Energy Minimization in Cooperative Relay Networks with Sleep Modes

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Abstract—**We consider a cooperative relay network where a source node communicates to a destination node with the help of multiple relay nodes (RNs). RNs are assumed to be able to work in either active mode or sleep mode. To minimize the total energy consumption, working modes of RNs and power allocation need to be optimized. Based on the time scales of mode transition, we formulate two different energy minimization problems. In the case of fast transition, where the transition can be applied in each frame, the problem is shown to be a supermodular minimization problem. We propose a relaxation based algorithm with a provable performance bound. In the case of slow transition, where the working modes of RNs are fixed, the problem is proven to be NP-hard even for the single relay selection case. Approximation algorithms are proposed based on the supermodular structure of the problem. Simulation results show that the proposed algorithms perform close to optimal, and significant energy saving can be achieved. The effect of the transition cost and the comparison between fast transition and slow transition are also presented.**

I. Introduction

Since the energy demands of information and communication technology have grown dramatically, green communications have been proposed to greatly improve the energy efficiency in a variety of communication systems [1]–[4]. In wireless networks, energy efficient communication techniques typically focus on minimizing the transmission energy, without considering the circuit energy. In [5], the authors show that transmission energy dominates the total energy consumption only when the transmission range is long. As wireless networks become denser and wireless devices become smaller, circuit energy will become comparable to, or even dominate energy consumption. Since wireless nodes consume circuit energy even when they do not transmit or receive, sleep-based energy saving mechanisms have been proposed [6]–[12]. When wireless devices are in sleep mode, the radio equipment is turned off and the circuit energy consumption becomes negligible.

In this paper, we consider energy efficient communication in a cooperative relay network, where a source node communicates with a destination node with the help of multiple relay nodes (RNs). The source node broadcasts data to RNs, some or all of which cooperatively forward the data to the destination node. Cooperative communication is a powerful technique in wireless networks [14], [15], which can provide spatial diversity by using a distributed virtual antenna array. Cooperative transmission has been shown to improve the throughput and energy efficiency [5], [16]. Opportunistic relay selection schemes are proposed to exploit the cooperative diversity with low complexity [20]–[24]. However, these studies mainly consider how to fully utilize the cooperative diversity under transmission power constraint. To the best of our knowledge, using the sleep mode for the energy saving has not been considered in cooperative relay networks.

In our energy consumption model, the total energy consumption consists of transmission energy and circuit energy. The transmission power is allocated among the source node and RNs so that a given source-to-destination rate requirement is satisfied. To save circuit energy, RNs are assumes to be able to work in either active mode or sleep mode. If there are more RNs in active mode, the transmission energy consumption can be lower due to cooperative gain, while the circuit energy consumption can higher. The circuit energy consumption due to mode transition is also considered in our model. To minimize the total energy consumption, the working modes of RNs and power allocation should be optimized.

The time scales for transition between modes can be quite different for different application scenarios. In a typical sensor network [7], the transition time of the sensor nodes is 2.45 ms from sleep mode to active mode and 0.25ms from active mode to sleep mode. In cellular networks, the transition time of mobile terminals is on the order of seconds [11], while base stations can change their states every couple of hours [12]. Depending on the time scales of mode transition, two different energy minimization (EM) problems are formulated in the paper. In the first case, the transition can be applied in each frame, and the *Fast Mode Transition* (FMT) problem is considered. In this case, the transition time must be shorter than the frame length. In the second case, the working modes of RNs are fixed, and the *Slow Mode Transition* (SMT) problem is considered. This happens when the transition time is much longer than the frame length.

The main contributions of this paper are as follows. First, the EM-FMT problem is formulated when the transition time is small, and we show that it is a supermodular minimization problem, which is NP-hard in general. In addition to a local greedy search algorithm, a relaxation based algorithm with provable performance bound is also proposed. Simulation results show that the algorithms perform close to optimal. Second, the EM-SMT problem is formulated when the transition cost is high, and we show that it is NP-hard even

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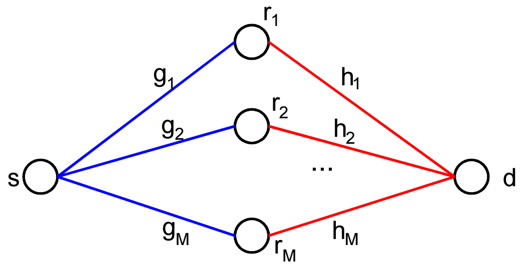

Fig. 1. Cooperative relay network model.

if we only consider single relay selection. With appropriate approximation, the problem can be solved by the algorithms developed for the EM-FMT problem. Last but not the least, we show that significant energy savings can be obtained by introducing sleep modes to cooperative relay networks, especially when the number of RNs is large. The simulation results also show that multiple relay selection perform better when RNs are clustered. The effect of transition cost and comparison between FMT and SMT are also presented.

