Designing Incentives for Wireless Relay Networks using Tokens

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***Abstract*—This paper proposes a novel system design for wireless relay networks formed of self-interested users that relies on token exchanges. Our emphasis in this paper is on developing optimal designs for token systems to be deployed in relay networks. The optimal designs aim to maximize the probability that the relay transmission will be executed by transceivers whenever they are requested to provide such services. We prove that the efficiency of the relay network heavily depends on issuing the optimal amount of tokens rather than an arbitrary amount. We formulate the design problem of the token system as a bi-level optimization problem. In the inner level optimization problem, we determine the transceivers' incentive-compatible strategies (i.e. the strategies that maximize the transceivers' own utilities). We prove that these strategies exhibit a simple threshold structure. The outer level problem determines the optimal token amount, which maximizes the overall relay network efficiency. We prove that the optimal amount of tokens tokens needs to be neither too small nor too large and depends on the threshold that the self-interested transceivers adopt in the inner level problem.**

I. Introduction

In many wireless scenarios, the transmission between two distant transceivers may have to be accomplished with the help of a relay node, due to degraded channel conditions and power constraints. The concept of relaying has been adopted in many wireless infrastructures to improve the transmission quality, such as multi-hop ad hoc networks, multi-hop cellular networks, wireless cooperative networks as well as next generation telecommunication standards (e.g. IEEE 802.16j). Importantly though, most existing works that address the resource allocation problem in wireless relay networks assume that all the participating devices will cooperate with each other (i.e. relay traffic for each other) in order to maximize the overall network utility. This assumption does not hold in many wireless networks where the participating devices (or their owners) are self-interested and may refuse to cooperatively relay traffic for other devices while incurring a cost (e.g. relay transmission power). Therefore, stimulating cooperation among selfish transceivers is a key problem to improve the network performance in wireless mobile networks using relays.

The literature has proposed various incentive schemes to stimulate cooperation in wireless relay networks. Much relies on reputation-based methods under which a transceiver is rewarded or punished based on its behavior in the system [1][2][3]. Mathematical analysis using game theory is made in [4][5] for the case that the interactions between a pair of transceivers goes on for infinite times. Various modified versions of Tit-For-Tat strategy are proposed in [6][7]. Repeated games are also used to build incentives based on reputation schemes for relay networks where nodes need to regularly update their partners due to mobility or environmental changes [8]. However, reputation-based schemes are only suitable in systems relying on a (centralized) infrastructure that is able to collect, process, and deliver information about the individuals behavior and transceivers need to be informed using this infrastructure about the reputations of their partners. In a wireless mobile network where interactions are mostly distributed, such information is hard to obtain.

Another way to foster cooperation is by using monetary pricing schemes [9][10]. These methods often focus on static networks where the interacting transceivers are fixed. Moreover, a key disadvantage of using monetary pricing is the absence of an associated reliable financial accounting and the impracticality of implementing centralized accounting to pay for decentralized services (i.e. relay cooperation among distributed transceivers). To cope with this problem, electronic tokens are proposed for incentive provision among transceivers [11][12], where transceivers pay tokens to the relay transceivers for relaying services. To support such a kind of distributed coordination, a secure token passing operation is proposed in [13]. However, most of the existing works rely on simulation methods to illustrate the efficiency of their proposed system designs and lack formalisms that are able to systematically assess the efficiency of the proposed designs. Most importantly, users' strategic behavior in token systems is not fully understood in a rigorous way neglecting which may significantly degrade the token system performance.

Analytical attempts to understand users' strategic behavior and its effect on the system efficiency are made in [14] in a "scrip" (i.e. a kind of token) system context. This is the closest work to this paper although there are several major differences. First, they use a model that is not suitable for the wireless relay system on which we focus in this paper because it considers a centralized scenario where all other users are able to respond to a user's request while in relay networks, transceivers are distributed. Second, we do not require that transceivers adopt threshold strategies, but instead, we rigorously prove that threshold strategies are the only incentive-

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compatible strategies (i.e. the strategy that transceivers may want to use in their self-interest).

In this paper, we provide incentives for transceivers in wireless mobile networks with relay support to provide relaying service using a token system due to its implementation simplicity and possibility to operate the network in an autonomous and distributed way. We design the token system for wireless relay networks to maximize the system efficiency, i.e. the probability that the relay transmission is carried out. The problem is then modeled as a bi-level optimization problem. First, we rigorously analyze the transceivers' incentive-compatible strategies. We prove that the only incentive-compatible strategies are threshold strategies. We also show the relationship between costs and the adopted thresholds. Second, we maximize the system efficiency given the transceivers' strategic behaviors studied previously. There is an optimal token supply that should be deployed in the network.

The rest of this paper is organized as follows: Section 2 builds the system model and describes the relay transmission process. Section 3 studies the incentive-compatible strategies for the transceivers and shows that only threshold strategies are incentive-compatible. Moreover, for each relaying cost, there exists a unique associated incentive-compatible threshold. Section 4 then determines the optimal token supply that maximizes the system efficiency. Section 5 provides simulation results. Finally, Section 6 concludes the paper.

II. System Model

A. Setup

We consider a dynamic wireless network with N wireless mobile transceivers and implicitly assume N is large because usually there are many transceivers in the network. Time is discrete. In each time period, a fraction of the transceivers need to receive data from their corresponding sources (e.g. the base station in a cellular network or another mobile transceiver in an ad hoc network) through relays due to bad direct channel conditions. We capture the demand for relay transmissions in the network by λ, which is the probability that a transceiver needs to receive data from its source using relay transmissions in each period. Hence, λ is the relay transmission demand rate and depends on the overall network condition.

