Joint Pricing and Cognitive Radio Network Selection: a Game Theoretical Approach

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***Abstract*—This paper addresses the joint pricing and network selection problem in cognitive radio networks, considering both the point of view of network users and the Primary Operator. The problem is formulated as a Stackelberg (leader-follower) game where first the PO sets the network subscription price to maximize its revenue. Then, users perform the network selection process, deciding whether to pay for having a guaranteed service, or use a cheaper, best-effort secondary network, where congestion and low throughput may be experienced. Such process is modeled as a population game to study the strategic interactions among a large number of agents.**

For our pricing and network selection game, we provide equilibrium and convergence properties, and derive optimal stable price and network selection settings. Numerical results illustrate that our game model captures the main factors behind cognitive network pricing and channel selection, thus representing a promising framework for the design and understanding of cognitive radio systems.

***Index Terms*: - Cognitive Radio Networks, Network Selection, Pricing, Population Game Model, Replicator Dynamics.**

I. Introduction

Cognitive radio networks (CRNs), also referred to as *xG* networks, are envisioned to deliver high bandwidth to mobile users via heterogeneous wireless architectures and dynamic spectrum access techniques [1], [2]. Such networks provide the capability to share the wireless channel with primary users in an opportunistic manner.

In CRNs, a *primary* (or licensed) user has a license to operate in a certain spectrum band; his access is generally controlled by the Primary Operator (PO) and should not be affected by the operations of any other unlicensed user. On the other hand, *secondary* users have no spectrum license, and they implement additional functionalities to share the licensed spectrum band without interfering with primary users.

In this work, we focus on a fundamental question concerning CRNs, i.e. whether it is better for a user to pay the Primary Operator for costlier, dedicated network resources with Quality of Service guarantees, or act as secondary user, facing little or no costs without any performance guarantee. Furthermore, we consider the pricing problem of POs, who must set access prices to maximize their revenues.

To answer the above question, we consider a cognitive radio scenario which consists of primary and secondary networks, as well as a large set of cognitive users that can choose either to act as secondary (unlicensed) users, sharing the spectrum holes left available by licensed users (through a secondary base station), or to act as primary users who access directly to the primary network through a primary base station, using a licensed band.

The joint pricing and cognitive radio network selection problem is modeled as a Stackelberg game, where first the Primary Operator sets the access price to attract as many users as possible, in order to maximize its revenue; then, users perform the network selection process, which is formulated as a *population game* [3]. Such games provide a general and powerful framework for characterizing the strategic interactions among large numbers of agents, whose behavior is modeled as a dynamic adjustment process. Therefore, in this paper we formulate the network selection process as a population game, and cognitive users' behavior is studied according to *replicator dynamics* [3], which well captures the behavior of users that adapt their choices and strategies based on the observed system's state.

We provide equilibrium and convergence properties of the proposed game, and derive optimal stable price and network selection settings. Numerical results obtained in different network scenarios illustrate that our evolutionary game captures the main factors behind cognitive network pricing and selection, thus representing a promising framework for the design and performance evaluation of cognitive radio systems.

The paper is organized as follows: related work is reviewed in Section II. Section III provides the main results on population games and replicator dynamics. The considered network model is described in Section IV, and the proposed game formulation of the network selection problem in CRNs is illustrated in Section V. Numerical results are presented in Section VI, while Section VII draws the conclusions.

II. Related Work

Several recent works, including those proposed in the networking context, have considered evolutionary games to study the behavior of network users [4], [5], [6], [7].

In [4], the authors consider a large number of non-cooperative mobile users that should (1) choose a subset of WLAN access points to connect to and multihome to and (2) split their traffic among the chosen access points. This problem is studied using a potential game model and replicator as well as Neumann-Nash dynamics.

