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Towards an operator for merging taxonomies

Amélie Cordier\textsuperscript{1} and Jean Lieber\textsuperscript{234} and Julien Stevenot\textsuperscript{234}

Abstract. The merging of knowledge bases is a fundamental part of the collaboration in continuous knowledge construction. This paper introduces an operator for merging similar taxonomies, i.e., taxonomies that share the major part of their contents. Taxonomies have been chosen for the low time and space complexity of the classical inferences defined on them. A limit of this language is that it does not incorporate negations, thus the union of taxonomies is never inconsistent, though it is meaningful to consider that their merging does not coincide with their union. Thus, a way to extend the taxonomies' language is presented to allow the definition of a merging operator. This operator is algorithmically simple for the part of their contents on which the taxonomies agree, confining complexity to the part on which they do not. So it allows a low time and space complexity merging on similar taxonomies.

1 INTRODUCTION

This work is part of the Kolflow project.\textsuperscript{5} Kolflow aims at investigating man-machine collaboration in continuous knowledge construction and this collaboration involves to make the conjunction of knowledge from different sources.

In \cite{8}, a continuous knowledge integration process (KCIP) is described in which semantic wikis are used as a way of representing knowledge. The semantic wiki used by Kolflow as use case for studying collaboration is WikiTaaable.\textsuperscript{6} To simplify, the formal part of WikiTaaable can be seen here as a taxonomy, where a taxonomy is a concept hierarchy\textsuperscript{7} organized by the subsumption relation\textsuperscript{8}.

In KCIP, there is a common stable version of WikiTaaable available on a web site so that anyone can download it, work on it and make some updates to make its own version of the wiki. This process will produce, at the same time, several versions of the same wiki which use similar vocabularies but which do not necessarily agree on everything. For example, the case could happen that one version has been modified by someone and says “A melon is a fruit” whereas another one, modified by someone else, says “A melon is a vegetable” (and the two knowledge bases share the concepts Vegetable and Fruit) as modelled in the figure 1 (where \(\subseteq\) is represented by an arrow).

In the current KCIP, both of these modifications will be included in a new version of WikiTaaable and submitted to the expert community\textsuperscript{9} and will be rejected if the experts consider that melons are either not fruits or not vegetables and all the other modifications possibly done at the same time will be lost.

So the merging of “A melon is a fruit” and “A melon is a vegetable” raises a problem. Indeed, if someone knows the concept Fruit and says that melons are vegetables without saying that melons are fruits, he/she probably means that melons are \textit{not} fruits.

\begin{figure}[h]
\centering
\begin{tikzpicture}
\node (Fruit) at (0,0) {Fruit};
\node (Vegetable) at (1,0) {Vegetable};
\node (PlantFood) at (2,0) {PlantFood};
\node (Melon) at (0,-1) {Melon};
\node (Melon2) at (1,-1) {Melon};
\node (PlantFood2) at (2,-1) {PlantFood};
\draw [->] (Fruit) -- (Vegetable);
\draw [->] (Fruit) -- (PlantFood);
\draw [->] (Melon) -- (PlantFood);
\end{tikzpicture}
\caption{Two taxonomies waiting to be merged.}
\end{figure}

The taxonomies form one of the simplest knowledge representation language and as such are interesting to study and to use because of the low time and space complexity of their classical inferences. But with the classical semantics, the conjunction of two taxonomies, i.e., the union of their formulas, cannot be inconsistent and, as such, cannot express all that a human could express like “Melons are not fruits”. For example, the conjunction of the two taxonomies seen in figure 1 is not inconsistent, it just means that melons are, at the same time, fruits and vegetables, as presented in figure 2.

So how to make arise some inconsistencies during the merging? A way of solving this issue is to increase the expressivity of the representation language but without significantly increasing its time and space complexity. To achieve this goal, this paper proposal is to add an axiom construct for modelling that melons are not fruits, in the case where a concept Fruit exists with the axiom Melon \(\subseteq\) Vegetable but without the axiom Melon \(\not\subseteq\) Fruit.

\textsuperscript{1}Université de Lyon 1, CNRS, LIRIS, UMR5205, F-69622, France; email: amelie.cordier@liris.cnrs.fr

\textsuperscript{2}Université de Lorraine, LORIA, UMR 7503 — Vandœuvre-lès-Nancy, F-54506, France

\textsuperscript{3}CNRS, LORIA, UMR 7503 — Vandœuvre-lès-Nancy, F-54506, France

\textsuperscript{4}Inria — Villers-lès-Nancy, F-54602, France

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\textsuperscript{6}http://wikitableau.lemma.fr

\textsuperscript{7}A concept represents a class of objects. For example Banana is the concept representing the set of all the bananas.

