Imitative Spectrum Access

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***Abstract*—In this paper, we study how secondary users can share the spectrum in a distributed fashion based on imitations. We propose an imitative spectrum access mechanism, where each secondary user estimates its expected throughput based on local observations, and imitates the channel selection of another user who achieves a higher throughput. We show that the imitative spectrum access mechanism converges to an imitation equilibrium, where no beneficial imitation can be further carried out on the time average. Numerical results show that when the number of users is large, the imitative spectrum access mechanism can converge to a Nash equilibrium, which is a special case of the imitation equilibrium.**

I. Introduction

Cognitive radio is envisioned as a promising technique to alleviate the problem of spectrum under-utilization [1]. It enables unlicensed wireless users (secondary users) to opportunistically access the licensed channels owned by legacy spectrum holders (primary users), and thus can significantly improve the spectrum efficiency [2].

A key challenge of the cognitive radio technology is how to share the spectrum resources in an intelligent fashion. Most of existing works in cognitive radio networks focus on exploring the *individual intelligence* of the secondary users. A common modeling approach is to consider that secondary users are *fully rational*, and model their interactions as noncooperative games (e.g., [3]–[6]). Nie and Comniciu in [4] designed a self-enforcing distributed spectrum access mechanism based on potential games. Niyato and Hossain in [5] studied a price-based spectrum access mechanism for competitive secondary users. Chen and Huang in [6] proposed a spatial spectrum access game framework for distributed spectrum sharing mechanism design with spatial reuse. The common assumption of all the above work is that secondary users adopt their channel selections based on best responses. To have full rationality, a user needs to have a high computational power to collect and analyze the network information in order to predict other users' behaviors. This is often not feasible due to the limitations of today's wireless devices.

In this paper, we will explore the *collective intelligence* of the secondary users based on social *imitation*. The motivation is to overcome the limited capability of today's wireless devices by leveraging the wisdom of secondary user crowds. Imitation is simple (following a successful action) and turns out to be an efficient strategy in many applications. Schlag in [7] used imitation to solve the multi-armed bandit problem. Lopes *et al.* in [8] designed an efficient imitation-based social learning mechanism for robots. Levine and Pesendorfer in [9] studied how to promote cooperation with imitation.

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Recently, Iellamo *et al.* in [10] proposed an imitation-based spectrum access mechanism for cognitive radio networks, by assuming that each secondary user has *complete information* of the channel characteristics and the available channel opportunities are equally shared among the users without any collision. In this paper, we relax this restrictive assumption and design an imitative spectrum access mechanism based on user's *local observations* such as the realized data rates and transmission collisions. The key idea is that each user applies the maximum likelihood estimation to estimate its expected throughput, and imitates another user's channel selection if that user's estimated throughput is higher. The main results and contributions of this paper are as follows:

- *Imitative Spectrum Access*: We propose a novel imitation-based distributed spectrum access mechanism, wherein each secondary user first estimates its expected throughput based on local observations, and chooses to imitate another better user.
- *Convergence to Imitation Equilibrium*: We show that the imitative spectrum access mechanism converges to the imitation equilibrium, wherein no beneficial imitation can be further carried out on the time average. Numerical results show that when the number of users is large, the imitative spectrum access mechanism converges to a Nash equilibrium, which is a special case of imitation equilibria.
- *Imitative Spectrum Access with User Heterogeneity*: We further design an imitation-based spectrum access mechanism with user heterogeneity, where different users achieve different data rates on the same channel. Numerical results show that the mechanism can also converge to an imitation equilibrium.

The rest of the paper is organized as follows. We introduce the system model and the maximum likelihood estimation in Sections II and III, respectively. We then present the imitative spectrum access mechanism and study its convergence in Sections IV and V, respectively. We next propose the imitative spectrum access mechanism with user heterogeneity in Section VI. We illustrate the performance of the proposed mechanisms in Section VII, and finally conclude in Section VIII.

II. System Model

We consider a cognitive radio network with a set $\mathcal{M} = \{1,2,...,M\}$ of independent and *stochastically heterogeneous* licensed channels. A set $\mathcal{N} = \{1,2,...,N\}$ of secondary users try to opportunistically access these channels, when the channels are not occupied by primary (licensed) transmissions. The system model has a slotted transmission structure as in Figure 1 and is described as follows.

- *Channel State*: the channel state for a channel m during a time slot is 0 if channel m is occupied by primary transmissions and 1 if channel m is idle.

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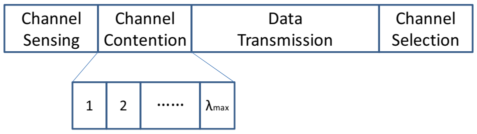

Fig. 1. Multiple stages in a single time slot.

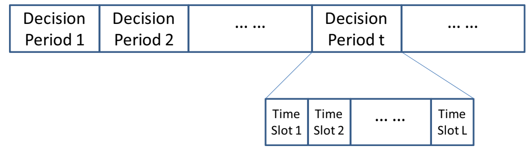

Fig. 2. The period structure of maximum likelihood estimation of various system parameters.

- *Channel State Changing*: for a channel m, we assume that the channel state is an i.i.d. Bernoulli random variable, with an idle probability $\theta_m \in (0, 1)$ and a busy probability $1-\theta_m$. This model can be a good approximation of the reality if the time slots for secondary transmissions are sufficiently long or the primary transmissions are highly bursty [11].
- *Heterogeneous Channel Throughput*: if a channel m is idle, the achievable data rate $b_m(\tau)$ by a secondary user in each time slot τ evolves according to an i.i.d. random process with a mean B_m, due to the local environmental effects such fading. All users achieve the same data rate on the same channel. In Section VI, we will further consider the heterogeneous user case.
- *Time Slot Structure*: each secondary user n executes the following stages synchronously during each time slot:
  - *Channel Sensing*: sense one of the channels based on the channel selection decision generated at the end of previous time slot. Access the channel if it is idle.
  - *Channel Contention*: use a backoff mechanism to resolve collisions when multiple secondary users access the same idle channel. The contention stage of a time slot is divided into $\lambda_{\max}$ mini-slots (see Figure 1), and user n executes the following two steps. *First*, count down according to a randomly and uniformly chosen integral backoff time (number of mini-slots) λ_n between 1 and $\lambda_{\max}$. *Second,* once the timer expires, transmit RTS/CTS messages if the channel is clear (i.e., no ongoing transmission). Note that if multiple users choose the same backoff value λ_n, a collision will occur with RTS/CTS transmissions and no users can successfully grab the channel.
  - *Data Transmission*: transmit data packets if the RTS/CTS message exchanges go through and the user successfully grabs the channel.
  - *Channel Selection*: choose a channel to access in the next time slot according to the imitative spectrum access mechanism (introduced in Section IV).

