Apports et Potentiels de la Programmation par Contraintes en Optimisation Globale sous Contraintes

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Outline

Motivations

Basics

A Global Constraint for Safe Linear Relaxation

Computing “sharp” upper bounds

Using CSP to boost safe OBR

A challenging finite-domain optimization application

Conclusion

The Problem

We consider the continuous global optimisation problem

$$\mathcal{P} \equiv \begin{cases} \text{min} & f(x) \\ \text{s.c.} & g_i(x) = 0,\ j = 1..k \\ & g_j(x) \leq 0,\ j = k+1..m \\ & \underline{\mathbf{x}} \leq x \leq \overline{\mathbf{x}} \end{cases}$$

with

- $\mathbf{X} = [\underline{\mathbf{x}}, \overline{\mathbf{x}}]$: a vector of intervals of R
- $f : R^n \rightarrow R$ and $g_j : R^n \rightarrow\ R$
- Functions f and g_j: are continuously differentiable on $\mathbf{X}$

Trends in global optimisation

- **Performance**

  Most successful systems (Baron, αBB, ...) use local methods and linear relaxations
  $\rightarrow$ **not rigorous** (work with floats)

- **Rigour**

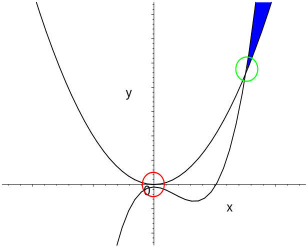
  Mainly rely on interval computation
  ... available systems (e.g., Globsol) are **quite slow**

- **Challenge:** to combine the advantages of both approaches in an **efficient** and **rigorous** global optimisation framework

Example of flaw due to a lack of rigour

Consider the following optimisation problem:

$$\begin{array}{ll} \min & x \\ \text{s. t.} & y - x^2 \geq 0 \\ & y - x^2 * (x-2) + 10^{-5} \leq 0 \\ & x, y \in [-10, +10] \end{array}$$



Baron 6.0 and Baron 7.2 find $\color{red}{0}$ as the minimum ...

Basics

- **Branch and Bound Algorithm**
- **Basics on Numeric CSP**

Branch and Bound Algorithm

- **BB Algorithm:**
  While $\mathcal{L} \neq \emptyset$ do   %$\mathcal{L}$ initialized with the input box
  - Select a box B from the set of current boxes $\mathcal{L}$
  - Reduction (filtering or tightening) of B
  - Lower bounding of f in box B
  - Upper bounding of f in box B
  - Update of $\underline{\mathbf{f}}$ and $\overline{\mathbf{f}}$
  - Splitting of B (if not empty)

- **Upper Bounding – Critical issue:**
  to prove the **existence** of a feasible point in a reduced box

- **Lower Bounding – Critical issue:**
  to achieve an **efficient pruning**

Numeric CSP

- $\mathcal{X} = \{x_1, \ldots, x_n\}$ is a set of variables
- $\mathbf{X} = \{\mathbf{X}_1, \ldots, \mathbf{X}_n\}$ is a set of domains
  ($\mathbf{X}_i$ contains all acceptable values for variable x_i)

  $\mathbf{X}_i = [\underline{\mathbf{x}_i}, \overline{\mathbf{x}_i}]$
- $\mathcal{C} = \{c_1, \ldots, c_m\}$ is a set of constraints

Numeric CSP: Overall scheme

A Branch & Prune schema:

1. **Pruning the search space**
2. **Making a choice to generate two (or more) sub-problems**

- The pruning step → **filtering techniques** to reduce the size of the intervals
- The branching step → **splits the intervals** (uses heuristics to choose the variable to split)

Local consistencies

- **2B–consistency** only requires to check the Arc–Consistency property **for each bound** of the intervals

  Variable x with $\mathbf{X} = [\underline{\mathbf{x}}, \overline{\mathbf{x}}]$ is 2B–consistent for constraint $f(x, x_1, \ldots, x_n) = 0$ if $\underline{\mathbf{x}}$ and $\overline{\mathbf{x}}$ are the leftmost and the rightmost zero of $f(x, x_1, \ldots, x_n)$

