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# Characterizing the Throughput Gain of Single Cell MIMO Wireless Systems with Full Duplex Radios

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**Abstract**—Using additional antennas and signal processing techniques to build a full-duplex radio is becoming a popular topic in the networking research community. However, additional antennas are used for MIMO (multiple input multiple output) transmissions, traditionally, and it is well-known that increasing the number of antennas on either transmitter or receiver side increases the MIMO link capacity. Therefore, deployment of extra antennas on a transceiver has twofold applications, first increasing the capacity of half-duplex MIMO link, second providing full-duplex capability for the radio. In this work, we look at the performance comparison between using multiple-antennas for capacity enhancement in a half-duplex MIMO link with that of utilizing them to build a full-duplex radio. Our results indicate that under certain conditions, using additional antennas for building full-duplex radio can provide performance boost compared to utilizing them to form a high capacity MIMO link.

## I. INTRODUCTION

Recently there has been considerable interest in building full-duplex radios [1]–[3]. Unlike half-duplex radios that need to separate their receptions and transmissions on different frequency bands (FDD) or different time slots (TDD), full-duplex radios are capable of receiving and transmitting simultaneously in the same frequency band. Hence, an ideal full-duplex radio can double the spectral efficiency compared to a half-duplex radio and increase the overall throughput of the wireless network.

However, implemented full-duplex (FD) radios receive and transmit over different antennas. Nevertheless, having new antennas on the radio for FD is not equivalent to having a new antenna for MIMO connections in terms of additional hardware requirement. In a half-duplex MIMO transceiver, each antenna is connected to two RF chains; one transmission chain and one reception chain. In FD radios only one RF chain is attached to each antenna while self-interference cancellation can be achieved with addition of simple RF components. The details of implementing an FD system are out of the scope of this paper (please see [1], [2], [4], [5]). The focus of this paper is on characterizing the potential theoretical gains achievable from utilizing FD radios.

Accordingly, *antenna conserved* and *RF-chain conserved* scenarios are considered to compare the performance of a half-duplex MIMO system with that of an FD system (Fig. 1). In *antenna conserved* scenario, we let MIMO and FD radios have the same number of antennas and for the FD radio a fraction

of the available antennas is used as transmit antennas while remaining antennas are used as receive antennas. Therefore, in *antenna conserved* scenario number of antennas are the same for the two radios while the number of RF chains for MIMO radios is twice that of FD radios. On the other hand, in *RF-chain conserved* scenario, the FD radio has the same number of receive and transmit RF chains as that of the MIMO radio. This means that to compare a  $2 \times 2$  MIMO link with an RF-chain conserved FD, the FD radio will have two transmit and two receive antennas per transceiver such that four RF chains are used on every FD and HD (Half-duplex) radios. From the hardware complexity standpoint, comparing MIMO with FD in the RF-chain conserved scenario is fair, since it would not increase the hardware complexity of the radio, which in turn increases exponentially with the number of transmit or receive RF chains. Besides, an additional antenna on a node translates to transmission of an additional stream of data which increases the joint decoding complexity of the MIMO receiver exponentially.

Two topologies are considered in this paper: point-to-point and point-to-multipoint communication topologies. We show through extensive simulation and analysis that an FD radio performance is inferior to that of a MIMO radio in most point-to-point scenarios in the antenna conserved scenario; however, the spectral efficiency can be potentially doubled in point-to-point communication when the RF-chain conserved scenario is considered. Hence, we conclude that employing FD radios in antenna conserved scenarios is not promising in terms of capacity, but it might be considered as a low-complexity competitor for the HD-MIMO. Given that the interference is more pronounced in point-to-multipoint setting, we only focus on RF-chain conserved FD radios for the case of point-to-multipoint communication. Our result indicates that FD radios can support more than twice the throughput over MIMO-HD in scheduled point-to-multipoint scenario; by benefiting from both additional capacity gain and scheduling flexibility that FD provides. Finally, our results favor the use of RF-chain conserved FD systems proposed in [4], [5] rather than the use of antenna conserved FD systems in [1], [2].

This paper is organized as follows: in Section II system model is briefly discussed, Section III includes the details of analysis and discussion of the results. In Section IV we conclude this paper.