A similar approach is presented in [5] to solve the network selection problem in heterogeneous wireless access networks (i.e., WMANs, cellular networks, and WLANs) considering users with different requirements. Evolutionary game theory is used to investigate the dynamics of user behavior. The solution given by the evolutionary game model is compared to the Nash equilibrium solution obtained from a non-cooperative game model. Finally, a set of algorithms (i.e., population evolution and reinforcement learning algorithms) are proposed to implement the evolutionary network selection game model.

Potential games and replicator dynamics are also used in [6] to study the non-cooperative routing problem in a general network topology. The routing problem is considered in the framework of a population game, and the evolution of the populations' size is studied using replicator dynamics.

In [7], the authors model the dynamics of a multiple-seller, multiple-buyer spectrum trading market as an evolutionary game [8], in which multiple primary users want to sell and multiple secondary users want to buy spectrum opportunities. Secondary users evolve over time, buying the spectrum opportunities that optimize their performance in terms of transmission rate and price.

An auction framework for the spectrum sharing problem in CRNs is proposed in [9]. The authors study analytically and numerically the spectrum auction mechanism, considering multiple primary and secondary users that are characterized by two-dimensional and non-continuous strategy (bid). Furthermore, they investigate the spectrum auction with licensed and free bands, and develop a distributed adaptive algorithm based on no-regret learning [10] to converge to a correlated equilibrium of the auction game.

The joint spectrum access and pricing problem has been studied in [11], for cognitive radio networks considering elastic traffic.

Unlike previous works, which study the interaction between two well-defined sets of users (primary and secondary ones) that *already* performed the choice of using the primary (licensed) or the secondary (unlicensed) network, our paper tackles a fundamental issue in CRNs. In fact, we model the users' decision process that takes place *before* such users enter the CRN, thus assessing the economic interest of deploying secondary (xG) networks. Such choice depends on the trade-off between *cost* and *performance guarantees* in such networks. At the same time, we derive the optimal price setting for a Primary Operator that plays before network users (Stackelberg approach), in order to maximize its revenue.

We use enhanced game theoretical tools, derived from population game theory, to model the network selection dynamics, providing convergence conditions and equilibrium settings.

III. Population Games: Introduction and Main Results

This section briefly introduces the game theoretic concepts and main theoretical results used in this paper. For more details on population games, the reader is referred to the book by W. H. Sandholm [3].

A. Population Games

A population game G, with Q non-atomic classes of players (i.e., network users) is defined by a mass and a strategy set for each class, and a payoff function for each strategy. By a non-atomic population, we mean that the contribution of each member of the population is very small. This is the case in our game, where a large set of users compete for CRN's bandwidth resources. We denote the set of classes by $\mathcal{Q} = \{1, \ldots, Q\}$, where $Q \geq 1$. The class q has mass m^q. Let S^q be the set of strategies available for players of class q, where $S^q = \{1, \ldots, s^q\}$. These strategies can be thought of as the actions that members of q could possibly take (i.e., connecting to the primary or the secondary network).

During the game play, each player of class q selects a strategy from S^q. The mass of players of class q that choose the strategy $n \in S^q$ is denoted by x_n^q, where $\sum_{n \in S^q} x_n^q = m^q$. We denote the vector of strategy distributions being used by the entire population by $x = \{x^1, \ldots, x^Q\}$, where $x^i = \{x_1^i, \ldots, x_{s^i}^i\}$. The vector x can be thought of as the state of the system.

The marginal payoff function (per mass unit) of players of class q who play strategy n when the state of the system is x is denoted by $F_n^q(x)$, usually referred to as *fitness* in evolutionary game theory, which is assumed to be continuous and differentiable. The total payoff of the players of class q is therefore $\sum_{n \in S^q} F_n^q(x) x_n^q$.

B. Replicator Dynamics

The replicator dynamics describes the behavior of a large population of agents who are randomly matched to play normal form games. It was first introduced in biology by Taylor and Jonker [12] to model the evolution of species, and it is also used in the economics field. Recently, such dynamics has been applied to many networking problems, like routing and resource allocation [4], [6].

