Correction to ”Estimation of errors in the inverse modeling of accidental release of atmospheric pollutant: Application to the reconstruction of the cesium-137 and iodine-131 source terms from the Fukushima Daiichi power plant”

Victor Winiarek, Marc Bocquet, Olivier Saunier, Anne Mathieu

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Victor Winiarek, Marc Bocquet, Olivier Saunier, and Anne Mathieu

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[1] In the paper "Estimation of errors in the inverse modeling of accidental release of atmospheric pollutant: Application to the reconstruction of the cesium-137 and iodine-131 source terms from the Fukushima Daiichi power plant" by Victor Winiarek et al. (*Journal of Geophysical Research*, *117*, D05122, doi:10.1029/2011JD016932, 2012) several equations should read as follows. One figure should also be replaced as shown below.

[2] Originally written with a zero background term ($\boldsymbol{\sigma}_b = \mathbf{0}$), the formulas were, in the revised manuscript, extended to the general case, with an unfortunate mistake in the normalization term in equation (9). In order for equation (9) to be a properly normalized probability density function (pdf), it should read

$$\begin{cases} \text{if} \quad \boldsymbol{\sigma} \geq \mathbf{0} & p(\boldsymbol{\sigma}) = \left(\int_{\mathbf{s} \geq 0} e^{-\frac{1}{2}(\mathbf{s}-\boldsymbol{\sigma}_b)^{\mathrm{T}}\mathbf{B}^{-1}(\mathbf{s}-\boldsymbol{\sigma}_b)} \mathrm{d}\mathbf{s} \right)^{-1} e^{-\frac{1}{2}(\boldsymbol{\sigma}-\boldsymbol{\sigma}_b)^{\mathrm{T}}\mathbf{B}^{-1}(\boldsymbol{\sigma}-\boldsymbol{\sigma}_b)} \\ \text{otherwise} & p(\boldsymbol{\sigma}) = 0. \end{cases}$$

In the case where $\mathbf{B}$ is diagonal, $\mathbf{B} = m^2\mathbf{I}_N$, the pdf reads

$$\begin{cases} \text{if} \quad \boldsymbol{\sigma} \geq \mathbf{0} & p(\boldsymbol{\sigma}) = \left(\prod_{i=1}^{N} \operatorname{erfc}\left(-\frac{[\sigma_b]_i}{m\sqrt{2}} \right) \right)^{-1} \dfrac{e^{-\frac{1}{2}(\boldsymbol{\sigma}-\boldsymbol{\sigma}_b)^{\mathrm{T}}\mathbf{B}^{-1}(\boldsymbol{\sigma}-\boldsymbol{\sigma}_b)}}{\sqrt{(\pi/2)^N |\mathbf{B}|}} \\ \text{otherwise} & p(\boldsymbol{\sigma}) = 0. \end{cases}$$

Function erfc is the complementary error function defined by $\operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^{+\infty} e^{-t^2} \mathrm{d}t$.

[3] As a consequence, equation (21) should read

$$\begin{aligned} p(\boldsymbol{\mu}|r,m) &= \left(\sqrt{(2\pi)^d |\mathbf{R}|} \int_{\mathbf{s} \geq \mathbf{0}} e^{-\frac{1}{2}(\mathbf{s}-\boldsymbol{\sigma}_b)^{\mathrm{T}}\mathbf{B}^{-1}(\mathbf{s}-\boldsymbol{\sigma}_b)} \mathrm{d}\mathbf{s} \right)^{-1} \\ &\quad \times \int_{\boldsymbol{\sigma} \geq \mathbf{0}} e^{-\frac{1}{2}(\boldsymbol{\mu}-\mathbf{H}\boldsymbol{\sigma})^{\mathrm{T}}\mathbf{R}^{-1}(\boldsymbol{\mu}-\mathbf{H}\boldsymbol{\sigma}) - \frac{1}{2}(\boldsymbol{\sigma}-\boldsymbol{\sigma}_b)^{\mathrm{T}}\mathbf{B}^{-1}(\boldsymbol{\sigma}-\boldsymbol{\sigma}_b)} \mathrm{d}\boldsymbol{\sigma} \\ &= e^{-\frac{1}{2}(\boldsymbol{\mu}-\mathbf{H}\boldsymbol{\sigma}_b)^{\mathrm{T}}\left(\mathbf{R}+\mathbf{HBH}^{\mathrm{T}}\right)^{-1}(\boldsymbol{\mu}-\mathbf{H}\boldsymbol{\sigma}_b)} \\ &\quad \times \left(\sqrt{(2\pi)^d |\mathbf{R}|} \int_{\mathbf{s} \geq \mathbf{0}} e^{-\frac{1}{2}(\mathbf{s}-\boldsymbol{\sigma}_b)^{\mathrm{T}}\mathbf{B}^{-1}(\mathbf{s}-\boldsymbol{\sigma}_b)} \mathrm{d}\mathbf{s} \right)^{-1} \\ &\quad \times \int_{\boldsymbol{\sigma} \geq \mathbf{0}} e^{-\frac{1}{2}(\boldsymbol{\sigma}-\boldsymbol{\sigma}_a)^{\mathrm{T}}\mathbf{P}_a^{-1}(\boldsymbol{\sigma}-\boldsymbol{\sigma}_a)} \mathrm{d}\boldsymbol{\sigma}. \end{aligned}$$