The remainder of the paper is organized as follows. In Section II, we describe the system model. In Section III and IV, energy minimization with fast mode transition and slow mode transition are discussed, respectively. The performance evaluation is given in Section V. Finally, Section VI concludes the paper.

II. System model

As illustrated in Fig. 1, we consider a cooperative relay network where a source node s communicates with a destination node d with the help of a set of RNs denoted by $\mathcal{M} = \{r_1, r_2, \ldots, r_M\}$. All RNs have a single antenna and satisfy a half-duplex constraint. Time is divided into frames of the length T, and the system bandwidth is W Hz. Assume the s and d are far enough away so that there is no direct link between them. Let g_i denote the channel gain from source s to r_i, and h_i denote the channel gain from r_i to destination d. We use $\{g_i\}$ and $\{h_i\}$ to denote the sets of channel gains. All the channels are assumed to be flat and block fading. The target is to meet a predefined rate requirement R bits/second from s to d for each frame.

A. Communication process

The communication process for each frame consists of three phases. The length of the time for three phases (T_1, T_2, and T_3) is assumed to be fixed, where $T_1 + T_2 + T_3 = T$. During the first phase, channel gains are measured and fed back to s. There have been many studies on how to collect channel state information (CSI) through control message exchanges in cooperative relay networks [22], [23]. This is not the focus of the paper, and we assume that the collected CSI is exact.

The next two phases are used for data transmission. During the second phase, s broadcasts data to RNs with transmission power P_s. The received signal at r_i is given by

$$y_i = g_i \sqrt{P_s} x_s + n_i, \ \forall\, i \in \mathcal{M},$$

where n_i is white Gaussian noise with mean zero and variance σ_i^2. Since the source-to-destination rate requirement is R, the minimum required transmission rate between s and RNs is RT/T_2. Define the set of RNs that can successfully decode the signal x_s as the *decoding set* $\mathcal{D}$, which is given by

$$\mathcal{D} = \left\{ r_i : WT_2 \log_2 \left(1 + \frac{|g_i|^2 P_s}{\sigma_i^2}\right) \geq RT \right\}. \quad (1)$$

During the third phase, some RNs in $\mathcal{D}$ are selected to cooperatively forward the data to d. Let the *cooperation set* $\mathcal{C} \subseteq \mathcal{D}$ represent the set of selected RNs. We assume the cooperative beamforming is used. The transmission power and phase adjustment of r_i are θ_i and P_i, respectively. Then, the received signal at d is given by

$$y_d = \sum_{i \in \mathcal{C}} h_i e^{j\theta_i} \sqrt{P_i} x_s + n_d,$$

where n_d is white Gaussian noise with mean zero and variance σ_d^2. To minimize the transmission power of the RNs, the optimal beamforming factors are given by

$$\theta_i = -\arg\{h_i\}, \ P_i = \frac{|h_i|^2 P_r}{\sum_{j \in \mathcal{C}} |h_j|^2}, \ \forall\, i \in \mathcal{C}, \quad (2)$$

where P_r is the total transmission power of all RNs in $\mathcal{C}$. To meet the given rate requirement, we must have

$$WT_3 \log_2 \left(1 + \frac{\sum_{i \in \mathcal{C}} |h_i|^2 P_r}{\sigma_d^2}\right) \geq RT. \quad (3)$$

B. Energy consumption model

The total energy consumption consists of both transmission energy and circuit energy. All the nodes consume transmission energy when they transmit data. We assume that when a node is in active mode, either transmitting or receiving, the circuit power is fixed and nonzero. The total energy consumption of node i is given by

$$E_i = P_i^t T_i^t + P_i^c T_i^c,$$

where P_i^t is the transmission power, P_i^c is the circuit power, T_i^t is the length of time when node i is transmitting, and T_i^c is the length of time when node i is in active mode. For simplicity, we do not consider the transmit energy consumption during the first phase. To reduce the total energy consumption, either the power or the time length needs to be reduced. In the next two sections, we discuss the energy minimization problem under fast mode transition and slow mode transition, respectively.