Transceivers are mobile. They move to various locations in different time periods and experience different channel conditions. We assume that the transceivers are anonymous and self-interested, meaning that they aim to maximize only their own utilities and do not care about the overall performance of the network. Because forwarding signals incurs costs (e.g. transmission power) to the transceivers who act as relays, self-interested transceivers do not want to help other transceivers by forwarding their traffic without having proper incentives. Hence, our focus is on designing such incentive mechanisms. Denote the action space $\mathcal{A} = \{0, 1\}$; $a = 0$ means "not relay" and $a = 1$ means "relay". For each relay transmission, fulfilling the transmission brings the receiving transceiver i a benefit b while the relay transceiver j incurs a cost c_j. We assume that

	Relay Transceiver	
Receiving Transceiver	$b, -c$	$0, 0$
	relay traffic	*not relay traffic*

Fig. 1. Relay transmission game.

the benefit is the same for all relay transmissions since it is an evaluation of a successful transmission. However, the costs to the relay transceiver j for different relay transmissions are not the same but depend on the specific channel conditions that the relay transceiver experiences to achieve a certain rate for the receiving transceivers. The cost incurred by relays when forwarding traffic can be due to many reasons, e.g. relay transmission power. In the analysis of this paper, we assume an abstract cost c which follows a certain distribution. However, since the cost at every time period is always positive ($c > 0$), the dominant strategy for the relay transceiver is always to not forward traffic in this simple gift-giving game (see Fig. 1).

B. Timing of the relay transmission

The conventional relay transmission process often involves two stages: relay selection and relay transmission. The relay selection has been shown to be critical for the relay network performance and much work has focused on this issue [15][16][17]. In this paper, we assume that there is no interference between relay transmissions. Therefore, the relay selection criterion is simply to choose the wireless transceiver which requires the least cost. Besides the usual two stages involved in the relay transmission when the transceivers are obedient, one more stage is necessary when transceivers are self-interested - the relay decision, which involves determining whether relaying the traffic is in the relays own best interest.

The timing of each relay transmission is illustrated in Fig. 2. After the relay selection stage, the receiving transceivers sends a relay request (REQ) to the relay transceiver. The relay transceiver decides whether or not to relay the traffic and sends back an positive or negative acknowledgement (ACK/NACK). If a positive acknowledgement is received by the receiving transceiver, it transfers a token to the relay receiver. Then, the relay transmission starts.

C. Token system

If the transceivers would only be involved in a single relay transmission as the relay, they will be reluctant to help forward the traffic since this leads only to a cost and no reward for themselves. However, because transceivers are active in the network for a long time, proper incentives can be provided to make them take into account future utilities when making decisions. We assume that the transceivers discount the future utility at a constant rate $\beta \in (0, 1)$.[1] One way to introduce such incentives to relay traffic for other transceivers

[1] Another interpretation of the discount factor β is the probability that the transceivers stay in the network. For example, if $\beta = 0.9$, the transceivers stay in the network with probability 0.9.

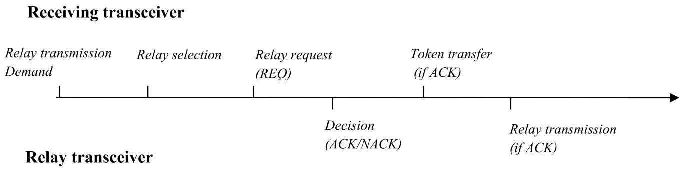

Fig. 2. Timing of the relay transmission.

is through the use of tokens, which are exchanged among transceivers to “buy” and “sell” relaying services. In each relay transmission, the receiving transceiver pays one token to the relay transceiver in exchange for forwarding the traffic. The token transaction takes place at the end of the decision stage of a relay transmission (see Fig. 2). An electronic token is transferred from the receiver to the relay once the receiver receives an ACK from the candidate relay transceiver. Secure token passing designs are proposed in several existing works, such as [13][14]. We assume that relay transceivers are not malicious and they will forward the traffic in the following transmission stage as long as it receives a valid electronic token. We view that at the acceptance of a token, a binding contract is established. Refusing relaying transmission violates the contract.[2]

The overlay token system enables simple deployment of the relay network with self-interested transceivers. (1) One token provides one unit relay transmission opportunity and has no intrinsic value outside of the relay network. This avoids many financial problems (such as fraud) that are associated with monetary incentive schemes. (2) No personal information of the others is required when a transceiver makes a decision. Hence, the system can be fully anonymous and more secure. In contrast, under a reputation-based incentive scheme, the transceivers inevitably need to know some identities - at least, the reputations of their interacting transceivers. (3) There have been some advanced techniques that enable secure electronic token transaction in a distributed way. Essentially no central entity is needed for the transactions.

D. Problem formulation

Despite all these advantages of the token system, maintaining a good system performance (i.e. a high probability that the relay transmission occurs when needed) still requires careful design by the system designer. In particular, the amount of tokens that are circulated in the system plays a critical role. A straightforward intuition is that the token system does not work if there are too few tokens in the network because few transceivers have the tokens to request relay transmission. We will also show that too many tokens are not helpful either, by studying the strategies that the self-interested transceivers use. Therefore, there must be a proper number of tokens that the designer must deploy in the network.

[2]One way to prevent malicious behaviors is to impose a severe punishment to the malicious transceivers, e.g. ostracism from the system. In order to focus on transceivers' rational behaviors, we do not consider malicious transceivers in this paper.