Suppose that k_m users choose an idle channel m to access. Then the probability that a user n (out of the k_m users) grabs the channel m is

$$\begin{aligned} g(k_m) &= Pr\{\lambda_n < \min_{i\neq n}\{\lambda_i\}\} \\ &= \sum_{\lambda=1}^{\lambda_{\max}} Pr\{\lambda_n = \lambda\} Pr\{\lambda < \min_{i\neq n}\{\lambda_i\} | \lambda_n = \lambda\} \\ &= \sum_{\lambda=1}^{\lambda_{\max}} \frac{1}{\lambda_{\max}} \left(\frac{\lambda_{\max} - \lambda}{\lambda_{\max}}\right)^{k_m - 1}, \end{aligned}$$

which is a decreasing function of the total contending users k_m. Then the expected throughput of a secondary user n choosing a channel m is given as

$$U_n = \theta_m B_m g(k_m). \tag{1}$$

Since our analysis is from secondary users' perspective, we will use terms "secondary user" and "user" interchangeably.

III. Expected Throughput Estimation

In order to imitate a successful action, a user needs to compare his and other users' performances (throughputs). In practice, many wireless devices only have a limited view of the network environment due to hardware constraints. To capture this reality, we first introduce the maximum likelihood estimation approach to estimate a user's expected throughput based on its local observations. To achieve an accurate estimation based on local observations, a user needs to gather a large number of observation samples. This motivates us to divide the spectrum access time into a sequence of *decision periods* indexed by $t(=1, 2, ...)$, where each decision period consists of L time slots (see Figure 2 for an illustration). During a single decision period, a user accesses the *same* channel in all L time slots. Thus the total number of users accessing each channel does not change within a decision period, which allows users to learn the environment.

According to (1), a user's expected throughput during decision period t depends on the probability of grabbing the channel $g(k_m(t))$ on that period, the channel idle probability θ_m, and the mean data rate B_m. Similarly to our work in [6] on the expected throughput estimation for distributed learning mechanism design, we will apply the maximum likelihood estimation (MLE) to get accurate estimations of there parameters for imitative spectrum access mechanism design, due to the efficiency and the ease of implementation of this method [12].

A. MLE of Channel Grabbing Probability $g(k_m(t))$

At the beginning of each time slot $l(= 1, ..., L)$ of a decision period t, we assume that a user n chooses to sense the same channel m. If the channel is idle, the user will contend to grab the channel according to the backoff mechanism in Section II. At the end of each time slot l, a user observes $S_n(t, l)$, $I_n(t, l)$, and $b_n(t, l)$. Here $S_n(t, l)$ denotes the state of the chosen channel (i.e., whether occupied by the primary traffic), $I_n(t, l)$ indicates whether the user has successfully grabbed the channel, i.e.,

$$I_n(t, l) = \begin{cases} 1, & \text{if user } n \text{ successfully grabs the channel} \\ 0, & \text{otherwise,} \end{cases}$$

and $b_n(t,l)$ is the received data rate on the chosen channel by user n at time slot l. At the end of each decision period t, each user n can collect a set of local observations $\Omega_n(t) = \{S_n(t,l), I_n(t,l), b_n(t,l)\}_{l=1}^{L}$. Note that if $S_n(t,l) = 0$ (i.e., the channel is occupied by the primary traffic), we set $I_n(t,l)$ and $b_n(t,l)$ to be 0.

When the channel m is idle (i.e., no primary traffic), $k_m(t)$ users contend for the channel according to the backoff mechanism in Section II. Then user n out of these $k_m(t)$ users grabs the channel with probability $g(k_m(t))$. Since there are a total of $\sum_{l=1}^{L} S_n(t,l)$ rounds of channel contentions in the period t and each round is independent, the total number of successful channel captures $\sum_{l=1}^{L} I_n(t,l)$ by user n follows the Binomial distribution. User n can then compute the likelihood of $g(k_m(t))$, i.e., the probability of the realized observations $\Omega_n(t)$ given the parameter $g(k(t))$ as

$$\mathcal{L}[\Omega_n(t)|g(k_m(t))] = \binom{\sum_{l=1}^{L} S_n(t,l)}{\sum_{l=1}^{L} I_n(t,l)} g(k_m(t))^{\sum_{l=1}^{L} I_n(t,l)} \times (1-g(k_m(t)))^{\sum_{l=1}^{L} S_n(t,l)-\sum_{l=1}^{L} I_n(t,l)}.$$

Then MLE of $g(k_m(t))$ can be computed by maximizing the log-likelihood function $\ln \mathcal{L}[\Omega_n(t)|g(k_m(t))]$, i.e., $\max_{g(k_m(t))} \ln \mathcal{L}[\Omega_n(t)|g(k_m(t))]$. By the first order condition, we obtain the optimal solution as $\tilde{g}(k_m(t)) = \sum_{l=1}^{L} I_n(t,l)/\sum_{l=1}^{L} S_n(t,l)$, which is the sample averaging estimation. When the length of decision period L is large, by the central limit theorem, we know that $\tilde{g}(k_m(t)) \sim \mathcal{N}\left(g(k_m(t)), \frac{g(k_m(t))(1-g(k_m(t)))}{\sum_{l=1}^{L} S_n(t,l)}\right)$, where $\mathcal{N}(\cdot)$ denotes the normal distribution.

B. MLE of Channel Idle Probability θ_m

We next apply the MLE to estimate the channel idle probability θ_m. Since the channel state $S_n(t,l)$ is i.i.d over different time slots and different decision periods, we can improve the estimation by averaging not only over multiple time slots but also over multiple periods.