III. Fast Mode Transition

As mentioned in the introduction, the fast mode transition (FMT) means that the transition between sleep mode and active mode is fast enough so that the transition can happen in a single frame. In this case, all RNs will be active during the first phase. As illustrated in Fig. 2, the RNs in the cooperation set $\mathcal{C}$ will be active during data transmission, while the other RNs will transit to sleep mode so that circuit energy can be saved. The RNs which transit to sleep mode during the second phase will transit back to active mode at the end of the third

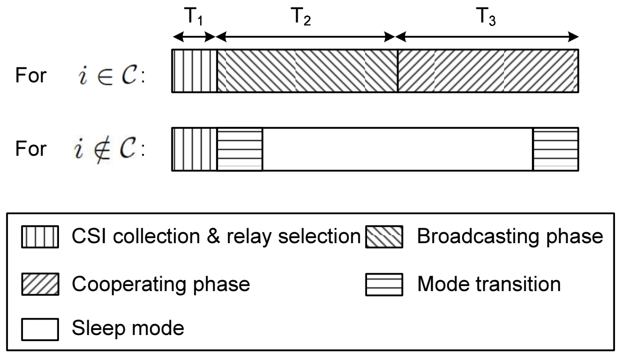

Fig. 2. Illustration of communication process and mode transition.

phase. We assume that the circuit power of r_i during mode transition is the same as P_i^c, and that the total transition time of r_i is denoted by T_i^m.

A. Problem formulation

Since all the RNs in $\mathcal{C}$ can successfully decode the received signal during the second phase, the minimum required transmission energy E_s depends on the RN with the worst channel gain in $\mathcal{C}$. According to (1), we have

$$E_s = \max_{i \in \mathcal{C}} \frac{\omega_i}{|g_i|^2},$$

where $\omega_i = (2^{RT/WT_2} - 1)\sigma_i^2 T_2$. During the third phase, the RNs in $\mathcal{C}$ cooperatively forward the data to d. According to (3), the minimum transmission energy of RNs is given by

$$E_r = \frac{\omega_d}{\sum_{i \in \mathcal{C}} |h_i|^2},$$

where $\omega_d = (2^{RT/WT_3} - 1)\sigma_d^2 T_3$. This equation implies that every RN in $\mathcal{C}$ contributes to the transmission energy saving, even if its channel gain is small. Since s and d are always active in each frame, $T_s^c = T_d^c = T$, we do not consider the circuit energy consumption of s and d in the formulation. For $i \in \mathcal{M}$, we have

$$T_i^c = \begin{cases} T, & \text{if} i \in \mathcal{C}, \\ T_1 + T_i^m, & \text{if} i \notin \mathcal{C}. \end{cases} \tag{4}$$

Given $\{g_i\}$ and $\{h_i\}$ in each frame, the energy minimization with fast mode transition (EM-FMT) problem can be formulated as

$$\min_{\mathcal{C} \subseteq \mathcal{M}} \left(\max_{i \in \mathcal{C}} \frac{\omega_i}{|g_i|^2} + \frac{\omega_d}{\sum_{i \in \mathcal{C}} |h_i|^2} \right) + \sum_{i \in \mathcal{C}} E_j^c, \tag{5}$$

where

$$E_j^c = P_i^c (T - T_1 - T_j^m).$$

Here, the fixed part of circuit energy consumption is subtracted from (4). From the objective function in (5), if the cooperation set $\mathcal{C}$ is larger, the circuit energy consumption will increase, E_s may also increase, but E_r will decrease. Hence, it is nontrivial to find the optimal cooperation set.

Since the number of all feasible cooperation sets is $2^M - 1$, the complexity of exhaustive search is extremely high when M is large. If the problem is to select a single RN, as in [20], [21], [24], there are only M possible cooperation sets, the complexity to find the optimal solution is $O(M)$.

B. Supermodularity

Let $H(\mathcal{C})$ be the objective function in (5). The following proposition characterizes the special structure of $H(\mathcal{C})$.