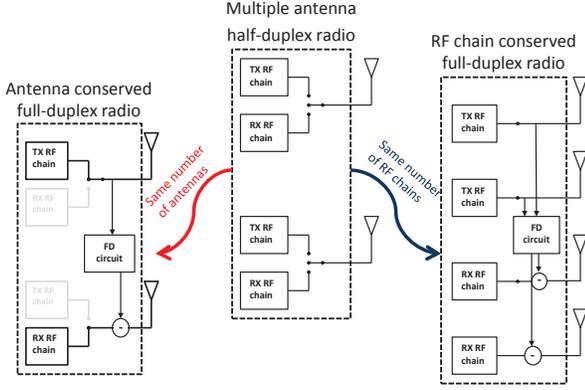


Fig. 1. Two FD radio scenarios: Antenna conserved FD (left) and RF-chain conserved FD (right)

## II. SYSTEM MODEL

The main challenge of building a full-duplex radio is self-interference cancellation. The transmitted signal from the nearby transmit antenna of an FD radio acts as a strong interferer for the attenuated received signal from another radio. Usual digital domain interference cancellation methods are not enough for self-interference cancellation and higher measures need to be taken [4]. In spite the fact that canceling all the self-interference in an FD radio is practically impossible, we make the optimistic assumption of having ideal FD radios in this work. The implication is that an upper bound on the performance of an FD radio is studied in this work. If an ideal FD radio can not provide gain over the HD radio, we can not expect gain from employing FD radios in practice since residual self-interference will degrade their performance. On the other hand, if the analyses under this assumption illustrate gain from employing FD radios over HD radios, the calculated gain will be an upper bound on the practical performance of FD radios.

### A. Point-to-Point

As the performance measure in the physical layer, we choose to work with capacity of the wireless channel. For point-to-point scenario, we approach the problem by considering different levels of feedback from the receiver to the transmitter, and hence studying outage capacity, open-loop capacity and closed-loop capacity for each link.

RF chain conserved scenario trivially doubles the capacity of the point-to-point links for ideal FD. The interesting scenario is then the antenna conserved scenario. Suppose that we have two nodes, node  $A$  with  $N$  antennas and node  $B$  with  $M$  antennas. To activate an antenna-conserved FD connection, node  $A$  and  $B$  choose  $N_1$  and  $M_1$  antennas for transmission, respectively, therefore,  $N - N_1$  and  $M - M_1$  antennas for reception.

Since the capacity of the HD links can be different from node  $A$  to node  $B$  and in the reverse direction, we assume a time sharing between the two links and the average of the two

capacities is considered as the capacity of the HD-MIMO link, while the FD transmissions can take place in both directions simultaneously. Therefore, the capacity gain of point-to-point FD with respect to point-to-point HD is defined as,  $G_{FD} = \frac{C_{A \rightarrow B}^{FD} + C_{B \rightarrow A}^{FD}}{0.5(C_{A \rightarrow B}^{HD} + C_{B \rightarrow A}^{HD})}$ , where  $C_{i \rightarrow j}$  is the capacity of the link from transmitter node  $i$  to receiver node  $j$ .

### B. Point-to-Multipoint

For point-to-multipoint scenario, a scheduled cellular system is considered, where the scheduler has perfect knowledge about the channel state information. In a single cell, there are  $n$  clients, each with  $M$  antennas and one  $N$ -antenna base station (BS), with the channel matrix  $\mathbf{H}_i$  between Client  $i$  and the BS.

In this work we use the inverse of the average service rate of each link as the weight of the link for scheduling purposes. The average rate for link  $i$ ,  $Ra_i$  evolves as follows:

$$Ra_i(t+1) = \left(1 - \frac{1}{T_c}\right)Ra_i(t) + \frac{1}{T_c}R_i(t) \quad (1)$$

Where,  $R_i(t)$  is the supported rate of link  $i$  at time slot  $t$  and  $T_c$  is the averaging period. Therefore, the objective of the scheduler will be to maximize the following weighted sum rate at each time slot, subject to the corresponding power constraints:

$$\max \sum_{i=1}^n \frac{1}{Ra_i(t)} R_i(t) \quad (2)$$

## III. PERFORMANCE ANALYSIS

### A. Point-to-Point

For a MIMO point-to-point link we consider  $N$  transmit antennas and  $M$  receive antennas, with the  $M \times N$  channel matrix,  $\mathbf{H}$  between the transmitter and the receiver. In the sequel, different definitions of capacity for fading channels depending on the channel information and feedback available to the transmitter and the gain achievable by FD in each case are studied:

