Delay-Constrained Multi-hop CSMA Networks

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***Abstract*—The mean packet delay (MPD) of multi-hop wireless ad-hoc networks (WANET) has a major impact on the networks performance and quality of service. In this paper we analyze the performance of slotted carrier sense multiple access (CSMA) WANET under a MPD constraint. The MPD is determined by the mean number of hops between sources and destination pairs, the back-off probability and the outage probability. Using stochastic geometry analysis we perform a cross-layer optimization and present a simple expression for the area spectral efficiency (ASE) of the network as function of the MPD and the fading channel properties. Our main result is that the ASE grows linearly with the allowed MPD, $\overline{D}$. The linear scaling coefficient can also be evaluated from the ASE expression: For example, with mean source-destination distance of $\overline{L}$, a network that operates over Rayleigh fading channels with a path loss factor of 4 can achieve a maximum of $\frac{0.04\overline{D}}{\overline{L}^2}$ source-bits per second per hertz per square-meter. The accuracy of the presented results is demonstrated by numerical simulations.**

I. Introduction

The decentralized nature of a wireless ad-hoc network (WANET) makes it suitable for a variety of applications, offering simplicity, scalability and flexibility. Lacking the infrastructure, WANET relies on node cooperation in order to accomplish long range communication. The cooperation between nodes is typically achieved by a multi-hop approach, in which packets are delivered from sources to destinations using intermediate nodes that operate as relays. Increasing the number of relays between each source and destination also increases the mean packet delay (MPD). The MPD of WANET is a major parameter of WANETs, since it has an impact on the network performance, quality of service and stability. Hence, in most WANETs it is desirable to limit the MPD.

The performance of WANETs is mainly characterized by their area spectral efficiency (ASE), which measures the average rate of successful transmissions per unit area. Gupta and Kumar [1] studied the capacity scaling law of ALOHA WANETs and showed that the capacity with n randomly located users, scales as $O(\sqrt{n})$. In order to increase the ASE, many WANETs employ a medium access control (MAC) protocol that can reduce the probability of message collisions. Carrier sensing multiple access (CSMA), is one of the most popular access mechanism. In this work we focus on the slotted CSMA variant, [2], where all users transmissions are aligned to predetermined time slots.

Analysis of random networks was shown to be a useful tool for cross-layer optimization (e.g., [3], [4]). In such analysis the physical channel parameters are optimized jointly with the MAC parameters. Stamatiou and Haenggi [5] had taken this aprroach a step further, and optimized global routing parameters such as the number of hops per link. In that work they minimized the average network delay with respect to the number of hops and other system parameters in an ALOHA/TDMA WANET. Their analysis shows how to minimize the delay for a given data rate and node density. The work also includes a short analysis on the effect on the throughput when the link length go to infinity.

In this paper we take a different approach and perform cross layer optimization of the ASE under a MPD constraint for finite link length. We analyze the more general CSMA protocol (and results for ALOHA can be derived as a private case). Our analysis shows that for large MPD, the ASE of CSMA WANETs grows linearly with the MPD. We show that this linear behavior can be deduced even from the classic work of Gupta and Kumar [1]. The usefulness of the evaluated ASE and the accuracy of the results is also demonstrated in the numerical section.

The rest of this paper is organized as follows: Section II describes the system model; Section III gives the performance analysis of slotted CSMA networks with MPD constraint; Section IV gives numerical and simulation results and concluding remarks are given in section V.

II. System Model

We assume a decentralized wireless ad-hoc network utilizing a slotted CSMA protocol (e.g., [2]). In each slot part of the nodes establish new links in the networks. In this model a link represents the delivery of a packet from a source node to a destination node. We assume that the source nodes positions form a PPP on the plane, and that the distance between the source and destination nodes is randomly distributed. As shown below, the analysis depends only on the mean distance between sources and destinations, $\overline{L}$. Thus, in this work, a specific distribution is used only in the numerical results section. Each link is composed of several segments using intermediate nodes, referred to as relays. Assuming a reasonable routing protocol, these relays are assumed to be equidistantly placed on a line between the source and the destination nodes (see for example [5]). A packet is successfully delivered from node to node if the transmitter gets an access to the medium and the receiver decodes the message successfully. The relaying is performed by a detect and forward approach and outage-events are followed by a retransmission of the packet. A transmitter that tries to access the medium is referred to hereafter as active transmitter, and together with its receiver they form an active pair. Since in each slot only one transmitter from each link is

978-3-901882-45-6/2012

active, and assuming that all links are statistically independent, the active transmitters distribution at each slot is modeled by a two dimensional PPP with density of $\lambda_{\rm p}$.