Proposition 1. *$H(\mathcal{C})$ is a supermodular set function of $\mathcal{C}$.*

The proof is given in [29]. Thus, the subproblem is a supermodular minimization problem, which is NP-hard in general [25]. The problem is also non-monotone [26], since $\mathcal{C}_1 \subseteq \mathcal{C}_2$ does not imply $H(\mathcal{C}_1) \geq H(\mathcal{C}_2)$ or $H(\mathcal{C}_1) \leq H(\mathcal{C}_2)$. Approximation algorithms and approximability results have been provided in the literature [26], [27], which focus on problems with special structure of constraints.

C. Local Greedy Search

Since a supermodular function has the property of *diminishing returns*, we can use a local greedy search (LGS) algorithm to find a local optimal solution. Let a function $G(i, \mathcal{C})$ defined as

$$G(i, \mathcal{C}) = H(\mathcal{C}) - H(\mathcal{C} \cup i),$$

which represent the energy reduction if r_i is added into a cooperation set $\mathcal{C}$. During the search process, $G(i, \mathcal{C})$ will serve as the selection metric. To find a cooperation set with minimal energy consumption, we start with an empty cooperation set, and always add the RN r_i with the largest $G(i, \mathcal{C})$ into the current cooperation set, until all the remaining RNs have negative selection metric or no RNs are left. The details of the LGS algorithm are described in Algorithm 1.

Algorithm 1: **L**ocal **G**reedy **S**earch

$\mathcal{C} = \emptyset$, $\mathcal{E} = \mathcal{M}$;
while $\mathcal{E} \neq \emptyset$ **do**
  Find the RN r_i in $\mathcal{E}$ with the largest $G(i, \mathcal{C})$;
  if $G(i, \mathcal{C}) > 0$ **then**
    $\mathcal{C} = \mathcal{C} \cup i$, $\mathcal{E} = \mathcal{E} \setminus i$;
  else
    break;
  end
end

The LGS algorithm is not guaranteed to find the optimal solution. For example, consider a system of three RNs with the same ω_i and $|g_i|$, $\omega_d = 1$, $|h_1|^2 = 0.5$, $E_1^c = 0.5$, and $|h_2|^2 = |h_3|^2 = 0.4$, $E_2^c = E_3^c = 0.1$. The optimal cooperation set found by the LGS algorithm is $\{r_1\}$, while the optimal cooperation set is actually $\{r_2, r_3\}$. For each instance, the selection metric $G(r_i, \mathcal{C})$ will be calculated $O(M^2)$ times at most, thus the complexity of the LGS algorithm is $O(M^2)$.

D. Relaxation Based Algorithm

To get a better approximation algorithm with low computational complexity, we adopt the relaxation method, which is often used in integer programming problems [25]. To facilitate the relaxation, we first decompose the EM-FMT problem into M subproblems. Without loss of generality, we assume $\mathcal{M}$ is arranged in a descending order of $\omega_i/|g_i|^2$. Let $\mathcal{M}_i = \{r_1, \dots, r_i\}$, and RN r_i be the *critical relay node* of $\mathcal{M}_i$. When the decoding set $\mathcal{D} = \mathcal{M}_i$, $E_s = \omega_i/|g_i|^2$. Then we can focus on the following subproblem

$$\min_{\mathcal{C}\subseteq\mathcal{M}_i} \frac{\omega_d}{\sum_{j\in\mathcal{C}}|h_j|^2} + \sum_{j\in\mathcal{C}} E_j^c. \tag{6}$$

If the optimal cooperation set is $\mathcal{C}^\star$, and $k \in \mathcal{C}^\star$ is the critical relay node, $\mathcal{C}^\star$ will also be the optimal cooperation set of the subproblem related to M_k. Thus, among the M optimal solutions to the subproblems, the one with the minimum total energy consumption will be the optimal solution to the EM-FMT problem.

For each subproblem (6), we have an equivalent form as

$$\begin{aligned} &\min \frac{\omega_d}{\sum_{j\in\mathcal{M}_i}\mu_j|h_j|^2} + \sum_{j\in\mathcal{M}_i}\mu_j E_j^c \qquad (7)\\ &\text{s.t. } \ \mu_j \in \{0,1\},\ \forall\, j \in \mathcal{M}_i, \end{aligned}$$

where $\mu_j = 1$ means that the RN r_j is contained in cooperative set $\mathcal{C}$. Then, the 0-1 variables are relaxed to continuous variables

$$\begin{aligned} &\min \frac{\omega_d}{\sum_{j\in\mathcal{M}_i}\mu_j|h_j|^2} + \sum_{j\in\mathcal{M}_i}\mu_j E_j^c \qquad (8)\\ &\text{s.t. } \ 0 \le \mu_j \le 1,\ \forall\, j \in \mathcal{M}_i. \end{aligned}$$