Denote the transceiver strategy by $\sigma : \mathcal{S} \rightarrow \mathcal{A}$ which is a mapping from the system state space $\mathcal{S}$ to the relay action space $\mathcal{A}$. Transceivers may use different strategies and hence, we denote σ_i as transceiver i's strategy. Denote the total amount of tokens by W . The efficiency, which is the expected probability that the relay transmission successfully takes place, is denoted by $\mathbb{E}\{E(\sigma_1, \sigma_2, ..., \sigma_N, s|W)\}$, where $E(\sigma_1, \sigma_2, ..., \sigma_N, s|W)$ is the relay transmission probability when transceiver i uses action $\sigma_i(s), \forall i$ in state $s \in \mathcal{S}$. The objective of the system designer is to issue a proper number of tokens in the system such that the relay transmission probability is maximized when all transceivers play incentive-compatible strategies to maximize their own utilities. The problem can be formulated as a bi-level optimization problem where the inner level optimization problem is the transceiver's incentive problem (P1) and the outer level problem is the efficiency maximization problem (P2).

$$\begin{array}{lll} (P1) & \underset{\sigma_i}{\text{maximize}} & V_i\left(s|\sigma_i\left(s\right)\right) \\ & \text{subject to} & V_i\left(s|\sigma_i\left(s\right)\right) \geq V_i(s|\sigma_i'\left(s\right)), \\ & & \forall \sigma_i'\left(s\right) \neq \sigma_i\left(s\right) \\ (P2) & \underset{\alpha}{\text{maximize}} & \{\mathbb{E}\{E\left(\sigma_1, \sigma_2, ...\sigma_N, s|\alpha\right)\}\} \\ & \text{subject to} & \sigma_i \text{ solves } \ (P1), \forall i, \forall s \end{array} \quad (1)$$

III. Incentive-Compatible Transceiver Strategies

In this section, we investigate users' incentive problem when they are requested to provide the relay service.

A. Incentive-compatible strategy

As a first step, let us examine the structural properties of incentive-compatible transceiver strategies. Whether a relay transceiver wants to provide relay service depends on how its evaluation of having one more tokens. From a representative transceiver's point of view, the relaying decision should only depend on the token holding number k which it possesses and the instant relaying cost c. Let us first fix the cost c for the following analysis and study the value of having one more token at the cost c. The relay strategy then only depends on the token holding k. Denote the relay strategy by the function: $\sigma : \mathbb{N}_+ \rightarrow \mathcal{A}$ which maps the token holding to the action of whether or not forward the traffic. For each token holding k, $\sigma(k) = 1$ means to forward while $\sigma(k) = 0$ means to not forward. Keep in mind that a strategy for cost c prescribes the actions for all possible token numbers. If a relay strategy is incentive-compatible, meaning that the transceiver would like to follow the strategy if and only if it has the one-shot deviation property [18]:

Definition 1. *(Incentive-Compatible Strategy). A transceiver strategy σ is an incentive-compatible strategy if and only if $\forall k \in \mathbb{N}_+$,*

$$\begin{array}{l} \beta\left(V\left(k+1|\sigma\right) - V\left(k|\sigma\right)\right) \geq c, \text{if} \sigma\left(k\right) = 1 \\ \beta\left(V\left(k+1|\sigma\right) - V\left(k|\sigma\right)\right) < c, \text{if} \sigma\left(k\right) = 0 \end{array} \quad (2)$$

Where $V(k|\sigma)$ is the utility of having k tokens when the transceiver follows the strategy σ.

The incentive-compatible strategies make the transceiver always maximize its utility for all possible token numbers that it might have by following the strategy. The number of all possible transceiver strategies is large and hence, finding the incentive-compatible strategies is thus difficult. In the following we study whether a strategy σ is incentive-compatible and simply write $V(k)$ instead of $V(k|\sigma)$ for brevity.

B. Values and Marginal values

The value of holding tokens depends on the strategy that the transceiver uses. The utility functions are inter-dependent with each other as follows

$$\begin{aligned} V(0) &= (1-\lambda\sigma(0))\beta V(0) + \lambda\sigma(0)(-c+\beta V(1)) \\ V(k) &= \underbrace{(1-\lambda(1+\sigma(k)))\beta V(k)}_{\text{remain the same token number}} \\ &+ \underbrace{\lambda\sigma(k)(-c+\beta V(k+1))}_{\text{have one more token}} + \underbrace{\lambda(b+\beta V(k-1))}_{\text{have one less token}}, \forall k \geq 1 \end{aligned} \tag{3}$$

For $k \geq 1$, the first term is because that the transceiver remains having k tokens with probability $1-\lambda(1+\sigma(k))$; the second term is because that the transceiver gains one more token with probability $\lambda\sigma(k)$ as it becomes a relay transceiver and uses strategy $\sigma(k)$ in the next period; the third term is because the transceiver loses one token with probability λ as it becomes a receiver in the next period.

It is convenient to write the marginal utility $V(k+1)-V(k)$ of holding k tokens by $M(k)$. In the following, we study the property of the marginal utilities. For every transceiver strategy σ there exists $K_\sigma \geq 0$ such that for all $k \leq K_\sigma$,

$$\sigma(k) = 1, \forall k < K_\sigma, \text{ and } \sigma(K_\sigma) = 0 \tag{4}$$

Thus, K_σ is the smallest level k that makes $\sigma(k) = 0$, i.e. "not relay". However, for $k \geq K_\sigma$, $\sigma(k)$ can be arbitrary.

We first study the marginal utilities for $k \leq K_\sigma$. Note that for the special case $K_\sigma = 0, \sigma(0) = 0$. Due to the space limitation, please find all the omitted proofs in this section in [19].