Similarly with MLE of $g(k_m(t))$, we first compute one-period MLE of θ_m as $\hat{\theta}_m = \frac{\sum_{l=1}^{L} S_n(t,l)}{L}$. When the length of decision period L is large, we have that $\hat{\theta}_m$ follows the normal distribution with the mean θ_m, i.e., $\hat{\theta}_m \sim \mathcal{N}\left(\theta_m, \frac{\theta_m(1-\theta_m)}{L}\right)$.

We then average the estimation over multiple decision periods. When a user n finishes accessing a channel m for a total C periods, it updates the estimation of the channel idle probability θ_m as $\tilde{\theta}_m(C) = \frac{1}{C}\sum_{i=1}^{C} \hat{\theta}_m(i)$, where $\tilde{\theta}_m(C)$ is the estimation of θ_m based on the information of all C decision periods, and $\hat{\theta}_m(i)$ is the one-period estimation. By doing so, we have $\tilde{\theta}_m(C) \sim \mathcal{N}\left(\theta_m, \frac{\theta_m(1-\theta_m)}{CL}\right)$, which reduces the variance of one-period MLE by a factor of C.

C. MLE of Mean Data Rate B_m

Since the received data rate $b_n(t,l)$ is also i.i.d over different time slots and different decision periods, similarly with the MLE of the channel idle probability θ_m, we can obtain the one-period MLE of mean data rate B_m as $\hat{B}_m = \frac{\sum_{l=1}^{L} b_n(t,l)}{\sum_{l=1}^{L} I_n(t,l)}$, and the averaged MLE estimation over C periods as $\tilde{B}_m(C) = \frac{1}{C}\sum_{i=1}^{C} \hat{B}_m(i)$.

By the MLE, we can obtain the estimation of $g(k(t))$, θ_m, and B_m as $\tilde{g}(k_m(t))$, $\tilde{\theta}_m$, and $\tilde{B}_m$, respectively, and then estimate the true expected throughput $U_n(t) = \theta_m B_m g(k(t))$ as $\tilde{U}_n(t) = \tilde{\theta}_m \tilde{B}_m \tilde{g}(k_m(t))$. Since $\tilde{g}(k_m(t))$, $\tilde{\theta}_m$ and $\tilde{B}_m$ follow independent normal distributions with the mean $g(k_m(t))$, θ_m and B_m, respectively, we thus have $E[\tilde{U}_n(t)] = E[\tilde{\theta}_m \tilde{B}_m \tilde{g}(k_m(t))] = U_n(t)$, i.e., the estimation of expected throughput $U_n(t)$ is unbiased. In the following analysis, we hence assume that

$$\tilde{U}_n(t) = U_n(t) + \omega_n, \tag{2}$$

where $\omega_n \in (\underline{\omega}, \overline{\omega})$ is the random estimation noise with the probability density function $f(\omega)$ satisfying

$$f(\omega) > 0, \forall \omega \in (\underline{\omega}, \overline{\omega}), \tag{3}$$

$$E[\omega_n] = \int_{\underline{\omega}}^{\overline{\omega}} \omega f(\omega) d\omega = 0. \tag{4}$$

IV. Imitative Spectrum Access Mechanism

We now apply the idea of imitation to design an efficient distributed spectrum access mechanism, which utilizes user's local estimation of its expected throughput. We will show that the proposed imitative spectrum access mechanism can converge to an imitation equilibrium on the time average.

A. Imitative Spectrum Access

We apply the principle of imitation for distributed spectrum access mechanism design and propose the imitative spectrum access mechanism in Algorithm 1. The key idea is to let users imitate the actions of those users that achieve a higher throughput (i.e., Lines 11 to 14 in Algorithm 1). Such a mechanism based on local throughput comparison is simple and easy to implementation. Each user n at each period t first collects the local observations $\Omega_n(t) = \{S_n(t,l), I_n(t,l), b_n(t,l)\}_{l=1}^{L}$ (i.e., Lines 5 to 8 in Algorithm 1) and estimates its expected throughput with the MLE method as introduced in Section III (i.e., Line 9 in Algorithm 1). Then user n tries to carry out the imitation by randomly sampling another user's estimated throughput (i.e., Line 10 in Algorithm 1). Such a random sampling can be achieved in different ways. For example, user n can randomly generate an integer user ID n' from the set $\mathcal{N}\backslash\{n\}$ and broadcast a throughput enquiry packet including the enquired user ID n' over a common control channel[1]. Then the user n' will send back an acknowledgement packet including its locally estimated expected throughput to user n. Note that we are considering social spectrum sharing in this paper, and thus users are willing to share their local throughput information when being asked. The mutual reciprocity is a common phenomenon in many social activities [14].

B. Dynamics of Imitative Spectrum Access

We next study the evolution dynamics of the imitative spectrum mechanism. For the ease of exposition, we will focus

[1]Please refer to [13] for the details on how to set up and maintain a reliable common control channel in cognitive radio networks.

Algorithm 1 Imitative Spectrum Access

initialization:
  choose a channel a_n randomly for each user n.
end initialization

loop for each decision period t and each user n in parallel:
  for each time slot l in the period t **do**
    sense and **contend** to access the channel a_n.
    record the observations $S_n(t,l)$, $I_n(t,l)$ and $b_n(t,l)$.
  end for
  estimate the expected throughput $\tilde{U}_n(t)$.
  select another user n' randomly and **enquiry** its estimated throughput $\tilde{U}_{n'}(t)$.
  if $\tilde{U}_{n'}(t) > \tilde{U}_n(t)$ **then**
    choose channel $a_{n'}$ (i.e., the one chosen by user n') in the next period.
  else choose the original channel in the next period.
  end if
end loop

on the case that the number of users N is large. Numerical results show that the observations also hold for the case that the number of users is small (see Section VII-C for details).

In a large user population, it is convenient to use the population state $\boldsymbol{x}(t)$ to describe the dynamics of spectrum access. Here $\boldsymbol{x}(t) \triangleq (x_1(t), ..., x_M(t))$, and $x_m(t)$ is the fraction of users in the population choosing channel m to access at decision period t.