The nodes use equal transmission power and are equipped with a single antenna. The power received at receiver j from transmitter i is:

$$W_{i,j} = \rho X_{i,j}^{-\alpha} V_{i,j} \tag{1}$$

where ρ is the transmission power; $X_{i,j}$ and $V_{i,j}$ are the distance and the additional channel gain (e.g., due to fading) between the i-th transmitter and the j-th receiver, respectively and $\alpha > 2$ is the exponential decay factor. In this model the fading random variable, $V_{i,j}$, is independent and identically distributed (i.i.d) for all i, j, and is also statistically independent of all distance variables.

The considered slotted CSMA protocol is based on periodic slots. Each slot is partitioned into access and data sub-slots. During the access sub-slot, the active pairs are sequentially competing on access to the shared medium, using random priorities and control signaling (commonly termed request to send (RTS) and clear to send (CTS)). These messages are used by all network members to evaluate the amount of interference between users. For simplicity, the duration of the access sub-slot is assumed to be significantly smaller than the duration of the data sub-slot, and hence we neglect the overhead of the access sub-slot.

The k-th active pair becomes successful if the pair and all previous successful pairs satisfy:

$$W_{ij} < \delta, \ \forall\ i \neq j \tag{2}$$

where δ is the allowed interference threshold (note that the special case of $\delta \rightarrow \infty$ represents the slotted ALOHA protocol). At the end of the access sub-slot, all successful transmitters start to transmit data all through the data sub-slot. If (2) is not satisfied for pair k we say that a collision occurred, and pair k backs-off. In such case the data that was planned for transmission by transmitter k will be transmitted in a future slot in which pair k will become successful. Denoting by $\lambda_{\rm c}$ the density of the successful pairs, the probability for a pair to back-off is given by:

$$P_{\rm B} \triangleq 1 - \frac{\lambda_{\rm c}}{\lambda_{\rm p}}. \tag{3}$$

In the following we focus on the analysis of the achievable data-rates in the data sub-slots. We use the shift invariant property of the system, [6], to analyze the performance of the network taking user 0 as a probe receiver. Without loss of generality, we assume that the probe receiver is located at the origin, and that it is a part of the successful set. For notation simplicity, in the following we drop the probe receiver index. We also define the set $\bar{\mathcal{S}}$ as the set, of all successful pairs, excluding the probe pair.

The aggregate interference, measured at the probe receiver, can be written as:

$$I = \sum_{i \in \bar{\mathcal{S}}} W_i = \sum_{i \in \bar{\mathcal{S}}} \rho X_i^{-\alpha} V_i. \tag{4}$$

As WANETs typically work at the interference limited regime, in the following we neglect the contribution of the thermal noise. Without loss of generality, we set $\rho = 1$ and therefore the desired signal is received with power of $V_0 d^{-\alpha}$, where d denotes the transmitter-receiver distance. We assume that the coding rate is $\log_2(1 + \beta d^{-\alpha})$ where $\beta > 0$ is a design parameter. Using Shannon theory, [7], assuming single user decoder over a Gaussian codebook and sufficiently long block length, a transmission can be decoded successfully if the signal-to-interference (SIR) is larger than $\beta d^{-\alpha}$, and the outage probability is:

$$P_{\rm O} = \Pr\left(\frac{V_0 d^{-\alpha}}{I} < \beta d^{-\alpha}\right) = 1 - \Pr\left(I < \frac{V_0}{\beta}\right). \tag{5}$$

A useful measure of the network performance is the average data rate, defined as:

$$R \triangleq \lambda_{\rm c} \cdot (1 - P_{\rm O}) \cdot \log_2\left(1 + \beta d^{-\alpha}\right). \tag{6}$$

We assume that the routing algorithm aims to divide all links to equidistant hops, all of approximate distance, d. Assuming also a large MPD, the quantization effect on the number of hops can be neglected. Hence, the average number of hops per link is well approximated by $\frac{\overline{L}}{d}$, where $\overline{L}$ is the average distance between sources and their destinations.