Equation (22) should read

$$\begin{aligned} p(\boldsymbol{\mu}|r,m) = & \frac{e^{-\frac{1}{2}(\boldsymbol{\mu}-\mathbf{H}\boldsymbol{\sigma}_b)^{\mathrm{T}}\left(\mathbf{R}+\mathbf{HBH}^{\mathrm{T}}\right)^{-1}(\boldsymbol{\mu}-\mathbf{H}\boldsymbol{\sigma}_b)}}{\sqrt{(2\pi)^d \left|\mathbf{HBH}^{\mathrm{T}} + \mathbf{R}\right|}} \\ & \times \sqrt{(2\pi)^N |\mathbf{B}|} \left(\int_{\mathbf{s} \geq \mathbf{0}} e^{-\frac{1}{2}(\mathbf{s}-\boldsymbol{\sigma}_b)^{\mathrm{T}}\mathbf{B}^{-1}(\mathbf{s}-\boldsymbol{\sigma}_b)} \mathrm{d}\mathbf{s} \right)^{-1} \\ & \times \int_{\boldsymbol{\sigma} \geq \mathbf{0}} \frac{e^{-\frac{1}{2}(\boldsymbol{\sigma}-\boldsymbol{\sigma}_a)^{\mathrm{T}}\mathbf{P}_a^{-1}(\boldsymbol{\sigma}-\boldsymbol{\sigma}_a)}}{\sqrt{(2\pi)^N |\mathbf{P}_a|}} \mathrm{d}\boldsymbol{\sigma}. \end{aligned}$$

In the general case where $\mathbf{B}$ is not diagonal, we can use the GHK simulator to compute both the integral terms, as specified in Appendix A of the original paper. In the case where $\mathbf{B} = m^2\mathbf{I}_N$, the likelihood reads

$$\begin{aligned} p(\boldsymbol{\mu}|r,m) = & \frac{e^{-\frac{1}{2}(\boldsymbol{\mu}-\mathbf{H}\boldsymbol{\sigma}_b)^{\mathrm{T}}\left(\mathbf{R}+\mathbf{HBH}^{\mathrm{T}}\right)^{-1}(\boldsymbol{\mu}-\mathbf{H}\boldsymbol{\sigma}_b)}}{\sqrt{(2\pi)^d \left|\mathbf{HBH}^{\mathrm{T}} + \mathbf{R}\right|}} \\ & \times 2^N \left(\prod_{i=1}^{N} \operatorname{erfc}\left(-\frac{[\sigma_b]_i}{m\sqrt{2}} \right) \right)^{-1} \int_{\boldsymbol{\sigma} \geq \mathbf{0}} \frac{e^{-\frac{1}{2}(\boldsymbol{\sigma}-\boldsymbol{\sigma}_a)^{\mathrm{T}}\mathbf{P}_a^{-1}(\boldsymbol{\sigma}-\boldsymbol{\sigma}_a)}}{\sqrt{(2\pi)^N |\mathbf{P}_a|}} \mathrm{d}\boldsymbol{\sigma}, \end{aligned}$$

which can be seen as the product of the regular "Gaussian case" term (the first fraction) by a correction taking into account the positivity of the source term (the "normalization term" and the integral term). The integral term can be computed using the GHK simulator.