- **Box–consistency** :
  - $\rightarrow$ coarser relaxation of AC than 2B–consistency
  - $\rightarrow$ **better filtering**

  Variable x with $\mathbf{X} = [\underline{\mathbf{x}}, \overline{\mathbf{x}}]$ is Box–Consistent for constraint $f(x, x_1, \ldots, x_n) = 0$ if $\underline{\mathbf{x}}$ and $\overline{\mathbf{x}}$ are the leftmost and the rightmost zero of $\mathbf{F}(\mathbf{x}, \mathbf{X_1}, \ldots, \mathbf{X_n})$, the optimal interval extension of $f(x, x_1, \ldots, x_n)$

Filtering

- **2B–filtering Algorithms** $\rightsquigarrow$ **projection functions**
- **Box–filtering Algorithms** $\rightsquigarrow$ **monovariate version of the interval Newton method**
- Based on **Interval Arithmetic**

Limits of Interval Arithmetic

- **Wrapping effect**: overestimate by a unique interval the image of **f** over an interval vector
- **Dependency problem**: independence the different occurences of some variable during the evaluation of an expression

  Consider $X = [0, 5]$
  $\mathbf{X} - \mathbf{X} = [\mathbf{0} - \mathbf{5}, \mathbf{5} - \mathbf{0}] =$ **[-5,5]** instead of **[0,0] !**
  $\mathbf{X^2} - \mathbf{X} = [\mathbf{0}, \mathbf{25}] - [\mathbf{0}, \mathbf{5}] =$ **[-5,25]**
  $\mathbf{X}(\mathbf{X} - \mathbf{1}) = [\mathbf{0}, \mathbf{5}]([\mathbf{0}, \mathbf{5}] - [\mathbf{1}, \mathbf{1}])$
  $= [\mathbf{0}, \mathbf{5}][-\mathbf{1}, \mathbf{4}] =$ **[-5,20]**

Limits of Local Consistencies

- **A constraint is handled as a black-box** by local consistencies (2B,BOX,...)
  - No way to catch the dependencies between constraints (amplified by constraint decomposition)
  - Splitting is behind the success for small dimensions

- **Higher consistencies** (KB–filtering,Bound–filtering)
  $\longrightarrow$ capture some dependencies between constraints
  $\longrightarrow$ visiting numerous combinations

$\Rightarrow$ A **global constraint** to handle a **linear approximation** with LP solvers
$\longrightarrow$ safe linear relaxations

A Global Constraint for Safe Linear Relaxation

- works on **quadratic terms and bilinear terms**

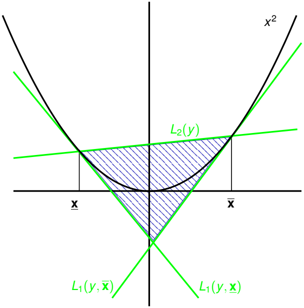
  $\longrightarrow$ to rewrite power terms and product terms

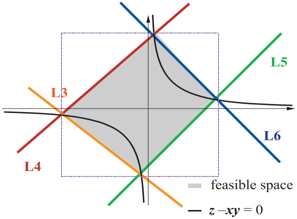
  - **quadrification technique** derived from Sheraldi techniques
  - **Critical issue:** to find a good trade off between a tight relaxation and the number of generated terms

- Quadratic terms and bilinear terms are approximated by tight redundant constraints

The QUAD process

- **Reformulation**
  - capture the linear part
    $\longrightarrow$ replace non linear terms by new variable
    eg x^2 by y_i
- **Linearisation**
  - introduce **redundant linear constraints**
    $\longrightarrow$ tight approximations (RLT)
- **Computing min(X) $= \underline{\mathbf{x}}_i$** and **max(X) $= \overline{\mathbf{x}}_i$** in *LP*