The optimal solution to problem (8) provides a lower bound on the original problem (7) by relaxing the constraint set. The Lagrangian of the problem (8) is given by

$$\begin{aligned} \mathcal{L}(\{\mu_j\},\{\alpha_j\},\{\beta_j\}) = &\frac{\omega_d}{\sum_{j\in\mathcal{M}_i}\mu_j|h_j|^2} + \sum_{j\in\mathcal{M}_i}\mu_j E_j^c \\ &+ \sum_{j\in\mathcal{M}_i}\alpha_j(\mu_j - 1) - \sum_{j\in\mathcal{M}_i}\beta_j\mu_j, \end{aligned}$$

where $\{\alpha_j\}, \{\beta_j\}$ are Lagrange multipliers. It is easy to verify that the relaxed problem (8) is a convex problem, and the KKT conditions are:

$$-\frac{\omega_d|h_j|^2}{\left(\sum_{j\in\mathcal{M}_i}\mu_j|h_j|^2\right)^2} + E_j^c + \alpha_j - \beta_j = 0,$$

$$\alpha_j(\mu_j - 1) = 0, \alpha_j \ge 0,$$

$$\beta_j\mu_j = 0, \beta_j \ge 0,$$

for all $j \in \mathcal{M}_i$. Then, we have

$$\text{If } \frac{\omega_d|h_j|^2}{E_j^c} > \left(\sum\nolimits_{j\in\mathcal{M}_i}\mu_j|h_j|^2\right)^2, \text{then} \quad \mu_j = 1, \tag{9}$$

$$\text{If } \frac{\omega_d|h_j|^2}{E_j^c} < \left(\sum\nolimits_{j\in\mathcal{M}_i}\mu_j|h_j|^2\right)^2, \text{then} \quad \mu_j = 0, \tag{10}$$

$$\text{If } \frac{\omega_d|h_j|^2}{E_j^c} = \left(\sum\nolimits_{j\in\mathcal{M}_i}\mu_j|h_j|^2\right)^2, \text{then } 0 < \mu_j < 1. \tag{11}$$

From (9), (10), and (11), we can observe that the righthand side can be considered as a threshold for RN selection. Let

$$\nu_j = \frac{\omega_d|h_j|^2}{E_j^c}.$$

If ν_j is larger than the threshold, r_j is selected, and vice versa. Since μ_j can be fractional, the solution to problem (8) cannot be directly applied to the original problem (6). To find a feasible solution, a standard approach is randomized rounding. Here we give a more efficient way to find a feasible solution, which uses ν_j as the RN selection metric.

First, RNs are sorted by ν_i in a descending order:

$$\hat{\mathcal{M}}_i = \{r_{[1]}, r_{[2]}, \dots, r_{[i]}\}.$$

From the previous analysis, if $r_{[i]}$ is selected, i.e. $\mu_i > 0$, then $r_{[j]}$ is also selected for all $j \le i$. Then we can add the RNs into cooperation set according to the order of $\hat{\mathcal{M}}_i$, and calculate μ_i when the RN r_i is selected. The procedure stops when we find any $\mu_i = 0$. The relaxation based algorithm (RBA) for each subproblem is summarized in Algorithm 2.

Algorithm 2: **R**elaxation **B**ased **A**lgorithm

```
M_i = {r_1, ..., r_i}, C_i = ∅ ;
Sort the RNs in M_i by ν_j as M̂_i = {r_[1], ..., r_[i]} ;
for j = 1 to i do
    Calculate μ_j according to (9), (10), and (11);
    if μ_j = 1 then
        C_i = C_i ∪ j;
    else if μ_j > 0 then
        C_i = C_i ∪ j, break;
    else
        break;
    end
end
```

For each subproblem, a sorting operation is needed. Thus, the computational complexity is $O(M\log(M))$. Therefore, the computational complexity of the RBA algorithm is $O(M^2\log(M))$, which is less than the LGS algorithm. However, the RBA algorithm is still an approximation algorithm, thus it is not guaranteed to find the optimal solution. For example, there is a system of two RNs with $\omega_d = 1$, $|h_1|^2 = 40$, $E_1^c = 10$, $|h_2|^2 = 3$, $E_2^c = 1$. In this case, the RBA algorithm will select r_1, as $|h_1|^2/E_1^c = 4 > |h_2|^2/E_2^c = 3$, however, r_2 is the optimal solution. Define

$$\delta_h = \frac{\max_i|h_i|^2}{\min_i|h_i|^2},\ \delta_p = \frac{\max_i E_i^c}{\min_i E_i^c}, \tag{12}$$

we now proof the following results about the worse case performance guarantee of RBA.

