all $t \geq T$, where the matrices $\underline{G_j}$, $\overline{G_j}$, $j = \overline{1, 3}$ are known. Therefore the following interval observer can be synthesized for (9):

$$
\begin{aligned}
\dot{\overline{r}} = {} & D\overline{r} + (\overline{G_1}^{+} - \overline{G_1}^{-})\hat{z}_1^{+} + (\underline{G_1}^{-} - \underline{G_1}^{+})\hat{z}_1^{-} + \\
& (\overline{G_1}^{+} + \underline{G_1}^{-})\overline{e}_1 + (\overline{G_2}^{+} - \overline{G_2}^{-})q_{n_1}^{+} + \\
& (\underline{G_2}^{-} - \underline{G_2}^{+})q_{n_1}^{-} + (\overline{G_2}^{+} + \underline{G_2}^{-})\overline{e}_2 + \\
& (\overline{G_3}^{+} + \underline{G_3}^{-})U,
\end{aligned} \tag{10}
$$

$$
\begin{aligned}
\dot{\underline{r}} = {} & D\underline{r} + (\underline{G_1}^{+} - \underline{G_1}^{-})\hat{z}_1^{+} + (\overline{G_1}^{-} - \overline{G_1}^{+})\hat{z}_1^{-} - \\
& (\overline{G_1}^{+} + \underline{G_1}^{-})\overline{e}_1 + (\underline{G_2}^{+} - \underline{G_2}^{-})q_{n_1}^{+} + \\
& (\overline{G_2}^{-} - \overline{G_2}^{+})q_{n_1}^{-} - (\overline{G_2}^{+} + \underline{G_2}^{-})\overline{e}_2 - \\
& (\overline{G_3}^{+} + \underline{G_3}^{-})U,
\end{aligned} \tag{11}
$$

the properties (4), (5) have been used to calculate (10), (11). Introducing the interval estimation errors $\overline{\epsilon} = \overline{r} - r$, $\underline{\epsilon} = r - \underline{r}$, we obtain

$$\dot{\overline{\epsilon}} = D\overline{\epsilon} + \overline{\varepsilon},\ \dot{\underline{\epsilon}} = D\underline{\epsilon} + \underline{\varepsilon},$$

where $\overline{\varepsilon} = (\overline{G_1}^{+} - \overline{G_1}^{-})\hat{z}_1^{+} + (\underline{G_1}^{-} - \underline{G_1}^{+})\hat{z}_1^{-} + (\overline{G_1}^{+} + \underline{G_1}^{-})\overline{e}_1 + (\overline{G_2}^{+} - \overline{G_2}^{-})q_{n_1}^{+} + (\underline{G_2}^{-} - \underline{G_2}^{+})q_{n_1}^{-} + (\overline{G_2}^{+} + \underline{G_2}^{-})\overline{e}_2 + (\overline{G_3}^{+} + \underline{G_3}^{-})U - G_1(\theta)(\hat{z}_1 + e_1) - G_2(q_{n_1} + e_2) - G_3(\theta)u$, $\underline{\varepsilon} = G_1(\theta)(\hat{z}_1 + e_1) + G_2(q_{n_1} + e_2) + G_3(\theta)u - (\underline{G_1}^{+} - \underline{G_1}^{-})\hat{z}_1^{+} - (\overline{G_1}^{-} - \overline{G_1}^{+})\hat{z}_1^{-} + (\overline{G_1}^{+} + \underline{G_1}^{-})\overline{e}_1 + (\underline{G_2}^{+} - \underline{G_2}^{-})q_{n_1}^{+} + (\overline{G_2}^{-} - \overline{G_2}^{+})q_{n_1}^{-} - (\overline{G_2}^{+} + \underline{G_2}^{-})\overline{e}_2 + (\overline{G_3}^{+} + \underline{G_3}^{-})U$. It is an arithmetic exercise to verify that under assumptions 1 and 2 (and the result of Theorem 1) the residual terms $\overline{\varepsilon}$ and $\underline{\varepsilon}$ are elementwise positive and bounded. Then using the results of monotone system theory from Smith (1995) we prove that for all $t \geq T$

$$\underline{r}(t) \leq r(t) \leq \overline{r}(t)$$

and the estimates $\underline{r}(t)$, $\overline{r}(t)$ are bounded, provided that