Reformulation for x^2

$\mathbf{y} = \mathbf{x^2}$ with $\mathbf{x} \in [-\mathbf{4}, \mathbf{5}]$

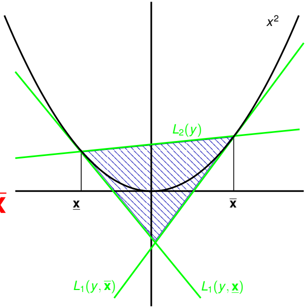
$\mathbf{L_1}(\mathbf{y}, \alpha) \equiv \mathbf{y} \geq \mathbf{2}\alpha\mathbf{x} - \alpha^{\mathbf{2}}$

$\mathbf{L_1}(\mathbf{y}, -\mathbf{4}) : \mathbf{y} \geq -\mathbf{8x} - \mathbf{16}$

$\mathbf{L_1}(\mathbf{y}, \mathbf{5}) : \mathbf{y} \geq \mathbf{10x} - \mathbf{25}$

$\mathbf{L_2}(\mathbf{y}) \equiv \mathbf{y} \leq (\underline{\mathbf{x}} + \overline{\mathbf{x}})\mathbf{x} - \underline{\mathbf{x}} * \overline{\mathbf{x}}$

$\mathbf{L_2}(\mathbf{y}) : \mathbf{y} \leq \mathbf{x} + \mathbf{20}$



Quad filtering algorithm

Function Quad_filtering (`IN`: $\mathbf{X}$, $\mathcal{C}$, ϵ) `return` $\mathbf{X}'$

1. ***Reformulation***
   $\rightarrow$ linear inequalities L_i for the nonlinear terms in $\mathcal{C}$
2. ***Linearisation/relaxation of the whole system***
   $\rightarrow$ a linear system LR
3. $\mathbf{X}'$:= $\mathbf{X}'$
4. ***Pruning***:
   While reduction of some bound $> \epsilon$ **and** $\emptyset \notin \mathbf{X}'$ **Do**
   4.1 Reduce the lower and upper bounds $\underline{\mathbf{x}}'_i$ and $\overline{\mathbf{x}}'_i$ of each ***initial*** variable $x_i \in \mathcal{X}$
   $\rightarrow$ Computing min and max of $\mathbf{X_i}$ with a LP solver
   4.2 Update the coefficients of L_i according to the new bounds

Issues in the use of linear relaxation

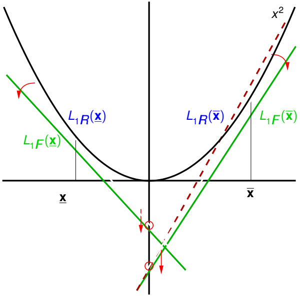
- Coefficients of linear relaxations are scalars
  $\Rightarrow$ computed with ***floating point numbers***
- Efficient implementations of the simplex algorithm
  $\Rightarrow$ use ***floating point numbers***
- All the computations with floating point numbers require ***right corrections***

Safe approximations of L_1

$$L_1(y,\alpha) \equiv y \geq 2\alpha x - \alpha^2$$

Effects of rounding:

- rounding of 2α
  - $\Rightarrow$ rotation on y axis
- rounding of α^2
  - $\Rightarrow$ translation on y axis



Correction of the Simplex algorithm

Consider the following LP :

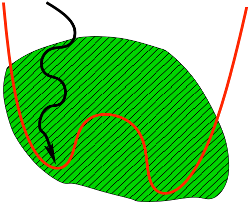
$$\text{minimise } c^T x$$

$$\text{subject to } \underline{\mathbf{b}} \leq Ax \leq \overline{\mathbf{b}}$$

- Solution $=$ vector $x_R \in R^n$
- LP solver computes a vector $x_F \in F^n \neq x_R$
- x_F is safe for the objective if $c^T x_R \geq c^T x_F$
- **Neumaier & Shcherbina**
  $\rightarrow$ cheap method to obtain a **rigorous bound** of the objective
  (use of the approximation solution of the dual)