In this work we study the spectral efficiency when the WANET's MPD is limited to a maximal allowed value, $\overline{D}$. The area spectral efficiency (ASE) is the maximum average packets rate, subject to the MPD constraint:

$$A(\overline{D}) \triangleq \max_{(\lambda_{\rm p}, \delta, \beta, d) \in \mathcal{G}(\overline{D})} \frac{R}{\overline{L}/d}. \tag{7}$$

where $\mathcal{G}(\overline{D})$ is the set of all allowed network parameters:

$$\mathcal{G}(\overline{D}) \triangleq \left\{(\lambda_{\rm p}, \delta, \beta, d) : D(\lambda_{\rm p}, \delta, \beta, d) \leq \overline{D}\right\} \tag{8}$$

and $D(\lambda_{\rm p}, \delta, \beta, d)$ is the MPD, defined as the average time for a packet to reach successfully from source to destination, measured in slot units. Averaging on all source-destination pairs, the MPD is given by:

$$D(\lambda_{\rm p}, \delta, \beta, d) = \frac{\overline{L}/d}{(1 - P_{\rm O})(1 - P_{\rm B})}. \tag{9}$$

III. Performance Analysis

A. Previous Results

The analysis of the slotted CSMA system exhibits several difficulties that need to be removed. We first adopt the approximation:

$$P_{\rm B} \simeq \pi \Phi \lambda_{\rm c} \delta^{-\frac{2}{\alpha}} \tag{10}$$

where

$$\Phi \triangleq E[V^{2/\alpha}]. \tag{11}$$

This approximation is based on a small BP assumption and a PPP model for the position of the successful pairs. This approximation was presented recently in [8] where it was shown to be very accurate even for high BP.

The more dominant difficulty is the distribution of the successful transmitters, which is quite complex. In this work, we adopt the approximation of the distribution of the successful transmitters as a PPP with density λ_{c} outside of the guard-zone of the probe receiver [3], [9], [10]. Denoting the interference received by the probe user in the approximating PPP by $\tilde{I}$, and using (10) the approximated ASE is given by:

$$\tilde{A}(\overline{D}) \triangleq \max_{\frac{\overline{L}/d}{\Pr\left(\tilde{I}<\frac{V_0}{\beta}\right)\left(1-\frac{\pi\Phi\lambda_{\mathrm{c}}}{\delta^{\frac{2}{\alpha}}}\right)} \leq \overline{D}} \frac{\lambda_{\mathrm{c}} d}{\overline{L}} \Pr\left(\tilde{I} < \frac{V_0}{\beta}\right) \log_2\left(1 + \frac{\beta}{d^{\alpha}}\right) \tag{12}$$

B. ASE with Mean Packet Delay Constraint

Theorem 1 below describes the dependence of the ASE on the system parameters including the channel path loss, the channel fading distribution, the average distance of links, and the MPD. Most notably, the theorem shows that the ASE grows linearly with the MPD. This linear dependence is of major importance in the joint design of the routing and the physical layers. Intuitively it can be stated (with just a little exaggeration) as "you can transmit as much as you want if each message can be delayed long enough". This throughput increase comes at the price of more active relay nodes, and hence is limited by the actual node density. Yet, it motivates a new design approach for joint optimization, in which the hop length is determined by the maximal allowed delay[1].

Although this interpretation of the results is new, the linear relation itself is not new. It can be deduced even from the classic work of Gupta and Kumar [1]. In their work they showed that the network capacity increases as $O(\sqrt{n})$. In order to achieve that, the single hop-distance (d) is required to scale as $O(\frac{1}{\sqrt{n}})$. Noting that the delay (D) is inversely proportional to the hop-distance (i.e., it scales as $O(\sqrt{n})$) one can conclude that the capacity scales linearly with the delay.

Theorem 1: The ASE with MPD constraint $\overline{D}$ and mean source-destination range of $\overline{L}$ is:

$$\tilde{A}(\overline{D}) = \left(\frac{\epsilon^{-\frac{2}{\alpha}} \log_2(1+\epsilon)}{\pi\Phi}\right) \cdot \frac{\overline{D}}{\overline{L}^2} \cdot \max_{\substack{\zeta>0 \\ 0\leq\eta<1}} \zeta(1-\eta) Q^2(\zeta,\eta) \tag{13}$$

where

$$\epsilon \triangleq \exp\left(\frac{\alpha}{2} + W\left(-\frac{\alpha}{2} e^{-\frac{\alpha}{2}}\right)\right) - 1 \tag{14}$$

$$Q(\zeta,\eta) \triangleq E\left[\left(\int_0^{V_0\zeta^{-\frac{\alpha}{2}}} \mathcal{L}^{-1}\left(\Xi(s)\right) dI\right)\right] \tag{15}$$

$$\Xi(s) \triangleq \exp\left(\eta\left(1 - e^{-s\eta^{-\frac{\alpha}{2}}}\right) - s^{\frac{2}{\alpha}} \gamma\left(1 - \frac{2}{\alpha}, s\eta^{-\frac{\alpha}{2}}\right)\right) \tag{16}$$

[1]Note that the actual delay also depends on the queuing delays, which are not analyzed herein. Yet, as can be noticed, assuming that different links utilize different intermediate nodes, for any MPD the queuing-delay is roughly identical because the optimal back-off and outage probabilities are independent of the MPD.

and $\gamma(a,x)$ is the lower incomplete gamma function, which is the solution to $\int_0^x t^{a-1}e^{-t}dt$, $\mathcal{L}^{-1}$ is the inverse Laplace transform and $W(\cdot)$ is the Lambert W function.