In the imitative spectrum access mechanism, each user n relies on its estimated expected throughput $\tilde{U}_n(t)$ to decide whether to imitate other user's channel selection. Due to the random estimation noise ω_n, the evolution of the population state $\{\boldsymbol{x}(t), \forall t \geq 0\}$ is hence stochastic and difficult to analyze directly. However, when the population of users N is large, due to the law of large number, such stochastic process can be well approximated ("averaged out") by its mean-field deterministic trajectory $\{\boldsymbol{X}(t), \forall t \geq 0\}$ [15]. Here $\boldsymbol{X}(t) \triangleq (X_1(t), ...X_M(t))$ is the deterministic population state. Let $P_i^j(\boldsymbol{X}(t))$ denote the probability that a user choosing channel i in the deterministic population state $\boldsymbol{X}(t)$ will choose channel j in next period. According to [15], we have

Lemma 1. *There exists a scalar δ, such that for any bound $\epsilon > 0$, decision period $t > 0$, and any large enough population size N, the maximum difference between the stochastic and deterministic population states over all periods is upper-bounded by ϵ with an arbitrary large enough probability, i.e.,*

$$Pr\{\max_{0 \leq \tau \leq t} \max_{m \in \mathcal{M}} |X_m(\tau) - x_m(\tau)| \leq \epsilon\} \geq 1 - e^{-\epsilon^2 \delta N}, \quad (5)$$

given that $\boldsymbol{X}(0) = \boldsymbol{x}(0)$ and the evolution of the deterministic population state $\{\boldsymbol{X}(t), \forall t \geq 0\}$ satisfies

$$X_j(t+1) = \sum_{i=1}^{M} X_i(t) P_i^j(\boldsymbol{X}(t)), \forall j \in \mathcal{M}. \quad (6)$$

The proof is similar to that in [15] and hence is omitted here. As illustrated in Figure 3, Lemma 1 indicates that the

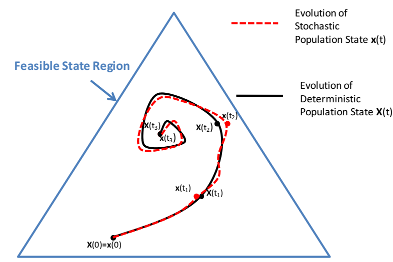

Fig. 3. Illustration of the approximation of stochastic population state $\boldsymbol{x}(t)$ by deterministic population state $\boldsymbol{X}(t)$

trajectory of the stochastic population state $\{\boldsymbol{x}(t), \forall t \geq 0\}$ is within a small neighborhood of the trajectory of the deterministic population state $\{\boldsymbol{X}(t), \forall t \geq 0\}$ when the user population N is large enough. Moreover, since the MLE is unbiased, if the mean-field deterministic dynamics $\{\boldsymbol{X}(t), \forall t \geq 0\}$ converge to an equilibrium, the stochastic dynamics $\{\boldsymbol{x}(t), \forall t \geq 0\}$ must also converge to the same equilibrium on the time average [15].

We now study the evolution dynamics of the deterministic population state $\{\boldsymbol{X}(t), \forall t \geq 0\}$ in (6). Let $U(m, \boldsymbol{X}(t)) = \theta_m B_m g(N X_m(t))$ denote the expected throughput of a user that chooses channel m in the population state $\boldsymbol{X}(t)$. Recall that in the imitative spectrum access mechanism, each user will randomly choose another user to imitate (if that user achieves a higher throughput). Thus, we can obtain the probability $P_i^j(\boldsymbol{X}(t))$ that a user n choosing channel i will imitate another user n' on channel j in the next period as

$$P_i^j(\boldsymbol{X}(t)) = X_j(t) Pr\{\tilde{U}(j, \boldsymbol{X}(t)) > \tilde{U}(i, \boldsymbol{X}(t))\}. \quad (7)$$

From (2), we have

$$\begin{aligned} &\tilde{U}(j, \boldsymbol{X}(t)) - \tilde{U}(i, \boldsymbol{X}(t)) \\ &\qquad = U(j, \boldsymbol{X}(t)) - U(i, \boldsymbol{X}(t)) + \omega_{n'} - \omega_n, \end{aligned} \quad (8)$$

where $\omega_n, \omega_{n'}$ are the random estimation noises with the probability density function $f(\omega)$. Let $\varpi = \omega_n - \omega_{n'}$, and we can obtain the probability density function of ϖ as

$$q(\varpi) = \int_{\underline{\omega}}^{\overline{\omega}} f(\omega) f(\varpi + \omega) d\omega. \quad (9)$$

We further denote the cumulative distribution function ϖ as $Q(\varpi)$, i.e., $Q(\varpi) = \int_{-\infty}^{\varpi} q(s) ds$. Then from (7) and (8), we have for any $j \neq i$,

$$P_i^j(\boldsymbol{X}(t)) = X_j(t) Q(U(j, \boldsymbol{X}(t)) - U(i, \boldsymbol{X}(t))), \quad (10)$$

and

$$P_i^i(\boldsymbol{X}(t)) = 1 - \sum_{j \neq i} X_j(t) Q(U(j, \boldsymbol{X}(t)) - U(i, \boldsymbol{X}(t))). \quad (11)$$

Based on (6), (10), and (11), we can obtain the evolution dynamics of the deterministic population state $\{\boldsymbol{X}(t)\}$ as

Theorem 1. *For the imitative spectrum access mechanism, the evolution dynamics of deterministic population state $\{\boldsymbol{X}(t), \forall t \geq 0\}$ are*

$$\begin{aligned} \dot{X}_m(t) = X_m(t) \sum_{i=1}^{M} X_i(t) \big(&Q(U(m, \boldsymbol{X}(t)) - U(i, \boldsymbol{X}(t)) \\ &- Q(U(i, \boldsymbol{X}(t)) - U(m, \boldsymbol{X}(t))\big), \end{aligned} \quad (12)$$

where the derivative is with respect to time t.