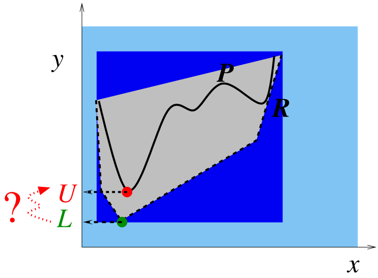
Computing “sharp” upper bounds

- **Upper bounding**
  - **local search**
    $\longrightarrow$ approximate feasible point x_{approx}
  - **epsilon inflation process and proof**
    $\longrightarrow$ provide a feasible box x_{proved}
  - compute $\bar{\mathbf{f}}^* = min(\bar{\mathbf{f}}(x_{proved}), \bar{\mathbf{f}}^*)$



- **Critical issue**: to prove the existence of a feasible point in a reduced box
  - Singularities
  - Guess point too far from a feasible region (local search works with floats)

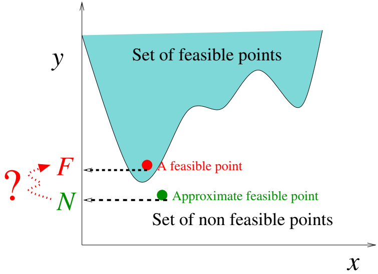
Using the lower bound to get an upper-bound



Branch&Bound step where *P* is the set of feasible points and *R* is the linear relaxation

Idea: modify the safe lower bound ...
to get an upper-bound !

Lower bound: a good starting point to find a feasible upper-bound ?



N, optimal solution of R, not a feasible point of P but (may be) **a good starting point**:

- BB splits the domains at each iteration:
  smaller box $\rightsquigarrow$ N nearest from the optima of P
- Proof process inflates a box around the guess point $\rightsquigarrow$ compensate the distance from the feasible region

Method

- Correction procedure to **get a better feasible point** from a given approximate feasible point

$\longrightarrow$ to exploit **Newton-Raphson for under-constrained systems** of equations (and Moore-Penrose inverse)

Good convergence when the starting point is nearly feasible

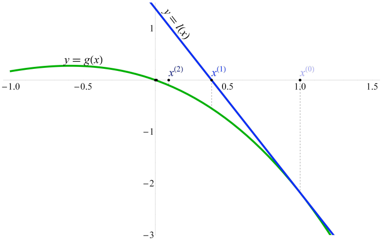
Handling square systems of equations

- $g = (g_1, \ldots, g_m) : R^n \longrightarrow R^m$ **($n = m$)**

  $\longrightarrow$ Newton-Raphson step:

  $$x^{(i+1)} = x^{(i)} - J_g^{-1}(x^{(i)})g(x^{(i)})$$

  Converges well if the exact solution to be approximated is **not singular**



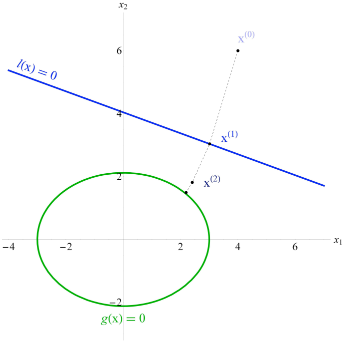
Handling under-constrained systems of equations

Manifold of solutions
$\rightarrow$ linear system $l(x) = 0$ is under-constrained
$\rightarrow$ **Choose a solution $x^{(1)}$** of $l(x) = 0$

Best choice:
Solution of $l(x) = 0$ close to $x^{(0)}$
Can easily be computed with the **Moore-Penrose inverse**:

$$x^{(i+1)} = x^{(i)} - A_g^+(x^{(i)})g(x^{(i)})$$

$A_g^+ \in R^{n \times m}$ is the Moore-Penrose inverse of A_g, solution of the equation which minimizes $||x^{(1)} - x^{(0)}||$)