Lemma 1. *For any transceiver strategy σ, the marginal utilities $M(k), \forall k \leq K_\sigma - 1$ satisfy*

1) $M(k) > 0$.
2) *$M(k)$ is either a decreasing sequence or there exists a unique K_t such that for $0 \leq k \leq K_t - 1$, $M(k)$ is a decreasing sequence and for $K_t \leq k \leq K_\sigma - 1$, $M(k)$ is an increasing sequence.*
3) *$M(k)$ is a decreasing sequence if*

$$M(k) \geq \frac{c}{\beta}, \quad \forall k \leq K_\sigma - 1 \tag{5}$$

Proof: See Appendix. ■

Lemma 1 shows how the marginal utilities look like. For any strategy, the marginal utilities below K_σ are positive, meaning that having more tokens always generates higher utility. More importantly, if the strategy is an incentive-compatible strategy, by the third part of this lemma, the marginal utility diminishes with the increase of token holding number. This implies that transceivers may not want to accumulate more tokens to some point if the marginal utility of having one more token falls below the current cost. However, because Lemma 1 only studies parts of the marginal utilities (below K_σ), we establish this threshold property in the next subsection by studying the general case.

C. Threshold property

We are now in a position to study which transceiver strategies can be incentive-compatible strategies.

Proposition 1. *Incentive-compatible strategies are threshold strategies, i.e. there exists K_{th}, such that*

$$\begin{aligned} \sigma(k) &= 1, \text{for} \quad k \leq K_{th} \\ \sigma(k) &= 0, \text{for} \quad k > K_{th} \end{aligned} \tag{6}$$

Proof: (sketch) Based on Lemma 1, it can be shown that if a strategy is an incentive-compatible strategy, the marginal utilities are also decreasing sequences. Therefore, there exists a threshold K_{th}. ■

The above proposition tells that the incentive compatible strategies can only be threshold strategies. This tremendously simplifies our analysis on transceivers' rational behaviors by only focusing on the thresholds. Our intuition also suggests that transceivers may like to use threshold strategies due to their simplicity and in fact, many research works make this assumption when they build their model. Different from these works, we start from arbitrary relay strategies and analytically show that threshold strategy is indeed the rational choice by the transceivers. Using our formalism, we can determine the thresholds that transceivers should want to use to optimize their performance. How to determine the threshold will be discussed in Section 4 when we study the optimal token supply by the system designer.

D. Varying costs

Now we take into account the impact of the instant c on the transceiver's decision problem. Because the cost to the relay transceivers are different over time due to changing locations and varying channel conditions, they may not want to use a constant threshold strategy. In the following, we study how the cost for relaying affects the choice of threshold.

Proposition 2. *For given λ, β, b and a threshold strategy σ_K with a threshold K, there exists $c^L, c^H (0 < c^L < c^H)$ such that $\forall c \in (c^L, c^H]$, σ_K is an incentive compatible strategy; otherwise, it is not.*

Proof: (sketch) We first see that $M(K_{th}-1) > c/\beta, M(K_{th}) < c/\beta$ is a necessary and sufficient condition for a strategy to be an equilibrium. This is established on the properties of marginal utilities. Define $F(c) = M(K_{th} - 1|\beta) - c/\beta, G(c) = M(K_{th}|\beta) - c/\beta$. Hence, the necessary and sufficient condition becomes $F(c) > 0, G(c) < 0$.

It can be shown that there exists a unique $c^H \in (0, b)$, such that $F(\beta) \geq 0, \forall c \in (0, c^H)$ and equality holds only for c^L. Next we show that there exists a unique $c^L \in (0, c^H)$ such that $G(c) \leq 0, \forall c \in (c^L, c^H)$ and equality holds only for c^H.

Therefore, there must exist an non-degenerate interval $[c^L, c^H]$ that makes a threshold strategy incentive-compatible. ■

Proposition 2 states that there is a corresponding continuous interval of cost values that makes a threshold strategy to be incentive compatible. It establishes the condition on the cost for a threshold strategy to be incentive compatible. However, what we are more interested in is that, for a given cost, whether there exists a (or some) threshold strategy to make it incentive compatible and this is not obvious by Proposition 2.

Proposition 3. $\exists c_0$, $\forall 0 < c \leq c_0$, *there exists a unique* $K > 0$, *such that* σ_K *is the incentive-compatible strategy for c.* $\forall c > c_0$, *the incentive-compatible strategy is* $\sigma \equiv 0$.

Proof: (sketch) It is sufficient to prove for two consecutive thresholds $K, K+1$, the corresponding end points of the incentive-compatible cost interval have the relation $c_{K+1}^L = c_K^H$. ■

Proposition 3 suggests that there is a mapping from the costs to the threshold strategies such that the threshold strategies are incentive-compatible. Importantly, the threshold is unique for any cost. Denote the mapping by $\mathcal{K} : (0, c_0] \to \mathbb{N}_+$. This is important for understanding the relay transceivers' strategic behaviors when they need to decide whether or not to forward traffic at a cost when they already have a certain number of tokens. Hence, the transceiver takes joint considerations of the number k of tokens that it already has and the cost that incurs by relaying the traffic. The optimal strategy is then a mapping from $\sigma : \mathbb{N}_+ \times (0, c_0] \to \mathcal{A}$, and

$$\sigma(k, c) = \begin{cases} 1, \text{if} & k < \mathcal{K}(c) \\ 0, \text{if} & k \geq \mathcal{K}(c) \end{cases} \tag{7}$$

IV. Optimal Token Supply

In the previous section, we show that relay transceivers do not cooperate, i.e. forward the traffic, all the time because they have incentives to stop accumulating tokens after accumulating a certain treasury. This suggests that if all transceivers already have lots of tokens, they stop forwarding traffic when they become relays. We already know that if there are too few tokens in the network, relay requests are seldom initiated because few transceivers have tokens to pay when they are receivers. Therefore, it seems there must be an optimal token supply in the network that maximizes the system efficiency, i.e. the probability that a relay transmission successfully takes place when needed.