1) *Outage Capacity*: Outage capacity is meaningful when there is no feedback between the receiver and the transmitter, and the transmitter wishes to choose a communication rate that guarantees a level certainty, given the channel coefficients statistics. The outage capacity for the outage probability of  $P_{out}$  is defined as follows:

$$C_{outage} = \{R : Pr(C < R) = P_{out}\} \quad (3)$$

where  $C$  is the instantaneous capacity of the communication link. For  $N = 2$ ,  $M = 2$  the outage probability of rate  $R$  for a Rayleigh fading channel has the following form:

$$P_{out}(R) = \int_0^\infty F_{\lambda_1} \left( \frac{2^R}{\lambda\gamma/2 + 1} - 1 | \lambda_2 = \lambda \right) f_\lambda(\lambda) d\lambda \quad (4)$$

where,  $F_{\lambda_1}(\lambda|\lambda_2) = e^{-\lambda \frac{1+(\lambda-\lambda_2+1)^2}{1+(1-\lambda_2)^2}}$  and  $f_\lambda(\lambda) = \frac{1}{2}e^{-\lambda}(1+(1+\lambda)^2)$  are the conditional cumulative distribution function (CDF) and unconditional probability distribution function (PDF) of the eigenvalues of the Wishart matrix  $\mathbf{H}\mathbf{H}^*$ . Dividing the antennas to one transmit and one receive antenna on each

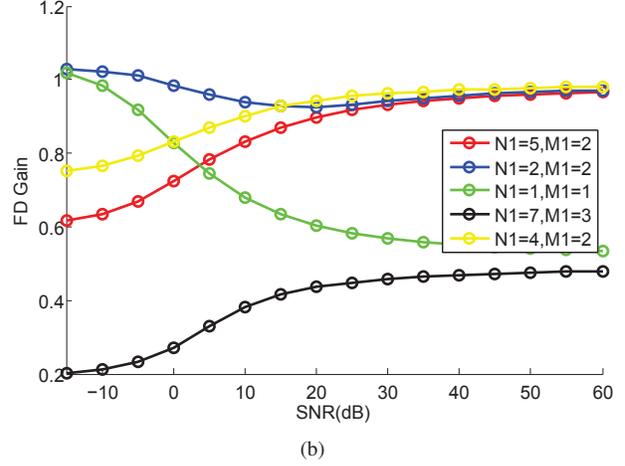
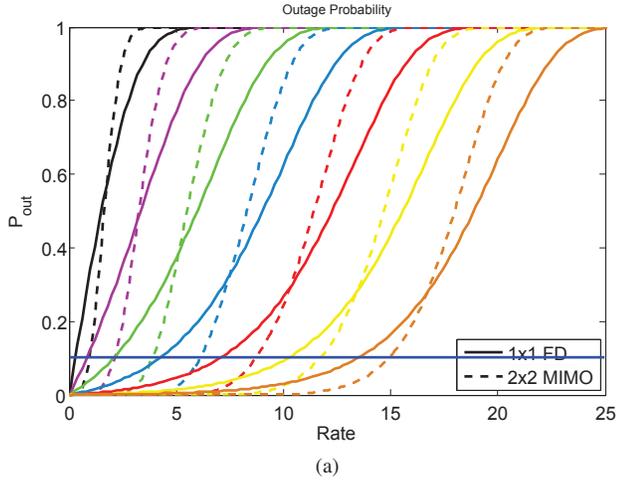


Fig. 2. a) Outage probability: From right to left, the SNR increases, b)  $(N, M) = (8, 4)$ ,  $P_{out} = 0.1$ .

node will result in the following outage probability for the FD connection:

$$P_{out}(R) = 1 - e^{-\frac{2R/2-1}{2}} \quad (5)$$

Given the above equations  $G_{FD}$  can be calculated. Higher number of antennas do not lend themselves to mathematical analysis for outage capacity and we will use simulation results characterize their performances.

The outage probability versus the rate is plotted in Fig. 2(a) for both  $2 \times 2$  MIMO and the corresponding FD links, and it is clear that for any given outage probability less than 10% MIMO links have higher outage rate. Fig. 2(b) plots outage capacity for different number of antennas and different antenna divisions and the outage probability of 10%. Therefore, except the negligible gain at very low SNRs with some choice of antenna division, antenna conserved FD link has lower outage capacity than the corresponding HD link.