Proof of Theorem 1. We start the proof by substituting:

$$\zeta \triangleq \eta\left(\delta\beta\right)^{\frac{2}{\alpha}}, \qquad \eta \triangleq \pi\Phi\lambda_{\mathrm{c}}\delta^{-\frac{2}{\alpha}}, \tag{17}$$

and showing that the probability of a successful reception is given by $E\left[\Pr\left(\tilde{I} \leq V_0/\beta\right)\right] = Q(\zeta,\eta)$ (note also that using (10) we have $P_{\mathrm{B}} \simeq \eta$). Equations (1) and (2) indicate that a probe receiver disables transmitters which satisfy $X_i^{-\alpha}V_i \geq \delta$. Therefore, transmitter i will be disabled if its distance from the probe receiver is smaller than $\left(\frac{V_i}{\delta}\right)^{1/\alpha}$. The characteristic function of the interference for 2-dimensional PPP, with density λ, a disabling radius A and a fading variable V is (see for example [9], [11]):

$$\Phi(s) = \exp\left(-\lambda \int_A^{\infty} E\left[1 - e^{-sVt^{-\alpha}}\right] 2\pi t dt\right). \tag{18}$$

Using the superposition property of the Poisson shot noise [12], the characteristic function of the normalized aggregate interference, $\tilde{I}$, can be written as:

$$\begin{aligned}\Phi_{\tilde{I}}(s) &= \exp\left(-\lambda_{\mathrm{c}} E_V\left[\int_{(\frac{V}{\delta})^{1/\alpha}}^{\infty} (1 - e^{-sVt^{-\alpha}}) 2\pi t dt\right]\right) \\ &= \exp\left(-\frac{\lambda_{\mathrm{c}}\delta^{-\frac{2}{\alpha}}}{\eta} E\left[V^{2/\alpha}\right] \int_{\sqrt{\eta}}^{\infty} (1 - e^{-s\delta\eta^{\frac{\alpha}{2}} r^{-\alpha}}) 2\pi r dr\right) \\ &= \exp\left(-\frac{1}{\pi} \int_{\sqrt{\eta}}^{\infty} (1 - e^{-\left(s\delta\eta^{\frac{\alpha}{2}}\right) r^{-\alpha}}) 2\pi r dr\right)\end{aligned} \tag{19}$$

where the second line used the substitution $r = t\sqrt{\eta}\left(\frac{\delta}{V}\right)^{1/\alpha}$ and the third line used (11) and (17). As shown in [9], $\Xi(s)$, defined in (16), can be also written as:

$$\Xi\left(s\right) = \exp\left(-\frac{1}{\pi} \int_{\sqrt{\eta}}^{\infty} (1 - e^{-sr^{-\alpha}}) 2\pi r dr\right) \tag{20}$$

and therefore equation (19) is equivalent to $\Phi_{\tilde{I}}\left(s\right) = \Xi(s\delta\eta^{\frac{\alpha}{2}})$. Defining $\xi(t) = \mathcal{L}^{-1}\{\Xi(s)\}$ and using the Laplace scaling property we can write:

$$\begin{aligned}\int_0^{\frac{V_0}{\beta}} \mathcal{L}^{-1}\left\{\Xi\left(s \cdot \delta\eta^{\frac{\alpha}{2}}\right)\right\} dt &= \frac{1}{\delta\eta^{\frac{\alpha}{2}}} \int_0^{\frac{V_0}{\beta}} \xi\left(\frac{t}{\delta\eta^{\frac{\alpha}{2}}}\right) dt \\ &= \int_0^{\frac{V_0}{\beta}\delta^{-1}\eta^{-\frac{\alpha}{2}}} \xi\left(t\right) dt.\end{aligned} \tag{21}$$