Because the transceiver population is usually very large, we approximate it by a continuum model (mass 1). Under this continuum model, the token supply is described by the average token number per transceiver α. Let $\eta_K(k, t)$ be the fraction of relay transceivers who has k tokens and the cost for whom to relay traffic is $\{c : \Theta(c) = K\}$ at some time t, then the fraction of relay transceivers who deny forwarding traffic is calculated by

$$\eta_d(t) = \sum_{i=0}^{\infty} w(i, t) \sum_{k \geq i} \eta_i(k, t) \tag{8}$$

where $w(i, t)$ is the fraction of relay transceivers the cost for whom to relay traffic is $\{c : \Theta(c) = i\}$. Let η_0 be the fraction of receiving transceivers who has 0 tokens and hence, they cannot request relay service from other transceivers when relay transmissions are needed.

$$\eta_o(t) = \sum_{i=0}^{\infty} w(i, t) \eta_i(0, t) \tag{9}$$

Therefore, the probability that the relay transmission successfully takes places is $E(t) = (1 - \eta_d(t))(1 - \eta_o(t))$.

Because the network is dynamic, $\eta_d(t), \eta_o(t)$ vary over time and are difficult to compute. However, we are able to explicitly derive the optimal token supply if the cost and the demand rate are homogeneous. By taking the homogeneous cost as the average cost for relaying traffic and the homogeneous demand rate as the average demand rate, we obtain a suboptimal token supply for the relay system while the complexity is significantly reduced. In the simulations, we will show the performance of this suboptimal choice of token supply compared to the optimal one.

A. Token holding distribution and optimal supply

For the homogeneous cost c, all transceivers use a same threshold strategy in all time periods. As we know from the last section, there is a unique threshold $K = \Theta(c)$ strategy that the transceivers adopt. Therefore, no transceivers hold more than K tokens. Hence, there are two feasibility conditions that the token distribution must satisfy

$$\sum_{k=0}^{K} \eta(k, t) = 1, \sum_{k=0}^{K} k\eta(k, t) = \alpha \tag{10}$$

Moreover, it is simply that $\eta_o = \eta(0), \eta_d = \eta(K)$.

If the current token distribution is η and the transceivers follow the threshold K strategy, the token distribution in the next time period can be calculated in a straightforward way.

$$\begin{aligned} \eta(0, t+1) &= \lambda(1 - \eta(K, t))\eta(1, t) \\ &\quad + (1 + \lambda(\eta(K, t) - 1))\eta(0, t) \\ \eta(k, t+1) &= \lambda(1 - \eta(0, t))\eta(k-1, t) \\ &\quad + \lambda(1 - \eta(K, t))\eta(k+1) \\ &\quad + (1 + \lambda(\eta(0, t) + \eta(K, t) - 2))\eta(k, t), \\ &\qquad 1 \leq k \leq K-1 \\ \eta(K, t+1) &= \lambda(1 - \eta(0, t))\eta(K-1, t) \\ &\quad + (1 + \lambda(\eta(0, t) - 1))\eta(K, t) \end{aligned} \tag{11}$$

The transceivers with k tokens in the next period are consisted of three parts: (1) transceivers who have tokens in the current period become the relay transceivers and get a token; (2) transceivers who have $k-1$ tokens in the current period become the receiver transceivers and lose a token; (3) transceivers who have $k+1$ tokens and do not get or lose tokens. If the token distribution remains the same in the next period, then we say the token distribution is invariant, i.e.

$$\eta(k, t+1) = \eta(k, t), \forall k \tag{12}$$

The next proposition characterizes the invariant distribution.

Proposition 4. *If all transceivers follow a same incentive-compatible threshold strategy with the threshold K, the invariant token holding distribution η satisfies,*

$$\eta(k) = \left(\frac{1-\eta(0)}{1-\eta(K)}\right)^k \eta(0), \forall k = 0, 1, ..., K \tag{13}$$

which is independent of the demand rate λ.

Proof: From the first equation in (11),

$$\eta(1) = \frac{1-\eta(0)}{1-\eta(K)}\eta(0) \tag{14}$$

From the second equation in (11),

$$\eta(k) = \frac{2-\eta(0)-\eta(K)}{1-\eta(K)}\eta(k-1) - \frac{1-\eta(0)}{1-\eta(k)}\eta(k-2) \tag{15}$$

Using the third equation in (11) and substitute (14) , we obtain the result. ■

The invariant token distribution is a restricted geometric distribution which only depends on the threshold that transceivers use. There are no closed form expressions of $\eta(0)$ and $\eta(K)$ and hence, we cannot derive the closed form expression for the system efficiency. Fortunately though, (13) provides sufficient information to find the optimal token supply to maximize the system efficiency.

Proposition 5. *If all transceivers follow a same incentive-compatible threshold strategy with the threshold K, the token supply α that maximizes the system efficiency E, i.e. the probability that relay transmission successfully takes place when needed, is $K/2$ per transceiver on average. Moreover, the maximized efficiency is*

$$E = \left(1 - \frac{1}{K+1}\right)^2 \tag{16}$$

Proof: It is convenient to first solve the following maximization problem

$$\begin{aligned} \text{maximize} \quad & (1-x_1)(1-x_2) = 1 - x_1 - x_2 + x_1 x_2 \\ \text{subject to} \quad & x_1(1-x_1)^K = x_2(1-x_2)^K \\ & 0 \le x_1, x_2 \le 1 \end{aligned} \tag{17}$$

To solve this problem, set $f(x) = x(1-x)^K$, a straightforward calculus exercise shows that if $0 \le x_1 \le 1/(K+1) \le x_2 \le 1$ and $f(x_1) = f(x_2)$ then,

(a) $x_1 + x_2 \ge 1/(K+1)$ with equality achieved only at $x_1 = x_2 = 1/(K+1)$.