Proof: From (6), (10), and (11), we have

$$\begin{aligned}\dot{X}_m(t) &= X_m(t+1) - X_m(t)\\ =&\sum_{i\neq m} X_i(t)P_i^m(\boldsymbol{X}(t)) - (1-P_m^m(\boldsymbol{X}(t)))X_m(t)\\ =&X_m(t)\sum_{i\neq m} X_i(t)Q(U(m,\boldsymbol{X}(t)) - U(i,\boldsymbol{X}(t)))\\ &-X_m(t)\sum_{j\neq m} X_j(t)Q(U(j,\boldsymbol{X}(t)) - U(m,\boldsymbol{X}(t))),\end{aligned}$$

which completes the proof. ■

V. Convergence of Imitative Spectrum Access

We now study the convergence of the imitative spectrum access mechanism. Let $\boldsymbol{x}^* \triangleq (x_1^*, ..., x_M^*)$ denote the equilibrium of the imitative spectrum access, and a_n^* denote the channel chosen by user n in the equilibrium $\boldsymbol{x}^*$. We first introduce the definition of *imitation equilibrium.*

Definition 1 (**Imitation Equilibrium**). *A population state $\boldsymbol{x}^* = (x_1^*, ..., x_M^*)$ is an imitation equilibrium if and only if for each user $n \in \mathcal{N}$,*

$$U(a_n^*, \boldsymbol{x}^*) \geq \max_{a\in\Delta(\boldsymbol{x}^*)\backslash\{a_n^*\}} U(a, \boldsymbol{x}^*), \tag{13}$$

where $\Delta(\boldsymbol{x}^*) \triangleq \{m \in \mathcal{M} : x_m^* > 0\}$ denotes the set of channels chosen by users in the equilibrium $\boldsymbol{x}^*$.

At an imitation equilibrium, no user can further improve its expected throughput by imitating another user. For the imitative spectrum access mechanism, we show that

Theorem 2. *For the imitative spectrum access mechanism, the evolution dynamics of deterministic population state $\{\boldsymbol{X}(t), \forall t \geq 0\}$ asymptotically converge to an imitation equilibrium $\boldsymbol{X}^*$ such that all users achieve the same expected throughput, i.e.,*

$$U(a_n^*, \boldsymbol{X}^*) = U(a_{n'}^*, \boldsymbol{X}^*), \forall n, n' \in \mathcal{N}. \tag{14}$$

Proof: To proceed, we first define the following function

$$V(\boldsymbol{X}(t)) = \sum_{m=1}^{M} \int_{-\infty}^{X_m(t)} \theta_m B_m g(Nz)dz. \tag{15}$$

We then consider the variation of $V(\boldsymbol{X}(t))$ along the evolution trajectory of deterministic population state $\{\boldsymbol{X}(t)\}$ in (12), i.e., differentiating $V(\boldsymbol{X}(t))$ with respective to time t,

$$\begin{aligned}\frac{dV(\boldsymbol{X}(t))}{dt} &= \sum_{m=1}^{M} \frac{dV(\boldsymbol{X}(t))}{dX_m(t)}\frac{dX_m(t)}{dt}\\ =&\sum_{m=1}^{M} \theta_m B_m g(NX_m(t))\frac{dX_m(t)}{dt}\\ =&\sum_{m=1}^{M} U(m,\boldsymbol{X}(t))X_m(t)\sum_{i=1}^{M} X_i(t)\\ &\times\Big(Q(U(m,\boldsymbol{X}(t)) - U(i,\boldsymbol{X}(t))\\ &- Q(U(i,\boldsymbol{X}(t)) - U(m,\boldsymbol{X}(t))\Big)\\ =&\frac{1}{2}\sum_{m=1}^{M}\sum_{i=1}^{M} X_m(t)X_i(t)\\ &\times(U(m,\boldsymbol{X}(t)) - U(i,\boldsymbol{X}(t)))\\ &\times\Big(Q(U(m,\boldsymbol{X}(t)) - U(i,\boldsymbol{X}(t))\\ &- Q(U(i,\boldsymbol{X}(t)) - U(m,\boldsymbol{X}(t))\Big).\end{aligned} \tag{16}$$

Since $f(\omega)$ is a probability density function satisfying $f(\omega) > 0, \forall\omega \in (\underline{\omega}, \overline{\omega})$ and $\int_{\underline{\omega}}^{\overline{\omega}} f(\omega)d\omega = 1$, it follows from (9) that $q(\varpi) > 0, \forall\varpi \in (\underline{\omega}-\overline{\omega}, \overline{\omega}-\underline{\omega})$ and $q(\varpi) = 0, \forall\varpi \notin (\underline{\omega}-\overline{\omega}, \overline{\omega}-\underline{\omega})$. Hence the cumulated probability function $Q(\varpi) = \int_{-\infty}^{\varpi} q(s)ds$ is strictly increasing for any $\varpi \in (\underline{\omega}-\overline{\omega}, \overline{\omega}-\underline{\omega})$, and further $Q(\varpi) = 0, \forall\varpi \in (-\infty, \underline{\omega}-\overline{\omega})$ and $Q(\varpi) = 1, \forall\varpi \in (\overline{\omega}-\underline{\omega}, +\infty)$. This implies that if $U(m,\boldsymbol{X}(t)) \neq U(i,\boldsymbol{X}(t)$,

$$\begin{aligned}&(U(m,\boldsymbol{X}(t)) - U(i,\boldsymbol{X}(t)))\\ &\times\Big(Q(U(m,\boldsymbol{X}(t)) - U(i,\boldsymbol{X}(t))\\ &- Q(U(i,\boldsymbol{X}(t)) - U(m,\boldsymbol{X}(t))\Big) > 0,\end{aligned} \tag{17}$$

and if $U(m,\boldsymbol{X}(t)) = U(i,\boldsymbol{X}(t)$,

$$\begin{aligned}&(U(m,\boldsymbol{X}(t)) - U(i,\boldsymbol{X}(t)))\\ &\times\Big(Q(U(m,\boldsymbol{X}(t)) - U(i,\boldsymbol{X}(t))\\ &- Q(U(i,\boldsymbol{X}(t)) - U(m,\boldsymbol{X}(t))\Big) = 0.\end{aligned} \tag{18}$$