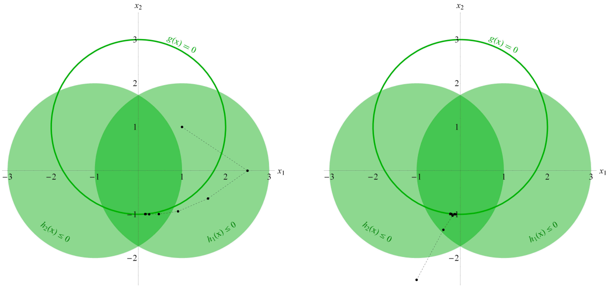


Handling under-constrained systems of equations and inequalities

- Under-constrained systems of equations and **inequalities**
  $\rightsquigarrow$ introduce **slack variables**

- **Initial values** for the slack variables have to be provided

  Slightly positive value
  $\longrightarrow$ to break the symmetry
  $\longrightarrow$ good convergence



A new upper bounding strategie

Function `UpperBounding`(`IN` $\mathbf{x}$, x^*_{LP}; `INOUT`$\mathcal{S}'$)

% $\mathcal{S}'$: list of proven feasible boxes
% x^*_{LP}: the optimal solution of the LP relaxation of $\mathcal{P}(\mathbf{x})$
$\mathcal{S}'$:= $\emptyset$
x^*_{corr} := FeasibilityCorrection(x^*_{LP}) % Improving x^*_{LP} feasibility
$\mathbf{x}_p$:= InflateAndProve(x^*_{corr}, $\mathbf{x}$)
if $\mathbf{x}_p \neq \emptyset$ **then**
    $\mathcal{S}'$:= $\mathcal{S}' \cup \mathbf{x}_p$
endif
return $\mathcal{S}'$

Experiments

- Significant set of benchmarks of the COCONUT project
- Selection of 35 benchmarks where Icos did find the global minimum while relying on an unsafe local search
- 31 benchmarks are solved and **proved** within a 30s time out
- Almost all benchmarks are solved in **much less time** and with **much more proven solutions**

Experiments (2)

Name	(n,m)	LS: $t(s)$	UB/LB: $t(s)$
alkyl	(14, 7)	-	1.54
circle	(3, 10)	1.98	0.84
ex14_1_2	(6, 9)	-	1.74
ex14_1_3	(3, 4)	-	0.42
ex14_1_6	(9, 15)	-	12.44
ex14_1_8	(3, 4)	-	-
ex2_1_1	(5, 1)	0.09	0.04
ex2_1_2	(6, 2)	-	0.24
ex2_1_3	(13, 9)	-	1.32
ex2_1_4	(6, 5)	0.52	0.43
ex2_1_6	(10, 5)	1.61	0.35
ex3_1_3	(6, 6)	1.03	0.29
ex3_1_4	(3, 3)	6.51	0.14
ex4_1_2	(1, 0)	18.84	17.03
ex4_1_6	(1, 0)	0.11	14.28
ex4_1_7	(1, 0)	0.07	0.01
ex5_4_2	(8, 6)	-	18.15
ex6_1_2	(4, 3)	0.51	0.52
ex6_1_4	(6, 4)	7.45	8.92
ex7_3_5	(13, 15)	-	-
ex8_1_6	(2, 0)	-	0.39
ex9_1_1	(13, 12)	-	-
ex9_1_10	(14, 12)	-	3.76
ex9_1_4	(10, 9)	-	0.49
ex9_1_5	(13, 12)	-	2.68
ex9_1_8	(14, 12)	-	3.76
ex9_2_1	(10, 9)	-	0.68
ex9_2_4	(8, 7)	2.94	0.69
ex9_2_5	(8, 7)	-	-
ex9_2_7	(10, 9)	-	0.68
ex9_2_8	(6, 5)	-	0.53
house	(8, 8)	-	0.90
nemhaus	(5, 5)	0.02	0.01

Using CSP to boost safe OBR

- **OBR** (optimal based reduction):
  known bounds of the objective function $\rightarrow$ **to reduce** the size of the domains