2) *Open-Loop Capacity*: When channel state information is available at the receiver (CSIR) and the receiver can feedback the rate to the transmitter, capacity can be defined as follows:

$$C_{CSIR} = \log \left( \mathbf{I}_{M \times M} + \frac{\rho}{N} \mathbf{H} \mathbf{H}^* \right) \quad (6)$$

Where  $\rho$  is the average SNR and  $|\mathbf{H}|$  is the determinant of matrix  $\mathbf{H}$ . Capacity expression in Eq. (6) can be approximated for very high, and very low SNRs as follows:

$$C_{CSIR} \approx \begin{cases} \log_2 e \frac{\rho}{N} \sum_{i=1}^N \sum_{j=1}^M |h_{ij}|^2 & \rho \ll 1 \\ \min(N, M) \log_2 \left( \frac{\rho}{M} \right) + \sum \log_2 \lambda_i & \rho \gg 1 \end{cases} \quad (7)$$

where  $\lambda_i$ s the eigenvalues of the Wishart matrix  $\mathbf{H} \mathbf{H}^*$ . Therefore, the average of the capacity in low and high SNRs is given by:

$$\mathcal{E} \{ C_{CSIR} \} \approx \begin{cases} M \rho \log_2 e & \rho \ll 1 \\ \min(N, M) \log_2(\rho) & \rho \gg 1 \end{cases} \quad (8)$$

Given the above approximations, the gain of FD over HD for low and high SNRs is:

$$G_{FD} \approx \begin{cases} \frac{(M-M_1)+(N-N_1)}{0.5(M+N)} = 2 - 2 \frac{N_1+M_1}{N+M} & \rho \ll 1 \\ \frac{\min(N_1, M-M_1) + \min(N-N_1, M_1)}{\min(N, M)} & \rho \gg 1 \end{cases} \quad (9)$$

Therefore, at very low SNRs  $G_{FD} \leq \frac{2(M+N-2)}{M+N}$  and gains of more than 1 over HD-MIMO link is achievable, while at high SNRs, we have  $G_{FD} \leq 1$  independent of the way the antennas are divided.

Fig. 3(a) depicts the average gain of FD-MIMO over HD-MIMO for different division strategies and on low, medium, and high SNRs, i.e.,  $\rho_{low} = -40dB$ ,  $\rho_{medium} = 15dB$ ,  $\rho_{high} = 60dB$ .

Fig. 3(a) illustrates that dividing the antennas for FD will not result in considerable capacity gain in medium SNRs. Please note that what is depicted here is the results for an ideal FD radio where there is no self-interference residue affecting the capacity of the FD link, which implies that in systems with rate feedback it is better to use HD-MIMO instead of dividing the antennas for FD at medium and high SNRs. At low SNRs, though, FD-MIMO provides gain as high as 60% in some division scenarios. A closer look at Eq. (9) predicts this gain and also shows that at low and high SNR, for the example of  $N = 8$ ,  $M = 4$ , the best divisions strategies are  $N_1 = 1$ ,  $M_1 = 1$  and  $N_1 = 4$ ,  $M_1 = 2$ , respectively, which requires an adaptive antenna division for acceptable gain at different SNRs, in order to exploit benefits of FD-MIMO.

3) *Closed-Loop Capacity*: When transmitter has channel state information, water filling algorithm is the optimal transmission strategy and results in the following closed-loop capacity expression:

$$C_{CSIT} = \sum_{i=1}^{\min(M, N)} \log \left( 1 + \frac{p_i}{N_0} \lambda_i \right) \quad (10)$$

At high SNRs, the optimum waterfilling policy is equal power over all antennas, and at low SNR the best policy will