Proposition 2. *Assume all* $\mathbf{h} = \{|h_i|^2\}, \mathbf{E}^c = \{E_i^c\}$ *are real numbers, then there is at most one RN* r_i *which has* $\mu_i \in (0, 1)$.

Let $\delta = \min\{\delta_h, \delta_p\}$, $OPT(\mathbf{h}, \mathbf{E}^c)$ be the optimal solution, $RBA(\mathbf{h}, \mathbf{E}^c)$ be the solution of the RBA. Then we have

$$\frac{RBA(\mathbf{h}, \mathbf{E}^c)}{OPT(\mathbf{h}, \mathbf{E}^c)} \leq 1 + \delta. \tag{13}$$

The proof is given in Appendix A.

The above proposition shows that if the RNs are similar to each other, either in the channel gains or circuit energy consumption, the RBA algorithm has a good performance guarantee. This means when RNs are spatially adjacent and the fading effect is small, or the circuit energy consumption of RNs has the same order of magnitude, the RBA algorithm has better performance guarantee.

IV. Slow Mode Transition

When the transition time T_i^m is larger than the frame length T, fast mode transition is infeasible. Therefore, we consider another case of slow mode transition (SMT), which means that the RNs will remain in some working modes for a long period of time. We assume that in the period of interest, the set of active RNs is fixed, which is denoted by $\mathcal{A}$. The frame structure is the same as in the case of FMT. The difference is that during the first phase, only the RNs in $\mathcal{A}$ are active, which means that

$$T_i^c = \begin{cases} T, & \text{if } i \in \mathcal{A}, \\ 0, & \text{if } i \notin \mathcal{A}. \end{cases}$$

The energy minimization with slow mode transition (EM-SMT) problem is formulated as:

$$\min_{\mathcal{A} \subseteq \mathcal{M}} \sum_{i \in \mathcal{A}} E_i^c + \mathbb{E}\left\{ \min_{\mathcal{C} \subseteq \mathcal{A}} \max_{i \in \mathcal{C}} \frac{\omega_i}{|g_i|^2} + \frac{\omega_d}{\sum_{i \in \mathcal{C}} |h_i|^2} \right\} \tag{14}$$

where $E_i^c = P_i^c T$ and the expectation is taken with respect to the channel gains $\{g_i\}, \{h_i\}$.

Compared with the EM-FMT problem, the EM-SMT problem considers the long term energy consumption. Since some RNs are in sleep mode in the period of interest, the spatial diversity due to relay selection is reduced, and the transmission energy consumption will be larger. On the other hand, there is less overhead in the case of SMT, thus in practice, it is unclear whether FMT or SMT is more efficient. For example, when all the channels are Gaussian, the EM-SMT problem is equivalent to the EM-FMT problem, and SMT will be more energy efficient than FMT, since no overhead is required for the RNs not in the cooperation set $\mathcal{C}$.

A. Problem Analysis

If we know the probability distribution of the channel gains $\{g_i\}, \{h_i\}$, the expectation in (13) can be calculated given the active set $\mathcal{A}$. The optimal total transmission energy consumption when the active set is $\mathcal{A}$ is given by

$$S(\mathcal{A}) = \mathbb{E}\left\{ \min_{\mathcal{C} \subseteq \mathcal{A}} \left(\max_{i \in \mathcal{C}} \frac{\omega_i}{|g_i|^2} \right) + \frac{\omega_d}{\sum_{i \in \mathcal{C}} |h_i|^2} \right\},$$

and the total energy consumption is given by

$$T(\mathcal{A}) = S(\mathcal{A}) + \sum_{i \in \mathcal{A}} P_i^c.$$

Similar to Proposition 1, we can have following results about the problem structure.