\textsuperscript{8}The subsumption between two concepts indicates the inclusion between the classes of objects they represent. It is denoted by \(\subseteq\). For example, the formula Banana \(\subseteq\) Fruit represents the knowledge bananas are fruits (the set of bananas is a subset of the set of fruits).

\textsuperscript{9}Some steps of the KCIP are not presented here because not directly related to our subject. For more detailed information on this process see \cite{8}.
Let \( \Delta \) be a merging operator. If the conjunction of all the knowledge bases \( \psi_1, \psi_2, \ldots, \psi_n \) is consistent, the result of the merging is their conjunction:

\[
\Delta(\{\psi_1, \psi_2, \ldots, \psi_n\}) \equiv \psi_1 \land \psi_2 \land \ldots \land \psi_n
\]

Else, minimal modifications of all the bases \( \psi_1 \mapsto \psi'_1, \psi_2 \mapsto \psi'_2, \ldots, \psi_n \mapsto \psi'_n \) such that \( \psi'_1 \land \psi'_2 \land \ldots \land \psi'_n \) is consistent have to be done, and:

\[
\Delta(\{\psi_1, \psi_2, \ldots, \psi_n\}) \equiv \psi'_1 \land \psi'_2 \land \ldots \land \psi'_n
\]

Some postulates of revision, inspired from the postulates of revision, are presented in [6].

3 Merging Taxonomies

3.1 Taxonomies

The term taxon is not the only concept in biology. This section is about the minimal change theory research field in which this paper aims at contributing. Two important notions of this field are belief revision and belief merging.

2.1 Revision of a Knowledge Base by Another One

Let \( \psi \) and \( \mu \) be two consistent knowledge bases. The revision of \( \psi \) by \( \mu \) consists in keeping all the knowledge from \( \mu \) and the maximal knowledge from \( \psi \) to obtain a consistent knowledge base.

In [1], some general postulates of belief revision have been proposed. These postulates have been reformulated in [5] for the particular case of revision in propositional logic. According to these postulates, if the conjunction of \( \psi \) and \( \mu \) is consistent, then the revision is equivalent to this conjunction. If \( \psi \land \mu \) is inconsistent, then minimal modifications \( \psi \mapsto \psi' \) have to be done such that \( \psi' \land \mu \) is consistent and the revision of \( \psi \) by \( \mu \) is \( \psi' \land \mu \). [7] presents a survey on belief revision.

2.2 Merging of Knowledge Bases

Let \( \psi_1, \psi_2, \ldots, \psi_n \) be \( n \) consistent knowledge bases. The merging of these knowledge bases consists in keeping as much as possible from them in order to obtain a consistent knowledge base. The difference with revision is that there is no a priori preference among the knowledge bases to be merged.
Either $\psi_1$ and $\psi_2$ are incomplete in the sense that the person in charge of the development of $\psi_1$ (resp., $\psi_2$) does not know whether melons are or are not vegetables (resp., fruits);

Or the persons in charge of the development of $\psi_1$ and $\psi_2$ are in disagreement: the former thinks that melons are fruits and are not vegetables, the latter thinks that melons are vegetables and are not fruits.

Therefore the merging of $\psi_1$ and $\psi_2$ should lead to a taxonomy $\psi$ satisfying one of the four possibilities:

(a) $\psi \models \text{Melon} \sqsubseteq \text{Fruit}$ and $\psi \models \text{Melon} \sqsubseteq \text{Vegetable}$

(b) $\psi \models \text{Melon} \sqsubseteq \text{Fruit}$ and $\psi \not\models \text{Melon} \sqsubseteq \text{Vegetable}$

(c) $\models \text{Melon} \sqsubseteq \text{Fruit}$ and $\psi \models \text{Melon} \sqsubseteq \text{Vegetable}$

(d) $\models \text{Melon} \sqsubseteq \text{Fruit}$ and $\psi \not\models \text{Melon} \sqsubseteq \text{Vegetable}$