Given x_n^q, which represents the proportion of players of class q that choose strategy n, as illustrated before, the replicator dynamics can be expressed as follows:

$$\dot{x}_n^q = x_n^q \Big(F_n^q(x) - \frac{1}{m^q} \sum_{n \in S^q} F_n^q(x) x_n^q \Big), \qquad (1)$$

where $\dot{x}_n^q$ represents the derivative of x_n^q with respect to time.

In fact, the ratio $\dot{x}_n^q / x_n^q$ measures the evolutionary success (the rate of increase) of a strategy n. This ratio can be also expressed as the difference in fitness $F_n^q(x)$ of the strategy n and the average fitness $\frac{1}{m^q} \sum_{n \in S^q} F_n^q(x) x_n^q$ of the class q.

C. Summary of results related to Replicator Dynamics

We now summarize the most notable results for the replicator dynamics (derived from [13], [14]), which help establishing the convergence of such dynamics to stable *Wardrop* equilibrium points.

Definition 1 The dynamics $\dot{x} = V(x)$ is said to be *positive correlated* (PC) if $\sum_{q \in Q} \sum_{n \in S^q} F_n^q(x) V_n^q(x) > 0$, whenever $V(x) \neq 0$.

Definition 2 A function $\Phi : X \rightarrow R$ is a *potential* for a game G if for every $i \in U$ and for every $x^{-i} \in X^{-i}$

$$\Phi(x, x^{-i}) - \Phi(z, x^{-i}) = u^i(x, x^{-i}) - u^i(z, x^{-i}), \forall x, z \in X^i,$$

where u^i represents the objective function (utility/cost) of user i.

G is called a *potential game* if there exists a continuously differentiable function $f : \mathcal{X} \rightarrow \mathcal{R}$ satisfying $\frac{\partial f(\mathbf{x})}{\partial x_i^q} = F_i^q(\mathbf{x})$ $\forall \mathbf{x} \in \mathcal{X}, i \in \mathcal{S}_q$ and $q \in \mathcal{Q}$.

Result 1 If V(x) satisfies PC, all Wardrop equilibria of G are stationary points of $\dot{x} = V(x)$.

Result 2 The replicator dynamics is PC.

Result 3 A potential game G, with dynamics $V(x)$ that is PC, has asymptotically stable stationary points.

For completeness, we briefly review hereafter a commonly used concept in the networking context: the Wardrop equilibrium [15]. Consider any strategy distribution $x^q = [x_1^q, \ldots x_{S_q}^q]$; there would be some elements which are non-zero and others which are zero. We call the strategies corresponding to the non-zero elements as the strategies used by class q.

Definition 3 A state $\hat{x}$ is a Wardrop equilibrium if for any class $q \in \mathcal{Q}$, all strategies being used by the members of q yield the same marginal payoff to each member of q, whereas the marginal payoff that would be obtained by members of q is lower for all strategies not used by class q.

Let $\widehat{\mathcal{S}}^q \subset \mathcal{S}^q$ be the set of all strategies used by class q in a strategy distribution $\hat{\mathbf{x}}$. A Wardrop equilibrium $\hat{\mathbf{x}}$ is then characterized by the following relation:

$$F_s^q(\hat{\mathbf{x}}) \geq F_{s'}^q(\hat{\mathbf{x}}) \ \forall s \in \widehat{\mathcal{S}}^q \text{ and } s' \in \mathcal{S}^q.$$

IV. Network Model

Having reviewed the mathematical tools used in our work, we now detail the network model, which is illustrated in Figure 1. We consider a cognitive radio wireless system which consists of an xG network that coexists with a primary network at the same location and on the same spectrum band.

Users arrive at this system sequentially, with interarrival times that are independent and identically distributed, and have finite mean λ^{-1}. Each arriving user must choose whether to join the primary network (paying a subscription cost) or the xG one (which has no subscription cost), based on criteria to be specified below, i.e., a combination of cost and QoS (service time/latency).