Proposition 3. *$T(\mathcal{A})$ is a supermodular set function of $\mathcal{A}$ if $S(\mathcal{A})$ is a supermodular function, that is to say*

$$S(\mathcal{A}_1 \cup k) - S(\mathcal{A}_1) \leq S(\mathcal{A}_2 \cup k) - S(\mathcal{A}_2), \tag{15}$$

for any $\mathcal{A}_1 \subseteq \mathcal{A}_2 \subseteq \mathcal{M}$, $k \in \mathcal{M} \backslash \mathcal{A}_2$.

If $S(\mathcal{A})$ is supermodular set function of $\mathcal{A}$, then we can develop efficient greedy search algorithms to find provable solution to the EM-SMT problem. However, (15) does not always hold. Although $S(\mathcal{A})$ is not supermodular in general, we can show that in many cases, it can be approximated as a supermodular function.

Proposition 4. *If the transmission energy consumption of source node E_s is the same for all $\mathcal{A}$, then $S(\mathcal{A})$ is supermodular.*

The proof is given in [29]. The condition in Proposition 4 holds in the scenario when the number of RNs is large. In this case, adding one more RN into $\mathcal{A}$ would only marginally change the E_s. Another scenario is when the transmission power during the second phase is fixed, then the condition also holds. In these cases, $S(\mathcal{A})$ is a supermodular function, and then we can use approaches similar to those for the EM-FMT problem to find good solutions.

For the general case, it is hard to find an approach which is applicable to all EM-SMT problems with different channel distributions. The following summarize the hardness result of the EM-SMT problem.

Proposition 5. *The EM-SMT problem with single relay selection is NP-hard, since any set covering problem can be reduced to an EM-SMT problem.*

The proof is given in [29].

B. Heuristic Algorithms

Since it is difficult to find the optimal solution, we propose heuristic algorithms to find a suboptimal solution. The main idea is to approximate the expectation. By directly taking expectation over the random variables, the EM-SMT problem can be approximated as

$$\min_{\mathcal{A} \subseteq \mathcal{M}} \sum_{i \in \mathcal{A}} E_i^c + \min_{\mathcal{C} \subseteq \mathcal{A}} \left(\max_{i \in \mathcal{C}} \frac{\omega_i}{\mathbb{E}|g_i|^2} \right) + \frac{\omega_d}{\sum_{i \in \mathcal{C}} \mathbb{E}|h_i|^2} \tag{16}$$

It should be noted that the transmission energy consumption is underestimated here. A better approximation can lead to an algorithm with better performance. The problem (16) has the same structure as the EM-FMT problem. Thus, it can be solved using the algorithms proposed in Section III.

When the channels are Rayleigh distribution and the energy consumption will be infinite if only one RN is selected. There

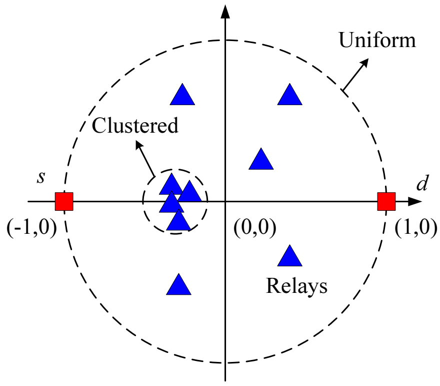

Fig. 3. Illustration of relay nodes deployment.

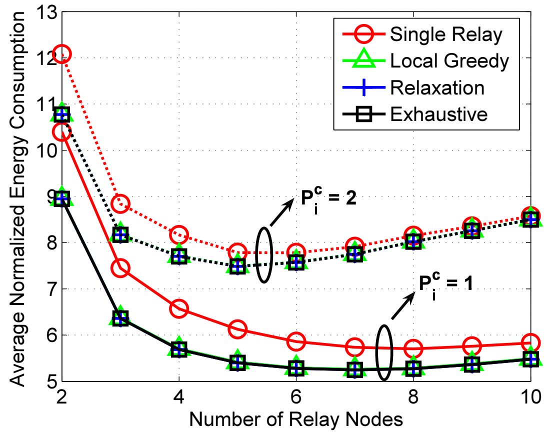

Fig. 4. Total normalized energy consumption of different algorithms for the EM-FMT problem, with clustered deployment. Cluster location is (-0.5, 0), and circuit power P_i^c is 1 or 2.