3.3 $CS_\mu(\psi)$ and $MCS_\mu(\psi)$

Let $\mu$ and $\psi$ be two $\mathcal{L}_7$ knowledge bases, such that $\mu$ is consistent. Let $CS_\mu(\psi)$ be the set of knowledge bases $\varphi$ such that $\mu \subseteq \varphi \subseteq \psi \cup \mu$ and $\varphi$ is consistent (CS stands for “consistent subsets”). $CS_\mu(\psi) \neq \emptyset$ since $\mu \in CS_\mu(\psi)$. Among the elements of $CS_\mu(\psi)$, the largest ones for inclusion constitute $MCS_\mu(\psi)$ ($MCS$ stands for maximal consistent subset). If $\psi \cup \mu$ is consistent, then $MCS_\mu(\psi) = \{\psi \cup \mu\}$.

For example (using the notations of the previous sections), if $\psi = \text{dash}_1 \cup \text{dash}_2$, then $MCS_\psi(\psi)$ is composed of the four consistent knowledge bases (a), (b), (c), and (d).

3.4 Modelling the choice among several possibilities

As pointed out above, there may be several possibilities and so, it is necessary to make a choice among them. This possibility to make a choice is represented by a preorder $\leq$ on the knowledge bases of $\mathcal{L}_7$ such that $\psi_1 \prec \psi_2$ means that $\psi_1$ is preferred to $\psi_2$ ($\psi_1 \prec \psi_2$ means that $\psi_1 \leq \psi_2$ and $\psi_2 \not\leq \psi_1$).$^{12}$ $\leq$ is assumed to be a total order up to the logical equivalence: it is reflexive and transitive, if $\psi_1 \leq \psi_2$ and $\psi_2 \leq \psi_1$ then $\psi_1$ and $\psi_2$ are equivalent, and for any $\psi_1$ and $\psi_2$, either $\psi_1 \leq \psi_2$ or $\psi_2 \leq \psi_1$. Therefore, if $S$ is a finite set of $\mathcal{L}_7$ knowledge bases, the minimal of $S$ for $\leq$ exists and is unique, modulo equivalence, and it is denoted by $\text{Min}_\leq(S)$.

Moreover, $\leq$ is assumed to prefer more specific knowledge bases, i.e., if $\psi_1 \subseteq \psi_2$ then $\psi_1 \leq \psi_2$. This property involves that $\text{Min}_\leq(\text{CS}_\mu(\psi)) = \text{Min}_\leq(\text{MCS}_\mu(\psi))$.

3.5 An operator for merging taxonomies

The merging operator presented in this section is inspired from the ideas of agreement and disagreement of two ontologies as introduced in [3]. Let $\psi_1, \psi_2, \ldots, \psi_n$ be a consistent

\[ \text{Min}_\leq(\text{CS}_\mu(\psi)) = \text{Min}_\leq(\text{MCS}_\mu(\psi)) \]

As pointed out by a reviewer, another idea is to use the majority merging rule stating that a preference is given to the piece of knowledge entailed by a majority of the $n$ knowledge bases to be merged (which makes sense if $n > 2$). However, in some situations, there is no strict majority (the number of knowledge bases entailing $A \sqsubseteq B$ is equal to the number of knowledge bases entailing $A \not\sqsubseteq B$ and the preorder $\leq$ can be used.)
knowledge bases of $L_T$ (e.g., two taxonomies) and $E = \{\psi_1, \psi_2, \ldots, \psi_n\}$. The notions introduced below are illustrated with figures 3 and 4.

The agreement $\alpha$ of $\psi_1, \psi_2, \ldots, \psi_n$ is constituted by the pieces of knowledge common to them, formally:

$$\alpha = \bigcap_i \hat{\psi}_i = \hat{\psi}_1 \cap \hat{\psi}_2 \cap \ldots \cap \hat{\psi}_n$$

$\alpha$ is necessary consistent (since $\alpha \subseteq \hat{\psi}_1$ that is consistent). Figure 7 shows a representation of $\alpha$.

$$\text{PlantFood}$$

```
Fruit --|-- Vegetable
```

```
Apple ---- Melon
```

Figure 6: $\alpha$ the agreement of the $\psi_1$ and $\psi_2$ of figures 3 and 4 represented without some of the edges that can be deduced by CWA.