$$\underline{r}(T) \leq r(T) \leq \overline{r}(T). \tag{12}$$

The former relation for the initial conditions can be easily satisfied since $||x|| \leq X$ under Assumption 1. Using the property (4) we get for all $t \geq T$:

$$
\begin{gathered}
\underline{z}_2(t) \leq z_2(t) = Pr(t) \leq \overline{z}_2(t), \qquad (13)\\
\underline{z}_2(t) = P^{+}\underline{r}(t) - P^{-}\overline{r}(t),\\
\overline{z}_2(t) = P^{+}\overline{r}(t) - P^{-}\underline{r}(t);\\
\underline{z}_1(t) \leq z_1(t) \leq \overline{z}_1(t),\\
\underline{z}_1(t) = \hat{z}_1(t) - \overline{e}_1,\ \overline{z}_1(t) = \hat{z}_1(t) + \overline{e}_1.
\end{gathered}
$$

Defining $\underline{z} = [\underline{z}_1^{\mathrm{T}}\, \underline{z}_2^{\mathrm{T}}]^{\mathrm{T}}$, $\overline{z} = [\overline{z}_1^{\mathrm{T}}\, \overline{z}_2^{\mathrm{T}}]^{\mathrm{T}}$and using (5) we finally formulate the interval estimates for the state x:

$$
\begin{gathered}
\underline{x} = \underline{S}^{+}\underline{z}^{+} - \overline{S}^{+}\underline{z}^{-} - \underline{S}^{-}\overline{z}^{+} + \overline{S}^{-}\overline{z}^{-} \qquad (14)\\
\leq x = S(\theta)z \leq \overline{S}^{+}\overline{z}^{+} - \underline{S}^{+}\overline{z}^{-} - \overline{S}^{-}\underline{z}^{+}\\
+\underline{S}^{-}\underline{z}^{-} = \overline{x},
\end{gathered}
$$

which is satisfied for all $t \geq T$. Thus the following theorem can be formulated.

Theorem 3. Let assumptions 1, 2, 3, 4 hold for the system (1). Then there exist the set of parameters λ_k, $k = \overline{0, n_1}$ in (3) and a constant $T > 0$ such that for all $t \geq T$ the interval estimate (14) is true, provided that the condition (12) is satisfied for (10), (11).

Remark 4. The assumptions 3 and 4 can be replaced with the following one: there exists a vector function $f(\theta) \in \mathbb{R}^{n_2}$ such that

$$
\begin{gathered}
[A_{22}(\theta)z_2 + B_2(\theta)u] - f(\theta)[a_{12}(\theta)^{\mathrm{T}}z_2\\
+b_1(\theta)^{\mathrm{T}}u] = \Delta_1(\theta)z_2 + \Delta_2(\theta)u
\end{gathered}
$$

for some Hurwitz and Metzler matrix function $\Delta_1 : \Theta \to \mathbb{R}^{n_2 \times n_2}$ and some $\Delta_2 : \Theta \to \mathbb{R}^{n_2 \times m}$. Next, the result of Theorem 3 can be obtained using the same technique and an interval observer from the paper Rassi *et al.* (2010).

Implicitly the conditions of Theorem 3 means that the interval observer (10), (11) has to be activated for $t \geq T$ only. The time T can be detected on-line using the property:

$$\sup_{t \geq T} |q_0(t) - \psi(t)| \leq \vartheta,$$

where $\vartheta > 0$ is a constant dependent on discretization step used for computation of (3) ($\vartheta = 0$ under assumption that the differential equation (3) is solved without a computational error), see Levant (2003).

4. FAULT DETECTION

In this section we consider the problem of fault detection for the case when the fault is modeled by the vector of scheduling parameters $\theta(t)$: this case corresponds to an internal fault and it is the most difficult case, but this approach can also be applied to actuators and sensors faults. Following assumption 1 let us assume that in the nominal mode $\theta(t) \in \Theta$, when the state interval estimation is ensured by Theorem 3. In a faulty mode the vector of scheduling parameters $\theta(t)$ leaves the set Θ, which may be detected by the interval consistency check for the measured output ψ.