[4] These changes do not affect the results of the original paper where $\boldsymbol{\sigma}_b = \mathbf{0}$, since in this case the equations are still fully valid. These changes slightly affect the results of section 3.6.2 where a non-null first guess is used, and Figure 13 should be replaced as shown here. The changes in the retrieved source are very small. Besides, the main peaks occur at the same date and have the same magnitude, and the total released activity is unchanged at the given numerical precision.

[5] Finally, in Appendix A, equation (A1) should consistently read

$$\begin{aligned} p(\boldsymbol{\mu}|r,m) = & \frac{e^{-\frac{1}{2}(\boldsymbol{\mu}-\mathbf{H}\boldsymbol{\sigma}_b)^{\mathrm{T}}\left(\mathbf{R}+\mathbf{HBH}^{\mathrm{T}}\right)^{-1}(\boldsymbol{\mu}-\mathbf{H}\boldsymbol{\sigma}_b)}}{\sqrt{(2\pi)^d \left|\mathbf{HBH}^{\mathrm{T}} + \mathbf{R}\right|}} \\ & \times \sqrt{(2\pi)^N |\mathbf{B}|} \left(\int_{\mathbf{s} \geq \mathbf{0}} e^{-\frac{1}{2}(\mathbf{s}-\boldsymbol{\sigma}_b)^{\mathrm{T}}\mathbf{B}^{-1}(\mathbf{s}-\boldsymbol{\sigma}_b)} \mathrm{d}\mathbf{s} \right)^{-1} \\ & \times \int_{\boldsymbol{\sigma} \geq \mathbf{0}} \frac{e^{-\frac{1}{2}(\boldsymbol{\sigma}-\boldsymbol{\sigma}_a)^{\mathrm{T}}\mathbf{P}_a^{-1}(\boldsymbol{\sigma}-\boldsymbol{\sigma}_a)}}{\sqrt{(2\pi)^N |\mathbf{P}_a|}} \mathrm{d}\boldsymbol{\sigma}. \end{aligned}$$

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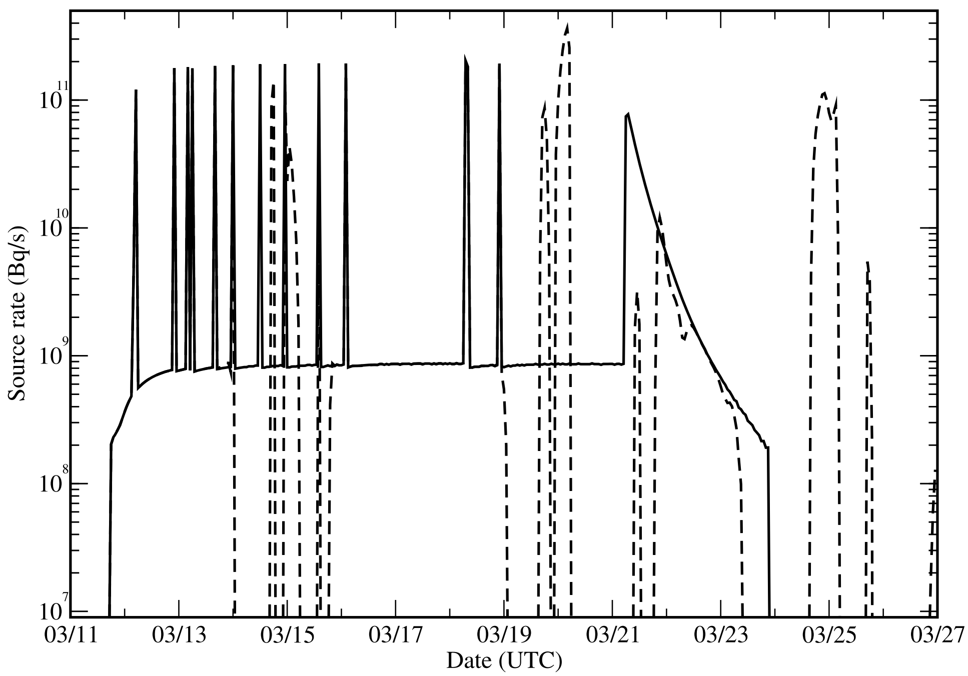

Figure 13. Reconstructed temporal profile of ^{137}Cs under semi-Gaussian assumptions with the maximum likelihood method, with the use of a non-null first guess. The full line represents the IRSN first guess profile. The dashed line represents the retrieved source term. Note that the retrieved profile coincides with the first guess profile for the first unobserved sequence of peaks, March 12 to March 14.