In our work, we use the population dynamics (and, in particular, *replicator* dynamics) to model the behavior of users that decide to which network they should access, since such dynamics well captures the behavior of users that adapt their choices and strategies based on the observed state of the system (in terms of costs and congestion, in our case).

To this aim, we consider a population game G with a non-atomic set of players ($q = 1$), which is defined by a *strategy set*

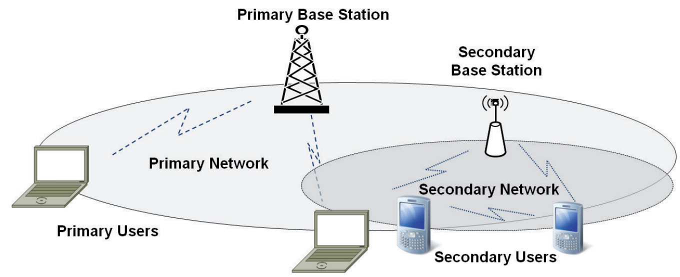

Fig. 1. CRN scenario with a primary network and a secondary (xG) network. Arriving users must decide whether to join the primary network, paying a subscription fee for guaranteed QoS, or the xG network (which has no subscription cost and no performance guarantees), based on the expected cost and congestion levels.

denoted by $\mathcal{S} = \{s_p, s_s\}$, identical for all players, and a *payoff function* for each strategy; s_p means that the player chooses the *primary* base station, and s_s that the player chooses the *secondary* base station, using the spectrum holes left free by primary users.

V. Cognitive Users' Behavior: Replicator dynamics

We use replicator dynamics to model and analyze the behavior of users that must decide whether to access the primary or secondary network.

More specifically, we focus on the cognitive radio scenario illustrated in the previous section, introducing replicator dynamics for the network selection game, and we determine the optimal price value (p^*) that should be set by the Primary Operator in order to maximize its revenue, as well as the network selection settings (X_P and $X_S = 1 - X_P$), i.e, the fraction of players that choose the primary and the secondary network, respectively.

Table I summarizes the basic notation used in our game model. The users' average arrival rate is denoted by λ. The average service rate of the primary and secondary base stations is denoted, respectively, by θ and μ; as a consequence, θ^{-1} and μ^{-1} represent the average service times.

We assume (like for example in Anshelevich et al. [16]) that the total cost incurred by a player is a combination of the cost for the player to access the network (which is equal to p for the primary network, and zero for the secondary network), and the service time (latency) experienced in such network.

TABLE I
Basic Notation

λ	Users' average arrival rate
μ	Service rate of the Secondary Base Station
θ	Service rate of the Primary Base Station
X_S	Fraction of Secondary Users
X_P	Fraction of Primary Users
p^*, p	(Optimal) Price charged by the PO to access its services
K	Constant, velocity of convergence
$\overline{N_u}$	Average number of users in the system

The goal of each user is therefore to *minimize* the sum of his cost and latency.

Hence, we can formalize our population game as follows:

$$\dot{X_S} = KX_S[\frac{-1}{\mu - \lambda X_S} - \Big(\frac{-X_S}{\mu - \lambda X_S} - X_P(\theta^{-1} + p)\Big)]$$
$$= KX_S[(1 - X_S)(\theta^{-1} + p - \frac{1}{\mu - \lambda X_S})], \quad (2)$$

where $\dot{X_S}$ represents the derivative of X_S with respect to time. This equation has the same structure as the replicator dynamics of equation (1): the first term ($F_n^q(x) \equiv \frac{-1}{\mu - \lambda X_S}$) corresponds to the delay perceived by users that choose to connect to the secondary network, using a M|M|1 approximation; the second term ($\frac{1}{m^q}\sum_{n \in S^q} F_n^q(x)x_n^q \equiv \frac{-X_S}{\mu - \lambda X_S} - X_P(\theta^{-1} + p)$) represents the average cost/delay incurred by the fraction X_S of secondary users (as explained before) and that experienced by the fraction X_P of primary users ($\theta^{-1} + p$, i.e., the service delay plus the price charged by the Primary Operator).