- **Refutation** techniques $\rightarrow$ **boosting safe OBR**

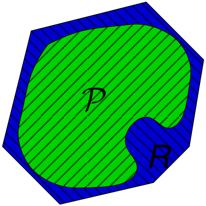
Lower bounding

- **Relaxing the problem**
  - linear relaxation R of $\mathcal{P}$

$$\begin{array}{ll} min & d^T x \\ s.t. & Ax \leq b \end{array}$$

  - LP solver $\rightarrow \underline{\mathbf{f}}^*$

$\rightarrow$ numerous splitting



- **OBR is a way to speed up the reduction process**

Optimality Base Reduction

- Introduced by **Ryoo and Sahinidis**
  - to take advantage of the **known bounds of the objective function** to reduce the size of the domains
  - uses a well known property of the **saddle point** to compute new bounds for the domains with the known bounds of the objective function

Theorems of OBR

- Let $[L, U]$ be the domain of f:
  - U is an **upper-bound of the intial problem $\mathcal{P}$**
  - L is a **lower-bound of a convex relaxation R of $\mathcal{P}$**

  If the constraint $\mathbf{x_i} - \overline{\mathbf{x}}_\mathbf{i} \leq \mathbf{0}$ is **active** at the optimal solution of R and has a corresponding multiplier $\lambda_i^* > 0$ (λ^* is the optimal solution of the dual of R), then

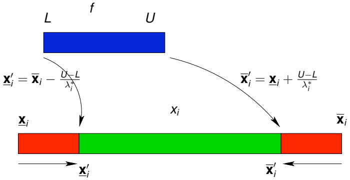
$$\mathbf{x_i} \geq \underline{\mathbf{x}}'_\mathbf{i} \text{ with } \underline{\mathbf{x}}'_\mathbf{i} = \overline{\mathbf{x}}_\mathbf{i} - \frac{\mathbf{U} - \mathbf{L}}{\lambda_\mathbf{i}^*}$$

  if $\underline{\mathbf{x}}'_i > \underline{\mathbf{x}}_i$, the domain of x_i can be shrinked to $[\underline{\mathbf{x}}'_i, \overline{\mathbf{x}}_i]$ **without loss of any global optima**

- similar theorems for $\underline{\mathbf{x}}_i - x_i \leq 0$ and $g_i(x) \leq 0$.

OBR: intuitions

- Ryoo & Sahinidis 96



$$x_i \geq \underline{\mathbf{x}}_i' \text{ with } \underline{\mathbf{x}}_i' = \overline{\mathbf{x}}_i - \frac{U-L}{\lambda_i^*}$$

- **does not modify the very branch and bound process**
- **almost for free !**

OBR Issues

- **Critical issue: basic OBR algorithm is unsafe**
  - it uses the dual solution of the linear relaxation
  - Efficient LP solvers work with floats $\longrightarrow$ the available dual solution λ^* is an **approximation** if used in OBR ...
    ... $\longrightarrow$ **OBR may remove actual optimum !**

- **Solutions:** two ways to take advantage of OBR
  1. **prove dual solution** (Kearfott): combininig the dual of linear relaxation with the Kuhn-Tucker conditions
  2. **validate the reduction** proposed by OBR with CP !

CP approach: intuition

- **Essential observation**: **if the constraint system**

$$\begin{array}{l} L \leq f(x) \leq U \\ g_i(x) = 0, \; i = 1..k \\ g_j(x) \leq 0, \; j = k+1..m \end{array}$$

  has no solution when the domain of x is set to $[\underline{\mathbf{x}}_i, \underline{\mathbf{x}}'_i]$, **the reduction computed by OBR is valid**

- **Try to reject $[\underline{\mathbf{x}}_i, \underline{\mathbf{x}}'_i]$** with **classical filtering techniques;**
  otherwise add this box to the list of boxes to process