Thus, using (15), (17), and (21), the probability for a successful reception can be written as:

$$E\left[\Pr\left(\tilde{I} \leq \frac{V_0}{\beta}\right)\right] = E\left[\int_0^{\frac{V_0}{\beta}} \mathcal{L}^{-1}\{\Phi_{\tilde{I}}\left(s\right)\} dt\right] = Q(\zeta,\eta). \tag{22}$$

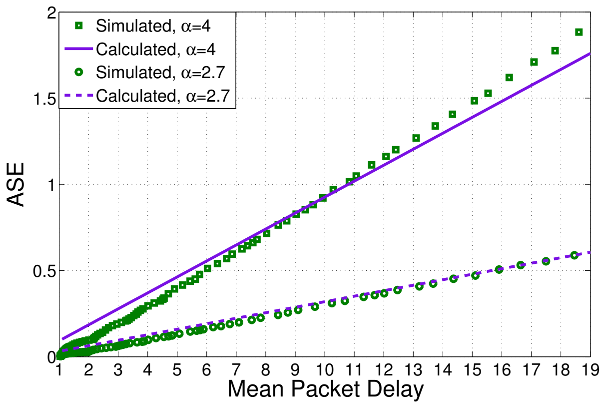

Fig. 1. Simulated and calculated ASE for Rayleigh fading channels with path loss exponents of $\alpha = 2.7, 4$

Next, we substitute (22) and (17) into (12):

$$\begin{aligned}\tilde{A}(\overline{D}) =& \left(\frac{1}{\overline{L}}\right) \max_{\frac{\overline{L}/d}{(1-\eta)Q(\zeta,\eta)} \leq \overline{D}} \lambda_c d \cdot Q(\zeta,\eta) \cdot \log_2\left(1 + \frac{\beta}{d^\alpha}\right) \\ =& \left(\frac{1}{\pi\Phi\overline{L}}\right) \max_{\frac{\overline{L}/d}{(1-\eta)Q(\zeta,\eta)} \leq \overline{D}} \frac{\zeta d}{\beta^{\frac{2}{\alpha}}} \cdot Q(\zeta,\eta) \cdot \log_2\left(1 + \frac{\beta}{d^\alpha}\right) \\ =& \left(\frac{1}{\pi\Phi\overline{L}}\right) \max_{\frac{\overline{L}/d}{(1-\eta)Q(\zeta,\eta)} \leq \overline{D}} \frac{\zeta}{d} \cdot Q(\zeta,\eta) \cdot \frac{\log_2(1+\epsilon)}{\epsilon^{\frac{2}{\alpha}}} \\ =& \left(\frac{\overline{D}}{\pi\Phi\overline{L}^2}\right) \max_{\zeta,\eta} \zeta(1-\eta)Q^2(\zeta,\eta) \cdot \max_{\epsilon} \frac{\log_2(1+\epsilon)}{\epsilon^{\frac{2}{\alpha}}}\end{aligned}$$

where the second line used (17), the third line used the substitution $\epsilon = \beta d^{-\alpha}$ and in the forth line we substituted the constraint.

The theorem follows by noticing that the right hand side maximization is achieved with $\epsilon = \exp\left(\frac{\alpha}{2} + W\left(-\frac{\alpha}{2}e^{-\frac{\alpha}{2}}\right)\right) - 1$ and the left hand side maximization can be calculated numerically. $\square$

IV. Numerical Results

In this section we evaluate the ASE expression, given in Theorem 1, and compare it to the ASE evaluated from a slotted CSMA simulation (as defined in (7)). We present results for Rayleigh fading channels with path loss exponents of $\alpha = 2.7, 4$. In the simulation we assumed that the location of the destination of each link is uniformly distributed on a disk of radius 1, centered at the link source (which results in $\overline{L} = 0.67$). Fig. 1 depicts the ASE as function of the mean packet delay, $\overline{D}$. The slopes of the ASE, calculated from Theorem 1, are 0.034 and 0.109 while the slopes of the ASE, measured from the CSMA simulation are evaluated as 0.032 and 0.093 for $\alpha = 2.7, 4$ respectively. One can observe that all curves increase linearly with the mean delay and that the ASE expression is very accurate.

V. Conclusions

In this paper we analyzed the performance of multiple-hop CSMA WANETs under mean delay constraint. We presented a useful simple expression that gives a good prediction of the ASE of CSMA WANET as function of the MPD. The expression indicates that the ASE scales linearly with the allowed delay. This relation was also demonstrated by simulations. The linear dependence of the network capacity with the delay is an interesting phenomenon that motivates the design of WANETs with large delays. Yet, further research is required to study the effects of routing-overhead, protocol-delay and queuing issues in large delay networks.

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