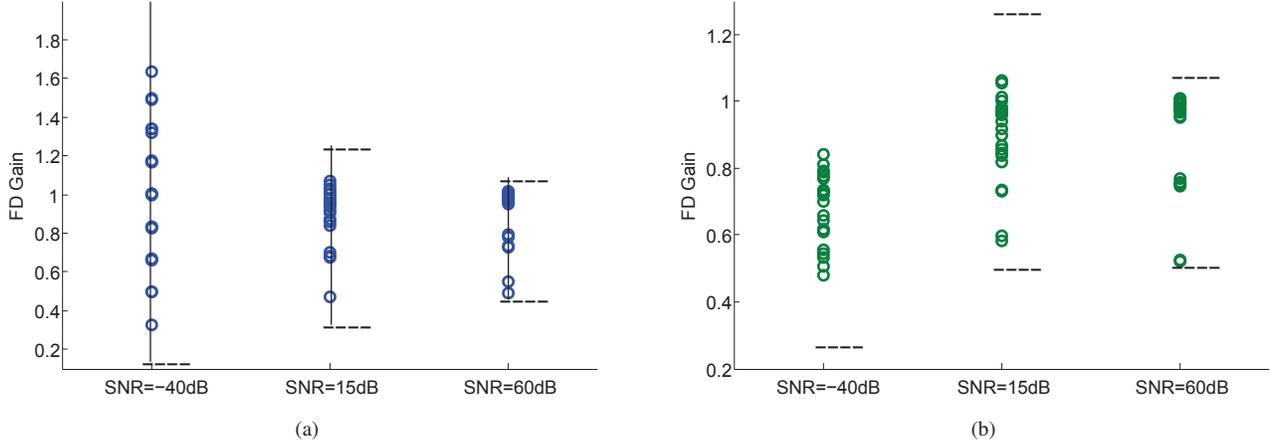


Fig. 3. a) Open-loop capacity, b) Closed-loop capacity

be transmitting with full power over the direction of the eigenvector with largest eigenvalue. Therefore, the following expressions represent the FD gain in the two extreme cases:

$$G_{FD} \approx \begin{cases} \frac{\lambda_{max}(\mathbf{H}_2^T \mathbf{H}_2^{T*}) + \lambda_{max}(\mathbf{H}_3 \mathbf{H}_3^*)}{\lambda_{max}(\mathbf{H} \mathbf{H}^*)} & \rho \ll 1 \\ \frac{\min(N_1, M - M_1) + \min(N - N_1, M_1)}{\min(N, M)} & \rho \gg 1 \end{cases} \quad (11)$$

Where,

$$\mathbf{H}_{M,N} = \begin{pmatrix} \mathbf{H}_{M_1, N_1}^1 & \mathbf{H}_{M_1, N - N_1}^2 \\ \mathbf{H}_{M - M_1, N_1}^3 & \mathbf{H}_{M - M_1, N - N_1}^4 \end{pmatrix} \quad (12)$$

Note that, FD gain approaches one for its maximum over different antenna divisions at high SNRs.

Fig. 3(b) illustrates  $G_{FD}$  for different ranges of the SNR and various antenna division strategies for closed-loop capacity. At very low and very high SNRs the average gain from FD-MIMO is at most 1, while  $G_{FD}$  at medium SNRs can be negligible.

In summary, antenna conserved FD link has similar capacity as the corresponding HD-MIMO at certain antenna groupings. This result is not very surprising, but from another point of view it does show that with lower complexity, i.e. lower number of RF chains and lower receiver complexity, an antenna conserved FD link can be as efficient as an HD-MIMO link. Besides, in low SNRs antenna conserved FD link can even have higher open-loop capacity than an HD-MIMO link.

Note that antennas are one of the resources that a wireless radio utilizes to increase the capacity, to the extent that at high SNRs the capacity is linearly scaled by the minimum number of transmit and receive antennas. Therefore, while the links are using the time and frequency resources in full with FD capability, they share their other resource: antennas. That is why we do not observe capacity enhancement for antenna conserved FD.

### B. Point-to-Multipoint

In a cellular setting, having FD radios allows concurrent downlink and uplink transmissions. In random access networks, e.g. IEEE 802.11, FD radios are shown to improve

fairness and alleviate the hidden terminal problem associated with such networks [1], [3]. In random access networks, spectrum is under utilized and studied benefits of having FD radios is associated to higher layer rather than PHY performance improvements, i.e. increase in the capacity of the network. In this section we look at a scheduled cellular communications, where the spectrum is utilized to its fullest in both HD-MIMO and FD scenarios and investigate the gains achievable from enabling FD capability.

To study a scheduled point-to-multipoint setting, we considered both antenna conserved and RF-chain conserved scenarios. As we expect from the point-to-point discussions, our results show that in cellular setting, conserving the number of antennas still falls short of providing considerable gain for the throughput of the network even when the self-interference cancellation is perfect. We skip presenting the results for antenna conserved scenario, and in this section we only consider RF-chain conserved scenario.

From the point of view of the BS, the uplink channel is a multiple access channel (MAC) and the downlink channel is a broadcast (BC) channel. Capacity achieving successive interference cancellation (SIC) for MAC and dirty paper coding (DPC) for BC are considered. For half-duplex scheduling problem, we assume that downlink and uplink transmissions are separated in time, such that each time slot is divided into two mini time-slots, each for one direction of the transmission. In this setting, problems of scheduling downlink and uplink transmissions become decoupled as opposed to FD scheduling problem with simultaneous uplink and downlink transmissions.