In particular, the speed of variation of X_S is proportional to the population size X_S (via the proportionality coefficient K), which models the willingness of the population to change strategy.

We observe that the arrival rate λ should be smaller than the service rate μ of the secondary network, since otherwise a positive fraction of users (in average) would be forced to use the primary network, and consequently the PO could set an arbitrarily large cost p to obtain infinite revenue. Note that, alternatively, the network access problem can be reformulated assuming that users decide to subscribe to the primary network services only if p does not exceed a maximum cost.

As stated in Section III, Wardrop equilibria are the stationary points of equation (2). As a consequence, the fraction of users that choose the xG network (secondary users) at the equilibrium ($\overline{X_S}$) is given by:

$$\overline{X_S} = \frac{\mu - \frac{1}{\theta^{-1} + p}}{\lambda}. \quad (3)$$

The average number of users in the system, $\overline{N_u}$, can be obtained using Little's theorem, which gives a correlation between the average user arrival rate, λ, the average time spent in the system by such users, T, and $\overline{N_u}$: $\overline{N_u} = \lambda T$.

Therefore, the revenue R (per unit time, i.e., T=1) obtained at the equilibrium by the Primary Operator is equal to:

$$R = p\overline{N_u}(1 - \overline{X_S}) = p(\lambda - \mu + \frac{1}{\theta^{-1} + p}). \quad (4)$$

The optimal price (p^*) the Primary Operator must set in order to maximize its revenue can be obtained solving the equation $\frac{\partial R}{\partial p} = 0$, and is given by the following expression:

$$p^* = \sqrt{\frac{\theta^{-1}}{\mu - \lambda}} - \theta^{-1}. \quad (5)$$

Note that expression (5) is valid only if $\theta^{-1} < \frac{1}{\mu - \lambda}$, since otherwise all users would choose the secondary network, even if $p = 0$. In other words: if $\theta^{-1} > \frac{1}{\mu - \lambda}$, then the PO's revenue is null for all p values, since no user will choose the primary network.

VI. Numerical Results

In this section, we analyze and discuss the numerical results obtained from simulating the evolutionary network selection game in different cognitive radio scenarios. More specifically, we evaluate the proposed game model in terms of stability and convergence, and we study the impact of different parameters (i.e., service rate θ and access price p) on the network selection process, and as a consequence, on the Primary Operator's revenue.

To this aim, we first consider a CRN scenario with μ=100, λ=80 and θ=40 users/(unit time). The parameter K in equation (2) is set to 1.

Figure 2 illustrates the convergence (expressed in steps needed in the replicator dynamics) of network users to a stationary solution, for two different prices set by the PO, i.e., p=0.01 and p=0.005. More specifically, the figure reports the fraction X_S of users that choose the secondary network. It can be observed that X_S increases for increasing p values, since more users will have an incentive to choose the secondary network instead of paying a high price to use the primary network's resources.

Note that, in this scenario, a large fraction of users (approximately 90% for $p = 0.01$, more than 80% for $p = 0.005$) choose the xG network in spite of the primary one, since the service rate (μ) of the former network is quite large with respect to the subscribed (guaranteed) rate of the primary network (θ).

Figure 3 shows, in the same scenario, the revenue obtained by the Primary Operator as a function of the price p charged for the access. It is interesting to notice that such revenue has a maximum, corresponding to the optimal price (p^*=0.01 in this scenario), while it is lower both for smaller and larger p values. It can be observed that the PO's revenue can change