CP algorithm

$\mathcal{L}_r$:= $\emptyset$ % set of potential non-solution boxes

for each variable x_i **do**
  Apply OBR
    and add the generated potential non-solution boxes to $\mathcal{L}_r$

for each box $\mathbf{B_i}$ in $\mathcal{L}_r$ **do**
  $\mathbf{B_i'}$:= **2B-filtering**($\mathbf{B_i}$)
  if $\mathbf{B_i'} = \emptyset$ **then** reduce the domain of x_i
  else $\mathbf{B_i''}$:= **QUAD-filtering**($\mathbf{B_i'}$)
      if $\mathbf{B_i''} = \emptyset$ **then** reduce the domain of x_i
      else add $\mathbf{B_i}$ to global list of box to be handled **endif**
  endif

Compute $\underline{f}$ with QUAD_SOLVER in X

Experiments

- Compares 4 versions of the branch and bound algorithm:
  - without OBR
  - with unsafe OBR
  - with safe OBR based on Kearfott's approach
  - with safe OBR based on CP techniques

  implemented with **Icos using Coin/CLP and Coin/IpOpt**
- On 78 **benches** (from Ryoo & Sahinidis 1995, Audet thesis and the coconut library)
- All experiments have been done on PC-Notebook/1Ghz.

Experimental Results (2): Synthesis

Synthesis of the results:

	$\Sigma_t(s)$	$\%saving$
no OBR	2384.36	-
unsafe OBR	881.51	63.03%
safe OBR Kearfott	1975.95	17.13%
safe OBR CP	**454.73**	**80.93%**

(with a timeout of 500s)

Safe CP-based OBR faster than unsafe OBR !

... because wrong domains reductions prevent the upper-bounding process from improving the current upper bound !!

Finite domains CSP & Global Optimisation

Handling software upgradeability problems

- A **critical issue** in modern operating systems
  - → Finding the "best" solution to install, remove or upgrade packages in a given installation.
  - → The complexity of the upgradeability problem itself is **NP complete**
  - → modern OS contain a huge number of packages (often more than **20 000** packages in a Linux distribution)
- **Several optimisation criteria** have to be considered, e.g., stability, memory efficiency, network efficiency
- **Mancoosi** project (FP7/2007-2013, `http://www.mancoosi.org/`)

Solving software upgradeability problems

Computing a final package configuration from an intial one

- A configuration states which package is installed and which package is not installed:
  - **Problem** (in CUDF): list of package descriptions (with their status) & a set of packages to install/remove/upgrade
  - **Final configuration**: list of installed packages (uninstalled packages are not listed)

- **Expected Answer**: **best solution** according to multiple criteria

A Problem: list of package descriptions & requests (1)

A package description provides:

- the **package name** and **package version**
  - $p_{i,j}$ = (package name p_i, package version v_j) is unique for each problem in CUDF
  - The $p_{i,j}$ are basic variables
    $\rightarrow$ **solvers have to instantiate $p_{i,j}$ with true or false**
- Package **dependencies** and **conflicts**: set of contraints between the $p_{i,j}$ (CNF formula)
- Provided **features:** if package p_1 depends on feature f_λ provided by q_1 and q_2, then installing q_1 or q_2 will fulfill p_1's dependency on f_λ.

A Problem: list of package descriptions & requests (2)

- **Requests** are:
  - **Commands/actions** on the initial configuration: install, remove and/or upgrade package instructions
    - **install p:** at least one version of p must be installed in the final configuration
    - **remove p:** no version of p must be installed in the final configuration
    - **upgrade p:** let p_v be the highest version installed in the initial configuration, then p'_v with $v' \geq v$ must be the only version installed in the final configuration
  - **Mandatory**: the final configuration must fulfill all the requests (otherwise there is no solution to the problem)
- **Requests** induce **additional constraints** on the problem to solve

Finding the best solution

- **Best solution**

  → multiple criteria, e.g.,
  - minimize the number of removed packages, and,
  - minimize the number of changed packages

- **Mono criteria optimization solvers**

  → using a **linear combination** of the criteria
  → solving each criteria sequentially

MILP model: handling dependencies

1. **Conjunction**:

$$\mathcal{D}epend(p_v) = \bigwedge_{i=1}^{n} p_i \;\rightsquigarrow\; \mathbf{-n * p_v} + \sum_{i=1}^{n} p_i >= 0$$

   if $p_v = 1$ (installed), then all $p_i = 1$; if $p_v = 0$ (not installed), then the p_i can take any value

2. **Disjunction**

$$\mathcal{D}epend(p_v) = \bigvee_{k=1}^{l_m} p_k \;\rightsquigarrow\; \mathbf{-p_v} + \sum_{k=1}^{l_m} p_k >= 0$$

   thus, if $p_v = 1$, at least one of the p_k will be installed.