(b) $x_1 x_2 \le 1/(K+1)$ with equality achieved only at $x_1 = x_2 = 1/(K+1)$.

Putting (a) and (b) together shows that the optimal solution to the maximization problem is to have $x_1 = x_2 = 1/(K+1)$ and the maximized objective function value is

$$\max(1-x_1)(1-x_2) = \left(1 - \frac{1}{K+1}\right)^2 \tag{18}$$

Now consider the threshold K strategy and let η be the corresponding invariant distribution. If we take $x_1 = \eta_o, x_2 = \eta_d$ then our characterization of the invariant distribution shows that $f(x_1) = f(x_2)$. By definition, $E = (1-x_1)(1-x_2)$ so

$$E = \left(1 - \frac{1}{K+1}\right)^2 \tag{19}$$

Taken together, these are the assertions which were to be proved. ■

Proposition 5 proves that there is an optimal token supply for the relay network where transceivers use the same threshold strategy: it is not too small that transceivers do not have tokens to make relay requests nor too large that transceivers decide not to provide relay service when they are needed for relaying. For the system designer to efficiently operate the relay network, it needs to understand the transceivers' strategic behaviors and issue the appropriate number of tokens.

V. Simulations

In this section, we provide several illustrative results to highlight the various design aspects of our proposed tokens framework. We assume that the network has $N = 1000$ wireless transceivers. The source transmission power is fixed P_s at 15dBm. For different relay transmissions, both the channel gain between the source and relay G_{sr} and the channel gain between the relay and the receiving transceiver G_{rd} are different. We assume that G_{sr} and G_{rd} follow normal distributions $\mathcal{N}(\bar{G}_{sr}, var_{sr})$ and $\mathcal{N}(\bar{G}_{rd}, var_{rd})$, respectively.

In the following simulations, the relay transmission demand is $\lambda = 0.2$ for all transceivers. The channel conditions are as follows: $\bar{G}_{sr} = \bar{G}_{rd} = 1, var_{sr} = var_{rd} = 0.2$. The normalization factor is $\rho = b/P_s$ to make the cost comparable to the benefit b. The target signal-to-noise ratio at the receiver transceiver side is $\Gamma_{min} = 5$. Transceivers discount future utilities at $\beta = 0.99$.

Fig. 3 illustrates the number of relay transmissions as the system goes for various token supplies W. The initial tokens are randomly distributed to the transceivers in the network. After a sufficient long period, the number of relay transmissions stays at a high level for $W = 5000$ and stays at a lower level for $W = 2000$ or $W = 10000$. This simulation also shows that for a given token supply, the efficiency is almost stable with small variance as the system evolves. The average number of relay transmissions over time is therefore a reasonable metric of the system efficiency. Furthermore, we show the average number of relay transmission in Fig. 4 for various token supplies. It shows that both too few tokens and too many tokens in the system degrade the system efficiency. The optimal token supply in our simulated scenario is about $W = 7000$ tokens and the optimal efficiency of the token system is about 85%.

The token system performance is then illustrated in Fig. 5 and Fig. 6 with respect to various target SNRs and relay transmission demand rates. Fig. 5 shows that the system becomes less efficient as the target SNR increases. For a high target SNR, the relay transceivers need to spend more relay transmission power and hence, they have fewer incentives to relay the traffic. Therefore, the thresholds that they adopt are

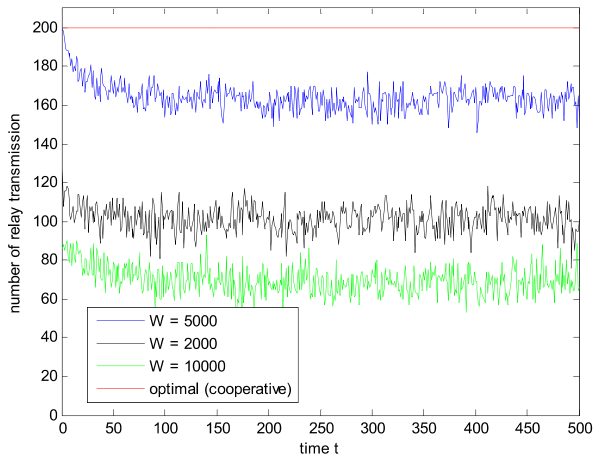

Fig. 3. Realtime number of relay transmission for various token supplies.

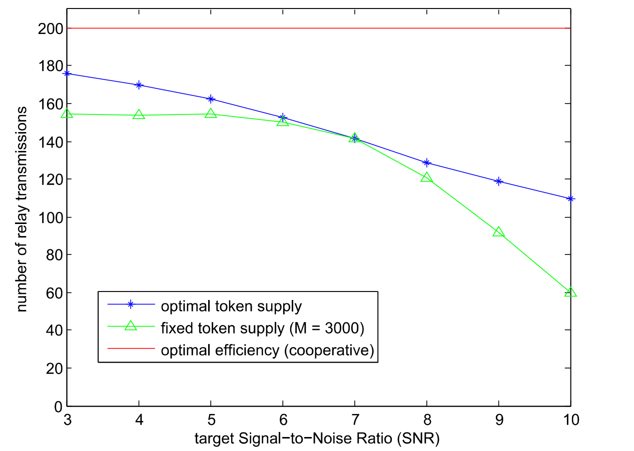

Fig. 5. Optimal token system efficiency for target signal-to-noise ratio (SNR).