For SIC [6], given the decoding order  $\pi_1 \rightarrow \pi_2 \rightarrow \dots \rightarrow \pi_n$  where user  $\pi_n$ , is decoded first by considering other users' signal as noise, and User  $\pi_1$  will be decoded last, the capacity of User  $\pi_i$  is given by:

$$C_{MAC}(\pi_i) = \log \frac{|\mathbf{I}N_0 + \sum_{j=1}^i \mathbf{H}_{\pi_j} \mathbf{P}_{\pi_j} \mathbf{H}_{\pi_j}^*|}{|\mathbf{I}N_0 + \sum_{j=1}^{i-1} \mathbf{H}_{\pi_j} \mathbf{P}_{\pi_j} \mathbf{H}_{\pi_j}^*|} \quad (13)$$

where  $\mathbf{P}_i$  is the covariance matrix of the transmitted signal for User  $i$ .

For DPC [6], given the coding order  $\mu_1 \rightarrow \mu_2 \rightarrow \dots \rightarrow \mu_n$ , where the User  $\mu_n$  is coded last, and User  $\mu_1$  is coded first, the capacity of user  $\mu_i$  is given by:

$$C_{BC}(\mu_i) = \log \frac{\left| \mathbf{I} N_0 + \mathbf{H}_{\mu_i} \sum_{j=i}^n \mathbf{Q}_{\mu_j} \mathbf{H}_{\mu_i}^* \right|}{\left| \mathbf{I} N_0 + \mathbf{H}_{\mu_i} \sum_{j=i+1}^n \mathbf{Q}_{\mu_j} \mathbf{H}_{\mu_i}^* \right|} \quad (14)$$

Where  $\mathbf{Q}_i$  is the covariance matrix of the transmitted signal of User  $i$ . Note that if different users are experiencing different noises, by normalizing the channel matrix to the noise covariance matrix, one gets a BC with unit noise variance for all users.

Power constraint on uplink transmissions is per user power constraint, where each user can consume power  $P_u$  when it is active, i.e.  $Tr(\mathbf{P}_i) \leq P_u$ . For downlink transmissions, the access point has a constraint on the total power it can consume while transmitting to different users, i.e.  $Tr(\mathbf{Q}_1 + \mathbf{Q}_2 + \dots + \mathbf{Q}_n) \leq P_d$ .

In each communications direction, the goal of the scheduler is to maximize the weighted sum rate at each time slot, subject to the corresponding power constraints:

$$\max \sum_{i=1}^n \frac{1}{Ra_i(t)} R_i(t) \quad (15)$$

where  $R_i(t)$  is the capacity supported by link  $i$  at time slot  $t$ . Resulting optimization problem for uplink scheduling is:

$$\begin{aligned} \max_{\{\pi_i\}_{i=0}^n, \mathbf{P}_i} & \quad \sigma_{\pi_i=0}^{n-1} \frac{1}{Ra_{\pi_i}(t)} C_{MAC}(\pi_i) \\ s.t. & \quad Tr(\mathbf{P}_i) \leq P_u \quad 0 \leq i < n \end{aligned} \quad (16)$$

Although, this optimization problem is in general a non-convex problem, it can be converted to a convex problem, simply by noting that the optimal decoding order for the MAC channel is given by  $\pi_1 \rightarrow \pi_2 \rightarrow \dots \rightarrow \pi_n$  such that  $w_{u,\pi_n} \leq w_{u,\pi_{n-1}} \leq \dots \leq w_{u,\pi_1}$  [7], where  $w_{u,i} = \frac{1}{Ra_i(t)}$  in uplink direction. With this ordering, links with higher weights face less interference and will be provided with higher rates. Given the optimal decoding order, Eq. (16) becomes:

$$\begin{aligned} \max_{\mathbf{P}_i} & \quad (w_{u,\pi_i} - w_{u,\pi_{i+1}}) \log \left| \mathbf{I} N_0 + \sigma_{j=1}^i \mathbf{H}_{\pi_j} \mathbf{P}_{\pi_j} \mathbf{H}_{\pi_j}^* \right| \\ s.t. & \quad Tr(\mathbf{P}_i) \leq P_u \quad 0 \leq i < n \end{aligned} \quad (17)$$

Capacity region of a BC channel is the same as the capacity region of its dual MAC channel with similar power constraints [6]. Therefore, using the dual MAC convex optimization problem, downlink scheduling problem can be solved efficiently.