From (16), (17), and (18), we have $\frac{dV(\boldsymbol{X}(t))}{dt} \geq 0$. Hence $V(\boldsymbol{X}(t))$ is non-decreasing along the trajectory of the dynamics (12). According to [16], the evolution dynamics of deterministic population state $\{\boldsymbol{X}(t), \forall t \geq 0\}$ asymptotically converge to a limit point $\boldsymbol{X}^*$ such that $\frac{dV(\boldsymbol{X}^*)}{dt} = 0$, i.e.,

$$\begin{aligned}&X_m^* X_i^* \left(U(m,\boldsymbol{X}^*) - U(i,\boldsymbol{X}^*)\right)\\ &\times\Big(Q(U(m,\boldsymbol{X}^*) - U(i,\boldsymbol{X}^*) - Q(U(i,\boldsymbol{X}^*) - U(m,\boldsymbol{X}^*)\Big) = 0.\end{aligned} \tag{19}$$

By (18), we must have $U(m,\boldsymbol{X}^*) = U(i,\boldsymbol{X}^*), \forall m, i \in \Delta(\boldsymbol{x}^*)$. Since $a_n^*, a_{n'}^* \in \Delta(\boldsymbol{x}^*)$, we thus have $U(a_n^*, \boldsymbol{X}^*) = U(a_{n'}^*, \boldsymbol{X}^*)$. ■

According to Lemma 1, we know that the stochastic imitative spectrum access dynamics $\{\boldsymbol{x}(t), \forall t \geq 0\}$ will be attracted into a small neighborhood around the imitation equilibrium $\boldsymbol{X}^*$. Moreover, since the imitation equilibrium $\boldsymbol{X}^*$ is also the mean field equilibrium of stochastic dynamics $\{\boldsymbol{x}(t), \forall t \geq 0\}$, the stochastic dynamics $\{\boldsymbol{x}(t), \forall t \geq 0\}$ hence converge to the imitation equilibrium $\boldsymbol{X}^*$ on the time average, i.e., $\lim_{t\to\infty} \frac{\sum_{\tau=1}^{t} x_m(\tau)}{t} = X_m^*, \forall m \in \mathcal{M}$.

Interestingly, when the set of chosen channels is the set of all channels, i.e., $\Delta(\boldsymbol{X}^*) = \mathcal{M}$, it is easy to verify that the imitation equilibrium $\boldsymbol{X}^*$ is a Nash equilibrium, since no user can improve its payoff by changing its channel unilaterally. Numerical results show that when the number of users N is large enough, the imitative spectrum access mechanism always converges to a Nash equilibrium. The reason is that the probability that a channel is not chosen by any users equals to $(M-1)^{-N}$, which decreases exponentially with the number of users N. On the other hand, when the number of users N is small, numerical results show that the imitative spectrum access mechanism also converges to an imitation equilibrium satisfying the condition in (13) on the time average. To understand this, we can consider the evolution of user distribution on channels as a Markov process. Then any user distribution that satisfies the definition of imitation equilibrium in (13) is an absorbing state.

VI. Imitative Spectrum Access With User Heterogeneity

For the ease of exposition, we have considered the case that users are homogeneous, i.e., different users achieve the same data rate on the same channel. We now consider the general heterogeneous case where different users may achieve different data rates on the same channel.

Let $b_m^n(\tau)$ be the realized data rate of user n on an idle channel m at a time slot τ, and B_m^n be the mean data rate of user n on the idle channel m, i.e., $B_m^n = E[b_m^n(\tau)]$. In this case, the expected throughput of user n is given as $U_n^m = \theta_m B_m^n g(k_m)$. For imitative spectrum access mechanism in Algorithm 1, each user carries out the channel imitation by comparing its throughput with the throughput of another user. However, such throughput comparison may not be feasible when users are heterogeneous, since a user may achieve a low throughput on a channel that offers a high throughput for another user.

To address this issue, we propose a new imitative spectrum access mechanism with user heterogeneity in Algorithm 2. More specifically, when a user n on a channel m randomly selects another user n' on another channel m', user n' informs user n about the estimated channel grabbing probability $\tilde{g}(k_{m'})$ instead of the estimated expected throughput. Then user n will compute the estimated expected throughput on channel m' as

$$\tilde{U}_n^{m'} = \tilde{\theta}_{m'} \tilde{B}_{m'}^n \tilde{g}(k_{m'}). \tag{20}$$

If $\tilde{U}_n^{m'} > \tilde{U}_n^m$, then user n will imitate the channel selection of user n'.

To implement the mechanism above, each user n must have the information of its own estimated channel idle probability $\tilde{\theta}_{m'}$ and data rate $\tilde{B}_{m'}^n$ of the unchosen channel m'. Hence we add an initial channel estimation stage in the imitative spectrum access mechanism in Algorithm 2. In this stage, each user initially estimates the channel idle probability $\tilde{\theta}_m$ and data rate $\tilde{B}_m^n$ by accessing all the channels in a randomized round-robin manner. This ensures that all users do not choose the same channel at the same period. Let $\mathcal{M}_n$ (equals to the empty set $\oslash$ initially) be set of channels probed by user n and $\mathcal{M}_n^c = \mathcal{M} \backslash \mathcal{M}_n$. At beginning of each decision period, user n randomly chooses a channel $m \in \mathcal{M}_n^c$ (i.e., a channel that has not been accessed before) to access. At end of the period, user n can estimate the channel idle probability $\tilde{\theta}_m$ and data rate $\tilde{B}_m^n$ according to the MLE method introduced in Section III. Note that there are L time slots within each decision period, and thus the user will be able to have a fairly good estimation if L is reasonably large. Then user n updates the set of probed channels as $\mathcal{M}_n = \mathcal{M}_n \cup \{m\}$. When all the channels are probed, i.e., $\mathcal{M}_n = \mathcal{M}$, the stage of initial channel estimation ends. Thus, the total time slots for this stage is ML.