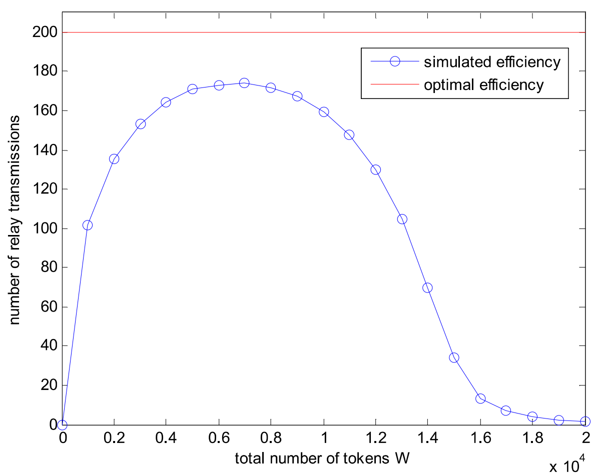

Fig. 4. Token system efficiency for various token supply.

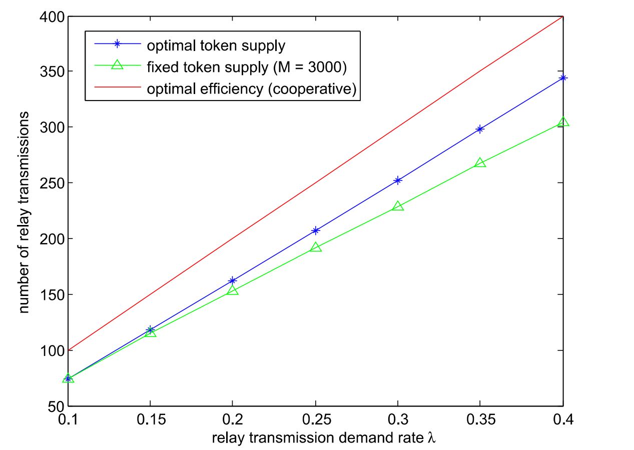

Fig. 6. Optimal token system efficiency for various relay transmission demand rate.

smaller than when the target SNR is smaller. Fig. 6 shows that the token system becomes more efficient when the traffic demand is larger because tokens become more valuable since they can be redeemed sooner in the future. In both figures, the protocol with a fixed token amount ($M = 3000$ in this simulation) is shown to be less efficient than the one with the optimal token supply. Therefore, it is important to choose the optimal token supply for different system parameters.

VI. Conclusions

In this paper, we propose a novel mechanism for providing self-interested transceivers with incentives to relay traffic for other wireless transceivers using a token system. The design of the token system is formulated as a bi-level optimization problem where the inner level optimization determines the transceivers' incentive-compatible strategy and the outer level optimization determines the optimal token supply in the network. Importantly, in this paper, we rigorously characterize the structural properties exhibited by the incentive-compatible strategies adopted by the transceivers and prove that they are threshold strategies. We also formally characterize the relation between the thresholds and the network parameters, such as the relay transmission cost. This threshold property allows a better understanding of transceivers' strategic behaviors when facing different costs. The token supply was often a neglected parameter when designing similar token systems in existing literature. Our findings in this paper emphasize that the token supply represents a critical design parameter affecting the system efficiency.

References

[1] S. Marti, T. J. Giuli, K. Lai, and M. Baker, "Mitigating routing misbehavior in mobile ad hoc network," in *Proc. ACM Mobicom*, 2000.

[2] S. Buchegger and J. Y. L. Boudec, "Performance analysis of the confidant protocol," *ACM MobiHoc*, 2002

[3] P. Michiardi and R. Molva, "Core: a collaborative reputation mechanism to enforce node cooperation in mobile ad hoc networks," in *Proc. IFIP-Comm. Multimedia Security Conf.*, 2002.

[4] P. Michiardi and R. Molva, "A game theoretical approach to evaluate cooperation enforcement mechanisms in mobile ad hoc networks," in *Proc. WiOPT'03*, 2003.

[5] J. Crowcroft, R. Gibbens, F. Kelly, and S. Ostring, "Modeling incentives for collaboration in mobile ad hoc networks," in *Proc. WiOPT'03*, 2003.

[6] V. Srinivasan, P. Nuggehalli, C. F. Chiasserini, and R. R. Rao, "Cooperation in wireless ad hoc networks," in *Proc. IEEE INFOCOM*, 2003.

[7] J. J. Jaramillo, R. Srikant, "A game theory based reputation mechanism to incentivize cooperation in wireless ad hoc networks," *Ad Hoc Networks*, 2010.

[8] Y. Chen and K. J. R. Liu, "Indirect reciprocity game modeling for cooperation stimulation in cognitive networks," *IEEE Trans. Communications*. Vol. 59, No. 1, Jan. 2011.

[9] J. Huang, Z. Han, M. Chiang, H. V. Poor, "Auction-based resource allocation for cooperative communications," *IEEE Journal on Selected Areas in Communications*, Vol. 26, No. 7, Sep 2008.

[10] Y. Xi and E. M. Yeh, "Pricing, competition, and routing in relay networks," 47th *Annual Allerton Conference on Communication, Control and Computing*, 2009.

[11] L. Buttyan, J-P Hubaux, 'Nuglets: a virtual currency to stimulate cooperation in self-organized mobile ad hoc networks," *EPFL technical report*, Jan. 2001.

[12] S. Zhong, J. Chen, Y. Yang, "Sprite: a simple, cheat-proof, credit-based system for mobile ad-hoc networks", *22rd Annual Joint Conference of the IEEE Computer and Communication Societies*, 2003 (Infocom 2003).

[13] A. Ciuffoletti, "Secure token passing at application level," *Future Generation Computer Systems*, Elsevier, Volume 26, Issue 7, Pages 1026-1031, July 2010.

[14] E. J. Friedman, J. Y. Halpern, and I. Kash, "Efficiency and nash equilibria in a scrip system for p2p networks," in *Proc. ACM conference on Electronic Commerce (EC'06)*, 2006.