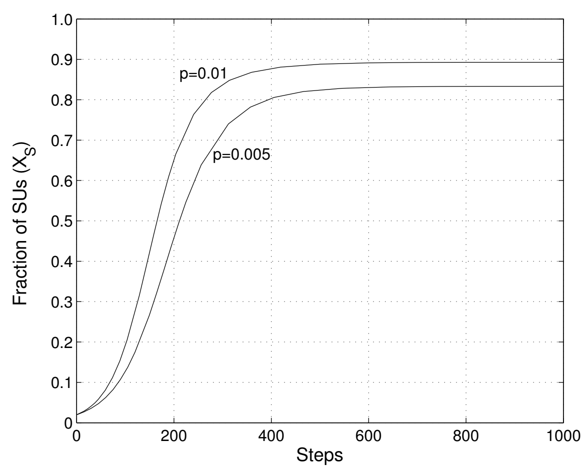

Fig. 2. Convergence of Secondary Users to the stationary points.

consistently based on the price p setting, so that an accurate choice of p must be performed. Our game model can help in deciding such setting.

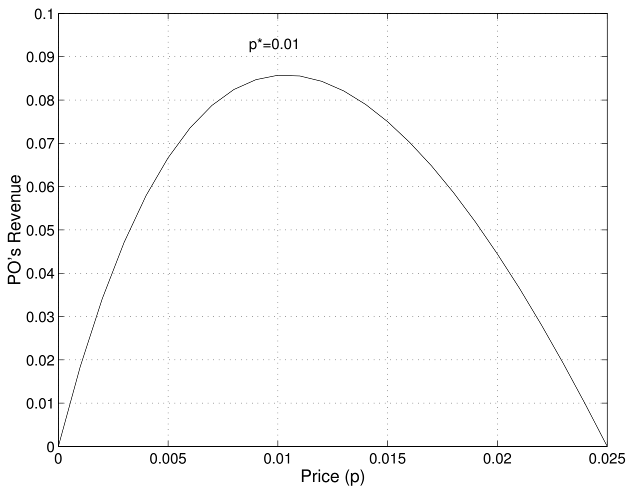

Fig. 3. Primary Operator's Revenue as a function of the access price p.

Finally, Figure 4 illustrates the optimal price p^* that the PO must charge in order to maximize its revenue, as a function of the service rate θ of the primary base station. The scenario is the same considered before, and it can be observed that such optimal price p^* increases consistently when θ increases from small values up to the maximum, which is obtained for $\theta = 80$. This is due to the fact that the primary network services become increasingly attractive for higher θ values; the PO can therefore charge higher prices to primary users.

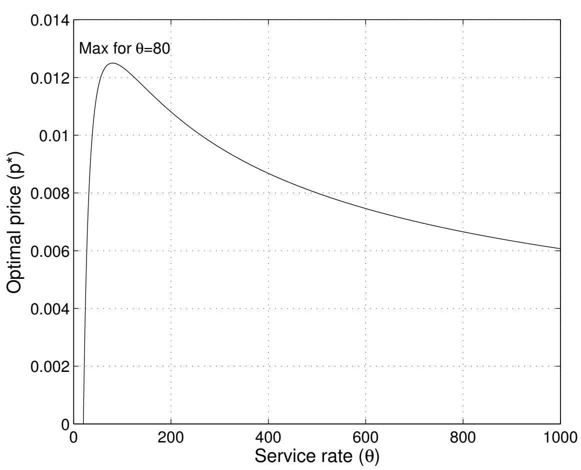

Fig. 4. Optimal price p^* (corresponding to maximum revenue for the PO) as a function of the primary base station's service rate θ.

VII. Conclusion

In this paper, we tackled a fundamental question related to Cognitive Radio Networks, i.e., the trade off between the cost savings that such networks promise to realize and the QoS degradation (with respect to reserved, licensed spectrum bands) due to the competition of secondary users for common network resources.

In particular, we considered the problem of joint pricing and network selection in CRNs. We modeled this problem using a leader-follower game, where the Primary Operator first sets the access price to maximize its revenue, and then users perform a network selection process, modeled using a population game and replicator dynamics. We derived optimal stable price and network selection settings, illustrating numerical examples in different network scenarios. Our game model captures the main factors behind cognitive network pricing and network selection, thus representing a promising framework for the design and understanding of cognitive radio systems.

Acknowledgments

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