MILP model: handling conflicts

Conflict property: a simple conjunction of packages
$\rightarrow$ inequality:

$$\mathbf{n'} * \mathbf{p_v} + \sum_{p_c \in \mathcal{C}onflict(p_v)} p_c <= \mathbf{n'}$$

where $\mathcal{C}onflict(p_v)$ is the set of package conflicting with p_v and $n' = \mathcal{C}ard(\mathcal{C}onflict(p_v))$

- $\rightarrow$ if p_v is installed, none of the p_v conflicting packages can be installed
- $\rightarrow$ if p_v is not installed, then the conflicting packages can freely be either installed or not

MILP model: handling multi criteria (1)

Assume the following 2 criteria:

- **First criterion**: **minimize the number of removed functionalities** among the installed ones

$$min \sum_{p \in F_{\mathcal{I}nstalled}} -p$$

where $F_{\mathcal{I}nstalled}$ is the set of installed functionalities

- **Second criterion**: **minimize the number of modifications;** if package *p*, version *i* is installed keep it installed, if package *p* version *u* it is not installed keep it uninstalled

$$min \sum_{p_i \in P_{\mathcal{I}nstalled}} -p_i + \sum_{p_u \in P_{\mathcal{U}ninstalled}} p_u$$

where $P_{\mathcal{I}nstalled}$ is the set of installed versioned packages and $P_{\mathcal{U}ninstalled}$ is the set of uninstalled versioned packages.

MILP model: handling multi criteria (2)

- Handling these criteria in a lexical order

  → **criteria are aggregated** in the following way:

$$\sum_{p \in F_{\mathcal{I}nstalled}} -\mathcal{C}ard(P) * p \;+ \sum_{p_i \in P_{\mathcal{I}nstalled}} -p_i \;+ \sum_{p_u \in P_{\mathcal{U}ninstalled}} p_u$$

where $P = P_{\mathcal{I}nstalled} \cup P_{\mathcal{U}ninstalled}$

Multiplying first criterion coefficients by $\mathcal{C}ard(P)$ lets any of them have a higher value than any combination of the second criterion

Experiments

- A set of **200 problems**, ranging from random problems to real one and from **20000 up to 50000 packages**
- **MILP solvers & Pseudo boolean solvers**

	IBM CPLEX 11.1	SCIP 1.2	WBO
Time out	0	0	1
No sol	58	58	58
Min time (s)	0.54	0.54	0.53
Max time (s)	7.83	193.73	300
Geometric Mean time (s)	**2.5**	**10.29**	**23.6**

- IBM **CP** : **could not find any solution within 300s**

Examples of optimization criteria (ongoing solver competition)

- **paranoid**:
  minimizing the packages removed in the solution
  &
  minimizing packages changed by the solution
- **trendy**:
  minimizing packages removed in the solution
  &
  minimizing outdated packages in the solution
  &
  minimizing package recommendations not satisfied
  &
  minimizing extra packages installed.

Open questions

- **How to boost CP ?**
  - Taking advantage of the dependency graph
  - Combining CP and MILP
- **Better handling of preferences ?**

Conclusion

+ CSP refutation techniques

- allow a **safe** and **efficient** implementation of OBR
- can **outperform standard mathematical methods**
- might be suitable for other unsafe methods

+ Safe global constraints

- provide an efficient alternative to local search: $\longrightarrow$ good starting point for a Newton method $\rightsquigarrow$ feasible region
- **drastically improve the performances** of the upper-bounding process

? CP and Robustness

? Large finite-domain optimization problems