[15] A.S. Ibrahim, A. K. Sadek, W. Su and K. J. R. Liu, "Cooperative communications with relay-selection: when to cooperate and whom to cooperate with?" *IEEE Trans. Wireless Communications*, Vol. 7, Issue. 7, Pages 2814-2827, July 2008.

[16] R. Madan, N. Mehta, A. Molisch, J. Zhang, "Energy-efficient cooperative relaying over fading channels with simple relay selection," *IEEE Trans. On Wireless Communication*s, vol. 7, no. 8, pp. 3013-3025, August 2008.

[17] J. Xu, S. Zhou, Z. Niu, "Interference-aware relay assignment in multi-source multi-destination wireless cooperative networks," *Asian-Pacific Conference on Communications (APCC)*, Oct. 2009.

[18] G. J. Mailath, L. Samuelson, "Repeated games and reputations - long-run relationships," *Oxford university press*, 2006.

[19] omitted proofs, http://www.ee.ucla.edu/∼jiexu/documents/wiopt12.pdf.

APPENDIX

For $k \leq K_\sigma$, the value functions are

$$\begin{array}{l} V(0) = (1-\lambda)\beta V(0) + \lambda(-c+\beta V(1)) \\ V(k) = (1-2\lambda)\beta V(k) + \lambda(-c+\beta V(k+1)) \\ +\lambda(b+\beta V(k-1)) \\ V(K_\sigma) = (1-\lambda)\beta V(K_\sigma) + \lambda(b+\beta V(K_\sigma-1)) \end{array} \tag{20}$$

These are second-order homogeneous difference equations. However, the solution expression is complicated and does not provide any direct results of the property of the marginal utilities. Therefore, we study these equations using an alternative way. Rearranging the terms and replacing with $M(k), \forall k \leq K_\sigma$ yields

$$\Phi M = u \tag{21}$$

where

$$\Phi = \begin{bmatrix} \phi_1 & \phi_2 & 0 & \cdots & 0 \\ \phi_2 & \phi_1 & \phi_2 & \ddots & \vdots \\ 0 & \phi_2 & \phi_1 & \phi_2 & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots \\ 0 & \cdots & 0 & \phi_2 & \phi_1 \end{bmatrix}, u = \begin{bmatrix} \lambda b \\ 0 \\ \vdots \\ 0 \\ \lambda c \end{bmatrix} \tag{22}$$

and

$$\phi_1 = 1-(1-2\lambda)\beta, \phi_2 = -\lambda\beta \tag{23}$$

(1) Suppose $\exists k^* \leq K_\sigma - 2$, $M(k^*) \leq 0$. We first show that neither $M(k^*-1)$ nor $M(k^*+1)$ is non-positive and then show they also cannot be positive. With this, we find a contradiction to conclude that $M(k) > 0, \forall k \leq K_\sigma$.

For "neither $M(k^*-1)$ nor $M(k^*+1)$ is non-positive", we study two cases.

Case 1: $0 \geq M(k^*-1) \geq M(k^*)$ (and $0 \geq M(k^*) \geq M(k^*+1)$). By (22),

$$M(k^*+1) = \frac{\phi_2 M(k^*-1) + \phi_1 M(k^*)}{-\phi_2} \tag{24}$$

$$\leq \frac{(\phi_1+\phi_2) M(k^*)}{-\phi_2} \leq M(k^*) \tag{25}$$

Recursively, it should be $M(K_\sigma - 1) \leq M(K_\sigma - 2) \leq ... \leq M(k^*) \leq M(k^*-1) \leq 0$. However, it is not true because otherwise we will get

$$\phi_1 M(K_{\sigma_1} - 1) = \lambda c - \phi_2 M(K_\sigma - 2) \tag{26}$$

$$> -\phi_2 M(K_\sigma - 2) \geq -\phi_2 M(K_\sigma - 1) \tag{27}$$

And hence, $\phi_s < -\phi_2$. This is a contradiction. With similar arguments, $0 \geq M(k^*) \geq M(k^*+1)$ is not true.

Case 2: $0 \geq M(k^*) \geq M(k^*-1)$ (and $0 \geq M(k^*+1) \geq M(k^*)$). This case is similar to the first case except that we go in the other direction of k.

The above two cases excludes the possibility that either $M(k^*+1)$ or $M(k^*-1)$ can be non-positive.

For "neither $M(k^*-1)$ nor $M(k^*+1)$ is positive", it is obviously not true because otherwise

$$\phi_2 M(k^*-1) + \phi_1 M(k^*) + \phi_2 M(k^*+1) < 0 \tag{28}$$

which contradicts (22). This completes the proof for the first part of this lemma.

(2) It is sufficient to prove there does not exist any $k^* \leq K_\sigma - 2$, such that

$$M(k^*-1) \leq M(k^*) \geq M(k^*+1) \tag{29}$$

Suppose (29) is true, then

$$M(k) = \frac{-\phi_2 M(k-1) - \phi_2 M(k+1)}{\phi_1} \tag{30}$$

$$\leq \frac{-2\phi_2}{\phi_1} M(k) < M(k) \tag{31}$$

which is a contradiction.

(3) By the second part of this lemma, if $M(k)$ is not a decreasing sequence, then it must be $M(K_\sigma - 1) \geq M(K_\sigma - 2)$. This is not true because otherwise

$$\lambda c = \phi_2 M(K_\sigma - 2) + \phi_1 M(K_\sigma - 1) \tag{32}$$

$$\geq (\phi_2 + \phi_1) M(K_\sigma - 1) \geq (\phi_2 + \phi_1)\frac{c}{\beta} > \lambda c \tag{33}$$

which is a contradiction. Therefore $M(k)$ is a decreasing sequence.