Algorithm 2 Imitative Spectrum Access With User Heterogeneity

initialization:
  choose a channel a_n randomly for each user n.
end initialization

loop for each user $n \in \mathcal{N}$ in parallel:
▷ ***Initial Channel Estimation Stage***
  while $\mathcal{M}_n \neq \mathcal{M}$ **do**
    choose a channel m from the set $\mathcal{M}_n^c$ randomly.
    sense and **contend** to access the channel m at each time slot of the decision period.
    record the observations $S_n(t,l)$, $I_n(t,l)$ and $b_n(t,l)$.
    estimate the channel idle probability $\tilde{\theta}_m$ and data rate $\tilde{B}_m^n$.
    set $\mathcal{M}_n = \mathcal{M}_n \cup \{m\}$.
  end while

▷ ***Imitative Spectrum Access Stage***
  for each time period t **do**
    sense and **contend** to access the channel m at each time slot of the decision period.
    record the observations $S_n(t,l)$, $I_n(t,l)$ and $b_n(t,l)$.
    estimate the expected throughput $\tilde{U}_n^{a_n}(t)$.
    select another user n' randomly and **enquiry** its channel grabbing probability $\tilde{g}(k_{a_{n'}})$.
    estimate the expected throughput $\tilde{U}_n^{a_{n'}}(t)$ based on (20).
    if $\tilde{U}_n^{a_{n'}}(t) > \tilde{U}_n^{a_n}(t)$ **then**
      choose channel $a_{n'}$ (i.e., the one chosen by user n') in the next period.
    else choose the original channel in the next period.
    end if
  end for
end loop

Numerical results show that the proposed imitative spectrum access mechanism with user heterogeneity can still converge to an imitation equilibrium satisfying the definition in (13), i.e., no user can further improve its expected throughput by imitating another user.

VII. Simulation Results

In this section, we evaluate the proposed imitative spectrum access mechanisms by simulations. We first implement the imitative spectrum access mechanism with user homogeneity (i.e., Algorithm 1). We consider a cognitive radio network consisting $M = 5$ Rayleigh fading channels. The channel idle probabilities $\{\theta_m\}_{m=1}^M = \{\frac{2}{3}, \frac{4}{7}, \frac{5}{9}, \frac{1}{2}, \frac{4}{5}\}$. The data rate on an idle channel m is computed according to the Shannon capacity, i.e., $b_m(\tau) = \zeta_m \log_2(1 + \frac{\eta_n g_m(\tau)}{n_0})$, where ζ_m is the bandwidth of channel m, η_n is the power adopted by uses, n_0 is the noise power, and $g_m(\tau)$ is the channel gain (a realization of a random variable that follows the exponential distribution

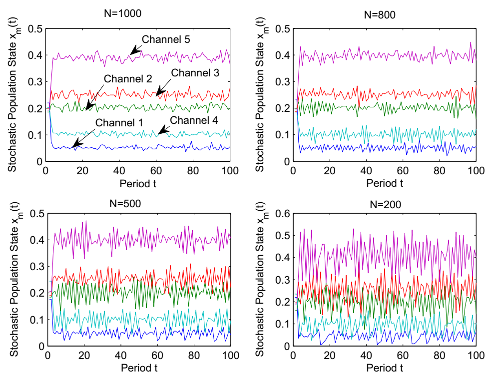

Fig. 4. Evolution of stochastic population state $\boldsymbol{x}(t)$ with user populations $N = 1000, 800, 500$, and 200, and the number of number of backoff mini-slots $\lambda_{\max} = 5000$

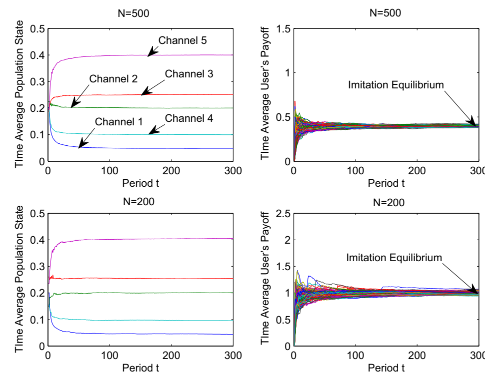

Fig. 5. Evolution of time average population state and user's payoff with user population $N = 500$ and 200, and the number of number of backoff mini-slots $\lambda_{\max} = 5000$

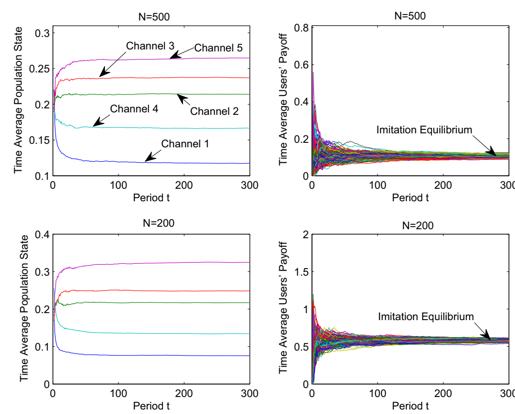

Fig. 6. Evolution of time average population state and user's payoff with user population $N = 500$ and 200, and the number of number of backoff mini-slots $\lambda_{\max} = 50$

with mean $\bar{g}_m$). In the following simulations, we set $\zeta_m = 10$ MHz, $n_0 = -100$ dBm, and $\eta_n = 100$ mW. By choosing different mean value $\bar{g}_m$, we have different mean data rates $\{B_m = E[b_m(\tau)]\}_{m=1}^{M} = \{15, 70, 90, 40, 100\}$ Mbps. We first implement the imitation spectrum access mechanism in cases with both large and small user populations.

A. Large User Populations With Large $\lambda_{\max}$

We first consider the case that the user population N is large. For the channel contention, we set the number of backoff mini-slots $\lambda_{\max} = 5000$, which is larger than the number of users N. This can be approximated by the asymptotic case $\lambda_{\max} \to \infty$, and the probability of grabbing the chosen channel $g(k) = \lim_{\lambda_{\max}\to\infty} \sum_{\lambda=1}^{\lambda_{\max}} \frac{1}{\lambda_{\max}} \left(\frac{\lambda_{\max}-\lambda}{\lambda_{\max}}\right)^{k-1} = \int_0^1 z^{k-1} dz = \frac{1}{k}$. According to (14), we can obtain the close-form solution of the imitation equilibrium $\boldsymbol{X}^*$ as

$$X_m^* = \begin{cases} \frac{\theta_m B_m}{\sum_{i\in\triangle(\boldsymbol{X}^*)} \theta_i B_i}, & \text{if } m \in \triangle(\boldsymbol{X}^*), \\ 0, & \text{if } m \notin \triangle(\boldsymbol{X}^*), \end{cases} \tag{21}$$

which provides a benchmark for the simulations. We set the length of the decision period $L = 500$, which achieves a good estimation of the expected throughput.