A more complex case includes the parametric faults in the unobservable subsystem (the z_2-subsystem (7)). In this case the calculated estimates (13) can be compared with some $\underline{z}_2^0$, $\overline{z}_2^0$ available *a priori* from assumption 1 (the state of the system is bounded and under nominal operating conditions it belongs to a known domain). Then the residual can be defined as follows:

$$d(t) = \begin{cases} 1 & \text{if } \underline{z}_2^0 > \overline{z}_2 \text{ or } \underline{z}_2 > \overline{z}_2^0, \\ 0 & \text{otherwise.} \end{cases}$$

Let us demonstrate the advances of the HOSM technique application to interval observers in fault detection on the example of a motor driven pendulum. It is required to detect a fault in the motor subsystem, while the the pendulum deviations are available for measurements only. The model has the following form:

$$
\begin{aligned}
\dot{x}_1 &= x_2,\ y = x_1 + v(t),\\
(ml^2 + J)\dot{x}_2 &= -\rho(\theta)x_2 + \kappa(\theta)x_3\\
&\quad -mgl\sin(x_1) + d_1(t),\\
L\dot{x}_3 &= -Rx_3 - \lambda(\theta)x_2 + u(t) + d_2(t),
\end{aligned}
$$

where x_1, x_2 are the pendulum angle and its velocity respectively, x_3 is the motor current, v is the measurement noise, d_1 and d_2 are state disturbances (unknown inputs with known bounds); m, l are the pendulum mass and length; $g = 9.8$; ρ is an uncertain friction coefficient; κ is the motor torque constant; λ is the motor back constant; R, L are resistance and inductance of the motor. The nominal values and bounds used for simulation are as follows:

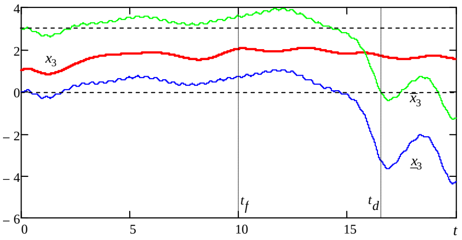

Fig. 1. The results of estimation and fault detection

$$m = 0.5,\ l = 1,\ J = 0.5,\ 1 \le \rho = 1$$
$$+0.5\sin(x_1)^2 \le 1.5,\ 1.5 \le \kappa$$
$$= 2 + 0.5\tanh(x_2) \le 2.5,\ R = 0.5,\ L = 1,$$
$$\lambda = 1,\ d_1(t) = \sin(t),\ d_2(t) = 0.2\cos(2t),$$
$$v(t) = 0.05\sin(15t),\ u(t) = 1.$$

For R, L and λ the admissible deviations are $\pm 10\%$. It is assumed that the fault appears at $t_f = 10$ sec, when the value λ degrades in 10 times from the nominal. It is known that the current has to be in the interval $[0, 3]$ under a healthy operating conditions.

To apply the developed approach we may differentiate twice the output to calculate the estimates of x_2 and $\dot{x}_2$, then an evaluation of x_3 is possible from the second equation. However, due to measurement noise v and disturbance d_1 presence (as well as the severe parametric uncertainty) it is more reasonable to differentiate once the output and apply the observers (10), (11) to the x_3 subsystem (i.e. $r = x_3$ in this example). Next, the obtained in (13) estimates on the variable x_3 can be compared with the values $\underline{x}_3^0 = 0$ and $\overline{x}_3^0 = 3$ in order to detect a fault presence in the motor dynamics.

The results of the system simulation and the curves x_3, $\underline{x}_3^0$, $\overline{x}_3^0$, $\underline{x}_3$ and $\overline{x}_3$ are shown in Fig. 1. As we can see, in this example the motor current belongs to the desired domain all simulation time (thus even its measurement would not help us to detect the presence of the fault). But the interval estimates leave the predefined domain in a finite time $t_d = 17.1$ sec, which implies a fault presence in the motor. The fault detection delay is originated by a serious parametric and signal uncertainty presented in the system.

5. CONCLUSION

The paper is devoted to application of the interval observers and the HOSM differentiation to fault detection and estimation for LPV systems. The HOSM techniques allow us to improve the estimation accuracy of an interval observer designed for LPV systems, or enlarge the class of LPV systems having an interval observer. That leads to fault detection accuracy and reliability improvement.

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