For scheduling in FD networks, uplink and downlink scheduling are entangled by the interference caused by uplink transmissions on downlink receptions. Note that for scheduling FD transmissions, the scheduler needs information about the channel between all pairs of users in order to predict the level of interference that active uplink transmissions will cause on active downlink receptions. Given all channel coefficients and

link weights, the scheduler should solve a non-convex problem, since the optimal order of coding/decoding for MAC/BC is unknown in this case. From this point on, we consider single antenna clients and  $N$ -antenna BS. One suboptimal solution is to use the optimal orderings for the corresponding MAC and BC which depend only on the weights of the links. This coding/decoding order results in the following scheduling problem:

$$\begin{aligned} \max_{\mathbf{P}, \mathbf{Q}} & \quad \sum_{i=1}^n (w_{u,\pi_i} - w_{u,\pi_{i+1}}) \log \left| \mathbf{I} N_0 + \sum_{j=1}^i \mathbf{H}_{\pi_j} p_{\pi_j} \mathbf{H}_{\pi_j}^* \right| \\ & \quad + \sum_{i=1}^n (w_{d,\mu_i} - w_{d,\mu_{i+1}}) \log \left| \mathbf{I} + \sum_{j=1}^i \tilde{\mathbf{G}}_{\mu_j}^* q_{\mu_j} \tilde{\mathbf{G}}_{\mu_j} \right| \\ s.t. & \quad p_i \leq P_u \quad \forall i \leq n \\ & \quad \sum_{i=1}^n q_i \leq P_d \end{aligned} \quad (18)$$

where  $p_i$  and  $q_i$  are the transmission power of Client  $i$  for uplink and dual downlink transmissions, respectively. Furthermore,  $g = 0$  if the clients are also FD capable and  $g = 1$  if clients are simple nodes with only HD capability. Adding the term  $gq_{\pi_i}$  when  $g = 1$  leaves out the schedules that contain simultaneous uplink and downlink transmissions from and to a single client.  $\tilde{\mathbf{G}}_i$  is the modified channel matrices from the access point to Client  $i$  after normalizing the channel matrix  $G_i$  to noise and interference power:

$$\tilde{\mathbf{G}}_i = \frac{1}{\sqrt{N_0 + \sum_{j=1}^n |h_{ji}|^2 p_j + gp_i}} \mathbf{G} \quad (19)$$

Where the term  $gp_i$  is placed to differentiate the two types of clients with a similar role to the role of the term  $gq_{\pi_i}$  in Eq. (18), and  $h_{ji}$  is the channel coefficient from client  $j$  to client  $i$ . Interdependence of  $\tilde{\mathbf{G}}$  with  $p_i$ 's makes this problem non-convex. We use numerical search to find the local optima of this problem. Therefore, solutions found for this problem through numerical search are local, and generally sub-optimal. Although suboptimal, we can still characterize the possible performance gains.

Fig. 4(a)(i) depicts the gain achievable with FD in a network with  $n = 2, 4, 8, 16$  clients and  $N = 1, 2, 4, 8$  antennas at the access point for RF chain conserved model. Increasing the number of antennas slightly decreases the gain provided by FD radios. Note that the FD gain is a combination of physical layer gain and link scheduling gain. Therefore, the FD gain could potentially reach values larger than two as it does in Fig. 4(a)(i). Furthermore, by increasing the number of antennas, the probability of having more clients scheduled for transmission/reception at each time-slot increases, because of additional degree of freedom provided by additional antennas. Since HD with DPC has no limit for the number of users scheduled in DL, average number of users scheduled simultaneously increases steadily when the number of antennas is increased. On the other hand, for FD scheduling, the scheduler needs to take into account both UL and DL transmissions and