The disagreement is intuitively defined as the pieces of knowledge that are not in agreement. This disagreement is defined as $\delta = \bigcup_i \delta_i$ where $\delta_i$ represents the pieces of knowledge of $\psi_i$ that are not in agreement with the $\psi_j$’s ($j \neq i$):

$$\delta_i = \hat{\psi}_i \setminus \alpha$$

Since $\psi_i$ is consistent, $\delta_i$ is also consistent. Figures 8 and 9 illustrate $\delta_1$ and $\delta_2$.

$$\text{PlantFood}$$

```
Fruit --|-- Vegetable
```

```
Apple ---- Melon
```

Figure 8: $\delta_1$.

```
Fruit --|-- Vegetable
```

```
Apple ---- Melon
```

Figure 9: $\delta_2$.

So, here, $\delta$ is the union of $\delta_1$ and $\delta_2$.

Then, a subset $\beta$ of $\delta$ has to be chosen. $\alpha \cup \beta$ has to be consistent and has to keep as much knowledge as possible, i.e., $\beta \in MCS_\alpha(\delta)$. If the choice is made according to $\leq$ (cf section 3.4) then:

$$\beta = \text{Min}_\leq(MCS_\alpha(\delta))$$

Finally, the result of the merging is a knowledge base of $L_T$ such that:

$$\Delta(E) = \beta$$

Figures 10 to 13 present the four possibilities for $\Delta(\psi_1, \psi_2)$, depending on the choice $\leq$.

$$\text{PlantFood}$$

```
Fruit --|-- Vegetable
```

```
Apple ---- Melon
```

Figure 10: Result of the merging after choosing (a).

$$\text{PlantFood}$$

```
Fruit --|-- Vegetable
```

```
Apple ---- Melon
```

Figure 11: Result of the merging after choosing (b).

$$\text{PlantFood}$$

```
Fruit --|-- Vegetable
```

```
Apple ---- Melon
```

Figure 12: Result of the merging after choosing (c).

$$\text{PlantFood}$$

```
Fruit --|-- Vegetable
```

```
Apple ---- Melon
```

Figure 13: Result of the merging after choosing (d).

3.6 Properties

First, $\Delta$ can be confronted to the postulates of $6$. These postulates are used for characterizing a merging operator in propositional logic, but can be reused in the $L_T$ formalism. These postulates deal with the merging of multisets of knowledge bases, but, since for the operator $\Delta$, the number of occurrences has no importance, we will consider only sets of knowledge bases.

These postulates are (for $E, E_1, E_2$ sets of knowledge bases; $\psi_1, \psi_2$: knowledge bases):

(A1) $\Delta(E)$ is consistent.

(A2) If $\bigwedge E$ is consistent then $\Delta(E)$ is equivalent to $\bigwedge E$.

(A3) If there is a bijection $F$ from $E_1$ to $E_2$ such that $F(\psi)$ is equivalent with $\psi$, then $\Delta(E_1)$ is equivalent to $\Delta(E_2)$ (this postulate states that the syntax is irrelevant for $\Delta$).

(A4) If $\psi_1 \cup \psi_2$ is not consistent, then $\Delta(\{\psi_1, \psi_2\}) \not\models \psi_1$.

(A5) $\Delta(E_1) \cup \Delta(E_2)$ || $\Delta(E_1 \cup E_2)$.

(A6) If $\Delta(E_1) \cup \Delta(E_2)$ is consistent, then $\Delta(E_1 \cup E_2) \models \Delta(E_1) \cup \Delta(E_2)$.

$\Delta$ satisfies (A1). Indeed, $\Delta(\{\psi_1, \psi_2, \ldots, \psi_n\}) \in MCS_\alpha(\delta)$ and thus is consistent.

13 This slightly differs from 3 where the agreement and the disagreement are not complementary.
\( \Delta \) satisfies (A2). To prove it, let us assume that \( \Delta \) is consistent. \( \Delta = \bigwedge \psi_i \) & \( \alpha \cup \delta \). Thus \( \alpha \cup \delta \) is consistent and so \( MCS_a(\delta) = \{ \alpha, \delta \} \). Hence \( \Delta(E) = \alpha \cup \delta \). Therefore, if \( \Delta \) is consistent then \( \Delta(E) = \bigwedge \psi_i \) which proves (A2).

\( \Delta \) satisfies (A3), which states the irrelevance of syntax. Indeed, for any knowledge bases \( \psi_1 \) and \( \psi_2 \) of \( \Delta \), \( \psi_1 \) is equivalent to \( \psi_2 \) if \( \psi_1 = \psi_2 \). Since \( \Delta \) is defined thanks to the \( \psi_i \)'s, \( \Delta(E) \) does not change when substituting a \( \psi_i \) by an equivalent knowledge base.