We implement the imitative spectrum access mechanism with the number of users $N = 1000, 800, 500$, and 200, respectively. Figure 4 shows the evolution dynamics of the stochastic population state $\boldsymbol{x}(t)$, which converge to a bounded neighborhood of the imitation equilibrium $\boldsymbol{X}^* = \left(\frac{\theta_1 B_1}{\sum_{i=1}^{M} \theta_i B_i}, ..., \frac{\theta_M B_M}{\sum_{i=1}^{M} \theta_i B_i}\right) = (0.05, 0.2, 0.25, 0.1, 0.4)$ in all cases. Moreover, as the number of users N increases, the amplitude of fluctuation (i.e., the size of neighborhood) around the imitation equilibrium $\boldsymbol{X}^*$ decreases. Since the set of chosen channels $\triangle(\boldsymbol{X}^*) = \mathcal{M}$, we know that the imitation equilibrium $\boldsymbol{X}^*$ is also a Nash equilibrium.

Figure 5 shows two population choices, $N = 500$ and 200. In both cases, the time average population state $\boldsymbol{x}(t)$ converges to the imitation equilibrium $\boldsymbol{X}^*$, and the time average user's estimated throughput $\tilde{U}_n(t)$ converges to the imitation equilibrium wherein all users achieve the same throughput as given in (14).

B. Large User Populations With Small $\lambda_{\max}$

We next implement the imitative spectrum access mechanism with the number of users $N = 500$ and 200, and the number of backoff mini-slots $\lambda_{\max} = 50$, which is much smaller than the number of users N. In this case, severe collisions in channel contention may occur and hence lead to a reduction in data rates for all users. The results are shown in Figure 6. We see that a small $\lambda_{\max}$ leads to a system performance loss (i.e., $\sum_{n=1}^{N} U_n < \sum_{m=1}^{M} \theta_m B_m$). However, the imitative spectrum access mechanism still converges to the imitation equilibrium such that all users achieve the same expected throughput. This verifies the effectiveness of the mechanism in the small $\lambda_{\max}$ case.

C. Small User Populations

We then consider the case that the user population N is small. We implement the imitative spectrum access mechanism with the number of users $N = 20$, 5, and 3, respectively. We show the evolution of time average population state $\boldsymbol{x}(t)$ and time average user's estimated throughput $\tilde{U}_n(t)$ in Figure 7. We observe similar results as in the large population case in Figure 5. For example, when $N = 20$ we have $\triangle(\boldsymbol{X}^*) = \{3, 5\}$ and all the users achieve the same average throughput

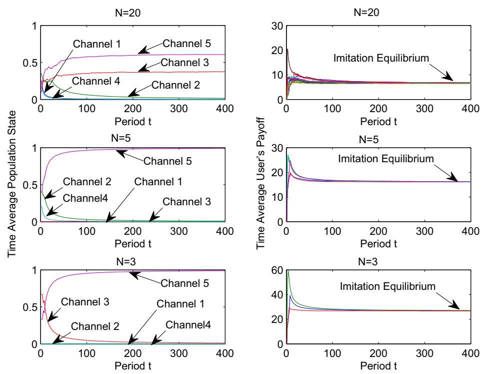

Fig. 7. Evolution of time average population state and user's payoff with user population $N = 20$, 5, and 3, and the number of number of backoff mini-slots $\lambda_{\max} = 50$

at the imitation equilibrium. This verifies the effectiveness of Theorem 2 in the small user population case.

When the number of users is small, we also observe that a subset of channels are utilized in the imitation equilibrium. This is because each user imitates the choice of another user, and thus one user's choice could have a significant impact in a small population. In other words, the gain of diversity is weaken when user population is small. To further improve the system performance in the small user population case, we can increase the gain of diversity by adding "innovation" (i.e., random channel exploration) into the imitative spectrum access mechanism. The design of the imitative spectrum access mechanism with innovation will be considered in a future work.

D. Imitative Spectrum Access With User Heterogeneity

We then implement the imitation spectrum access mechanism with user heterogeneity (i.e., Algorithm 2). The number of users $N = 500$ and 200, and the number of backoff mini-slots $\lambda_{\max} = 50$. We set the user specific data rate as $b_m^n(\tau) = h_n b_m(\tau)$, where h_n is the user specific transmission gain. In this case, we have user specific mean data rate as $B_m^n = h_n B_m$. In this simulation, the transmission gain h_n of each user n is randomly assigned from the set $\{2.0, 1.0\}$. The results are shown in Figure 8. The mechanism converges to the equilibrium wherein users of the same transmission gain achieve the same expected throughput and users of different transmission gains may achieve different expected throughputs. Moreover, we observe that the user distribution on channels in Figure 8 is the same as that in Figure 6. This implies that no user can further improve its expected throughput by imitating another user. That is, the equilibrium is an imitation equilibrium satisfying the definition in (13).

VIII. Conclusion

In this paper, we design imitation-based spectrum access mechanisms with user homogeneity and heterogeneity. We show that the proposed imitative spectrum access mechanisms can converge to an imitation equilibrium on the time average. Numerical results demonstrate that, when the number of users

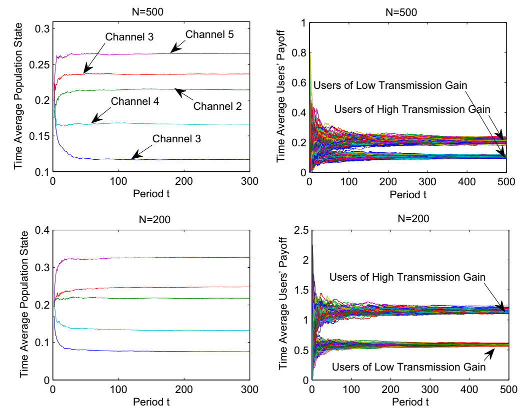

Fig. 8. Imitation spectrum access mechanism with user heterogeneity with user population $N = 500$ and 200, and the number of number of backoff mini-slots $\lambda_{\max} = 50$

is large, the convergent imitation equilibrium is equivalent to a Nash equilibrium.

An interesting direction of extending this paper is to take the spatial reuse into account. How to design an efficient imitation based spectrum access mechanism such that each users can only imitate neighboring users' channel selections is very challenging.

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