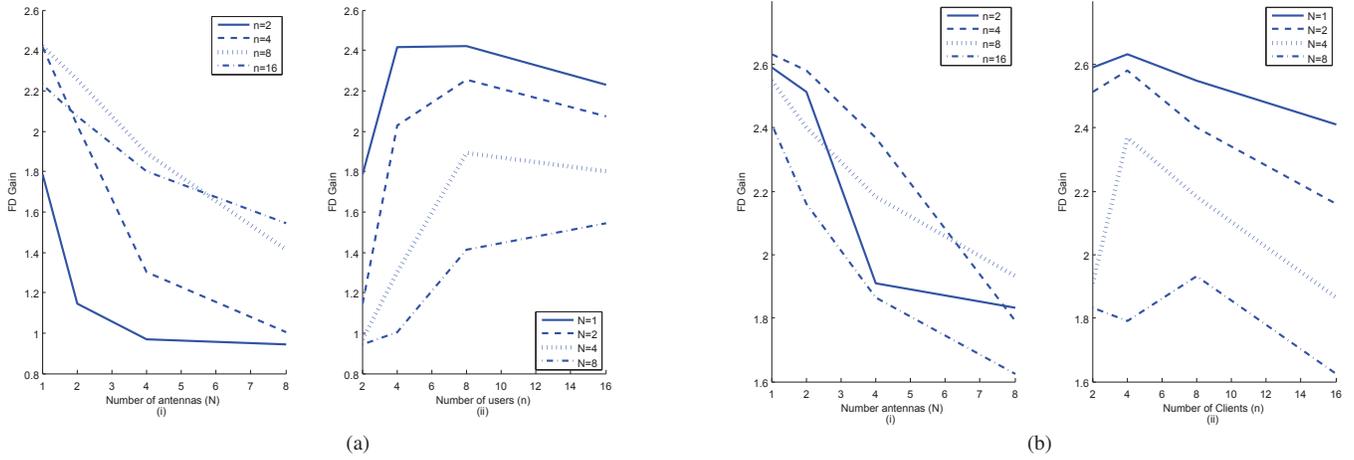


Fig. 4. a) Half-duplex clients, (i) for different number of antennas at the AP, (ii) for different number of clients, b) Full-duplex clients, (i) for different number of antennas at the AP, (ii) for different number of clients

the interference caused by UL transmissions on DL receptions. Therefore, the FD scheduler does not have the same degrees of freedom as the HD scheduler to schedule higher number of users, and hence, the lower gain for FD.

Fig. 4(a)(ii) illustrates the variations in the FD gain when the number of users are increased and the number of antennas at the BS is constant. Increasing the number of users increases the FD gain initially, and then slowly decreases the gain after it reaches its maximum. The reason is that, increasing the number of users gives FD an opportunity to choose more "non-interfering" simultaneous transmissions. It also gives the HD scheduler better chance to schedule users with good channel conditions. Furthermore, HD always schedules all the users for the uplink transmission with full power, as the optimal scheduling decision, while FD activates limited number of UL users, for their interference on DL receivers. Therefore, by increasing the number of users, the UL throughput of the HD scheduler increases more rapidly than that of FD scheduler. That is why the overall gain of the FD scheduler starts to decrease when the number of clients is more than a certain threshold. Fig. 4(b) illustrates variations of FD gain with respect to the number of users and  $N$ , when the clients also have FD radios. Compared to Fig. 4(a) FD gain reaches its maximum faster, because in networks with FD clients, the scheduler starts by scheduling higher number of UL clients compared to the other scenario, and therefore the gain saturates faster than the case that full-duplex connection to a single client is not allowed.

Fig. 4(b) and 4(a) illustrate the results for the two cases of having full-duplex clients and half-duplex clients, respectively. Note that the trend of the variations with respect to system parameters are similar in the two scenarios, while, as expected, network with full-duplex clients has higher throughput than that with half-duplex clients. Full-duplex clients are less sensitive to changes in the number of antennas, but start to lose their gain earlier when the number of users increases.

Despite the mutual interference between uplink transmis-

sions and downlink receptions for FD cellular setting, RF chain conserved FD results in considerable throughput gain as compared to the corresponding HD-MIMO system. However, increase in the number of users or in the number of antennas accentuates the severity of the mutual interference between the different directions of transmission and results in gradual descent in the gain achievable with FD.

#### IV. CONCLUSION

In this paper, the maximum theoretical capacity gains of utilizing FD radios are characterized for both point-to-point and point-to-multipoint communication systems. Results show that an FD radio formed by conserving the number of antennas (equal to that of the corresponding HD radio) will not provide capacity gains, unless the system has rate feedback and works at very low SNRs. This means to support higher gains by employing FD radios, it is necessary to place additional antennas on transceivers while conserving the number of RF-chains per radio. Such passive antenna elements do not increase the hardware complexity of the radio compared to the corresponding MIMO radio, while having similar computational complexity for the MIMO decoder as well. Nonetheless, in scheduled cellular setting, increasing the number of users or the number antennas reduces the total throughput gain of FD systems, although the gain remains substantial.

Further studies will be needed to characterize the performance of the FD scenario in point-to-multipoint setting when clients are equipped with multiple transmit and receive antennas. It will also be interesting to consider other practical schemes such as beamforming or precoding for multi-user communication and measure the gains achievable by full-duplexing.

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