(A4) is not satisfied by \( \Delta \) as the following counterexample shows. Let \( \psi_1 = \{ A \not\subseteq B \} \) and \( \psi_2 = \{ A \not\subseteq B \} \). Then \( \psi_1 = \{ A \subseteq B, B \not\subseteq A \} \) and \( \psi_2 = \{ A \subseteq B, B \not\subseteq A \} \). \( \psi_1 \lor \psi_2 = \{ A \subseteq B, B \not\subseteq A \} \). \( \psi_1 \land \psi_2 = \{ A \subseteq B, A \subseteq B, B \not\subseteq A \} \). \( \alpha = \{ B \subseteq A \} \), \( \beta = \{ A \subseteq B \} \), \( \delta = \{ A \not\subseteq B \} \). \( MCS_a(\delta) = \{ \{ A \subseteq B, B \not\subseteq A \} \} \).

Thus according to the choice performed by \( \leq \), \( \Delta(\{ \psi_1, \psi_2 \}) \models \psi_1 \) or \( \Delta(\{ \psi_1, \psi_2 \}) \models \psi_2 \). (A4) is called in \( \{ \} \) the fairness property: it states that \( \Delta \) should not make a preference between the knowledge bases to be merged. Our interpretation of the non fairness of our operator is that the \( \mathcal{L}_T \) language does not permit to express disjunctions and so, the operator has to make a choice (that is why it can be a total order). Indeed, let us consider \( \mathcal{L}_T \) the extension of \( \mathcal{L}_T \) with disjunction: if \( \psi_1 \) and \( \psi_2 \) are \( \mathcal{L}_T \) knowledge bases, then \( \psi_1 \lor \psi_2 \) is an \( \mathcal{L}_T \) knowledge base and \( \omega \) satisfies it if \( \omega \) satisfies \( \psi_1 \) or \( \omega \) satisfies \( \psi_2 \). Now, let \( \mathcal{V} \) be the merging operator defined by \( \mathcal{V}(E) = \{ \bigwedge MCS_a(\delta)(E) \} \); a set of \( \mathcal{L}_T \) knowledge bases, \( \mathcal{V}(E) \) an \( \mathcal{L}_T \) knowledge base). \( \mathcal{V} \) satisfies (A1), (A2), and (A3) (similar proofs than the proofs for \( \Delta \)) and it satisfies also (A4): Let \( \psi_1, \psi_2 \) two consistent \( \mathcal{L}_T \) knowledge bases such that \( \psi_1 \land \psi_2 \) is consistent. Thus, \( \alpha = \psi_1 \land \psi_2 \), \( \beta = \psi_1 \land \psi_2 \), \( \alpha \cup \beta = \psi_1 \land \psi_2 \), \( \alpha \cup \beta = \psi_1 \land \psi_2 \) are consistent, so there exist \( \phi_1 \) and \( \phi_2 \) such that \( \phi_1 \in MCS_a(\psi_1) \lor \phi_2 \in \psi_2 \), \( \phi_1 \lor \phi_2 = \psi_1 \land \psi_2 \) is inconsistent (since \( \phi_1 \lor \phi_2 \equiv \psi_1 \lor \psi_2 = \psi_1 \land \psi_2 \) that is inconsistent). Therefore \( \phi_1 \lor \phi_2 \models \neg \psi ) \lor \phi_2 \not\in \phi_1 \) (since \( \phi_1 \not\in \phi_2 \)). \( \phi_1 \lor \phi_2 \not\in \phi_2 \) (since \( \phi_2 \not\in \phi_1 \)). Hence, \( \neg \psi \lor \phi_2 \not\in \phi_2 \) for \( i \in \{ 1, 2 \} \). This is why the non fairness of \( \Delta \) is interpreted as a consequence of the necessity to make choices, in the \( \mathcal{L}_T \) formalism.

At this point, we have neither proven that \( \Delta \) satisfies (A5) and/or (A6), nor found any counterexample.

A detailed complexity analysis has still to be carried out. However, a naive algorithm for \( \Delta \) gives a polynomial complexity for the computation \( \alpha \) and \( \delta \) and an exponential complexity for the computation of \( MCS_a(\delta) \) (exponential in the size of \( \delta \)). Therefore, with this algorithm, the computation of \( \Delta \) is tractable when the taxonomies are similar. Indeed \( \delta = \bigcup \psi_i \), \( \bigcap \psi_i \) in the taxonomies are not shared by the taxonomies, so \( \delta \) can be used to characterize the dis-similarities of the \( \psi_i \)'s. Hence making frequent merging of taxonomies that have forked from the same taxonomy is useful.

4 CONCLUSION AND FUTURE WORK

This paper has presented an operator for merging similar taxonomies that satisfies a subset of the postulates defined in \cite{6}. There is still work to do in order to study its properties.

This operator is used to design an efficient algorithm for the merging when the taxonomies are similar, which is the case when they are originated from the same taxonomy and have not diverged for a too long time. This algorithm, in order to be efficient, should not compute \( \psi \) (this operation is too complex and is too time and space consuming: \( |\psi| = |\psi(\psi)|^2 - |\psi(\psi)| \)).

The design of such an algorithm involves that the relation \( \leq \) has to be specified. Indeed, the operator presented in this paper is based on the maximal consistent subsets of formulas issued from the conjunction of the knowledge bases to be merged.

A way to integrate this operator in the KCIP is to specify the \( \leq \) relation as following:

- In the current KCIP, any user can submit his/her own version of the knowledge base at any time. When a user submits his/her version, it is merged with another user version and the knowledge base obtained by this merging process has to pass some automatic test in order to determine if it worth to be submitted to the community of the experts.
- Now, when a user wants to merge his/her own version to the current knowledge base, once the operator has determined all the MCS, they can be used to make all the possibilities of result for the merging and these possibilities can be submitted to the tests currently in use. Then all the possibilities which have passed the test are presented to the user, who will choose which possibility is the closest of what he wants (the user will make the choice represented by \( \leq \) in our formalism). The choice done by the users can be stored for further reuse; this idea remains to be studied in details.

So, once this algorithm is efficiently implemented, it will be useful to the Kollof project. But Kollof does not limit itself to \( \mathcal{L}_T \) and there is a large spectrum of languages ranging from \( \mathcal{L}_T \) to, e.g., OWL DL. One advantage of \( \mathcal{L}_T \) is that its inferences are much less complex than OWL DL’s (e.g., the sumsubsumption test is linear for \( \mathcal{L}_T \) whereas it is \text{NExpTime} hard in OWL DL). The question we intend to address in future work is what the extensions of \( \mathcal{L}_T \) for which we will design a merging operator. Since \( \mathcal{L}_T \) can be considered as the fragment of RDFS with only one possible properties, \text{subClassOf} (corresponding to \( \subseteq \)), some larger fragments should be considered (using other properties). Indeed in the particular case of WikiTaalble, some properties are more used or important and some are easier to compute than other ones so one can think of a kind of anytime approach where the algorithm will consecutively consider the RDFS properties starting with \text{subClassOf}.

A kind of equivalent to the MCS is the \text{MUPS} that are used in the system Pellet: \footnote{http://clarkparsia.com/pellet/} this system contains a tool for debugging inconsistent ontologies which allows to find the \text{MUPS} \cite{4} of an inconsistent ontology. A \text{MUPS} (Minimal Unsatisfiability Preserving Sub-TBoxes) is a minimal subset of axioms which causes the inconsistency. If we find all the
MUPS of a knowledge base issued from the conjunction of two other ones, the set of all the possible consistent knowledge bases made from the conjunction of all the MUPS after deleting one formula on each of them, is equivalent to the MCS. As Pellet works on knowledge bases on OWL DL it could be a lead to pass from $\mathcal{L}_\mathcal{T}$ to OWL DL. It could also allows to compare our algorithm to the results of Pellet's debugging tool.

Finally, another future work (following a discussion with Pierre-Antoine Champin) is to study a similar merging operator based on another closed world assumption, a "disjointness assumption". This assumption for a tree-structured taxonomy $\psi$ means that if neither $\psi \models A \subseteq B$ nor $\psi \models B \subseteq A$ then $A$ and $B$ are disjoint ($\omega(A) \cap \omega(B) = \emptyset$). The definition for any taxonomy must be adapted (e.g., in figure 2 Fruit et Vegetable are not comparable by $\subseteq$, yet they should not be disjoint in order not to entail that there is no melon). The future work aim will be to see how this different closed world assumption modifies the belief merging operator.

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