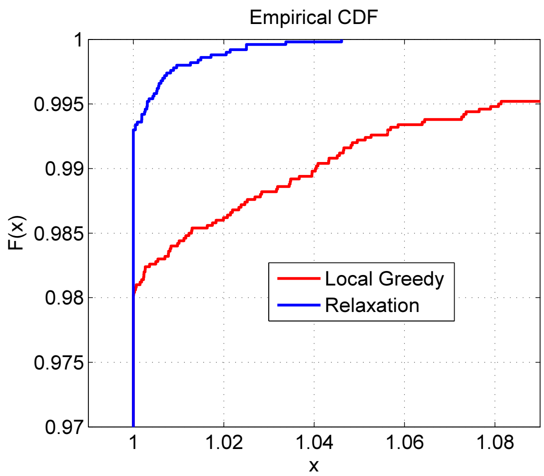

Fig. 5. The empirical cumulative distribution function of the performance ratio of LGA and RBA algorithms (The optimal solution is given by the exhaustive search).

should be two RNs at least to provide enough spatial diversity. Thus, a constraint $|\mathcal{A}| \geq 2$ is made to the above algorithms.

V. Performance Evaluation

In this section, simulation results are presented to evaluate the performance of the proposed algorithms. Throughout the simulation, the noise power is assumed to be 1. Both circuit power and transmission power are normalized. All the nodes are placed in a circle centered at the origin of the x-y plane with radius $r = 1$. The source node and destination node are located at $(-1, 0)$ and $(1, 0)$, respectively. All the channels are assumed to be Rayleigh fading, with the average power gain $h_{i,j} = 1/d^{\nu}$, where d is the distance between i and j, and $\nu = 3$ is the path loss exponent. The frame length T is normalized to 1. Unless specified, $T_1 = 0.1$, $T_2 = 0.5$, $T_3 = 0.4$, the circuit power is 1 unit, and the spectral efficiency requirement is 1 bits/s/Hz.

A. Fast Mode Transition

We first evaluate the proposed algorithms for the EM-FMT problem. We consider two types of relay node deployment: clustered and uniform, as shown in Fig. 3. In the clustered case, all the RNs are located near to each other and form a cluster. The position of the cluster moves along the line segment from s to d. In the uniform case, the RNs are uniformly distributed in the circle.

In Fig. 4, we compare the performance of single relay selection with multiple relay selection in the case of clustered deployment. For the multiple relay selection, three algorithms are compared, including Local Greedy Search (LGS), Relaxation Based Algorithm (RBA), and exhaustive search (Exhaustive). The results are averaged over 5000 cases. It shows that when the number of RNs increases, as much as half of the total energy can be saved with more RNs in the cooperation. Compared with single relay selection, multiple relay selection can save as much as 18% when circuit power is 1, and 10% when circuit power is 2, in the case of two RNs. This means cooperative beamforming is beneficial for energy saving, especially when the transmission power is the bottleneck of energy consumption. As the number of RNs becomes larger, the circuit energy consumption begins to dominate, and beyond a certain point the total energy consumption start to increase. Fig. 4 also shows that both the RBA algorithm and the LGS algorithm perform close to the optimal solution determined by exhaustive search, with the curves almost overlapping with each other. In Fig. 5, we compare the empirical cumulative distribution functions of the performance ratio of LGA and RBA algorithms. It shows that in most cases, the two algorithm perform exactly the same as the exhaustive search, but RBA algorithm has better worst case guarantee than the LGA algorithm.

In Fig. 6, the case of uniform deployment is considered. Here again, cooperative transmission results in substantial energy savings. The difference from the previous case is that the single relay selection performs quite close to the multiple relay selection, which means the benefit of cooperative beam-

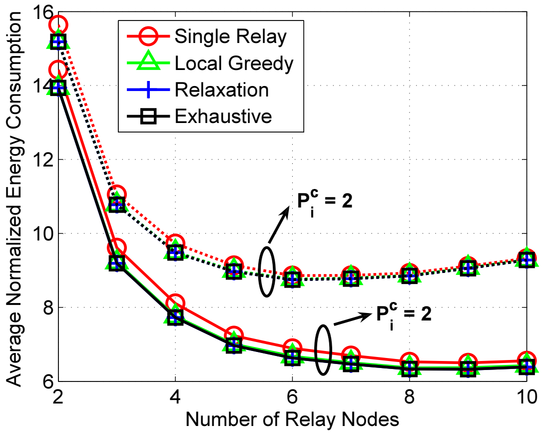

Fig. 6. Total normalized energy consumption of different algorithms for the EM-FMT problem, with uniform deployment. Circuit power P_i^c is 1 or 2.

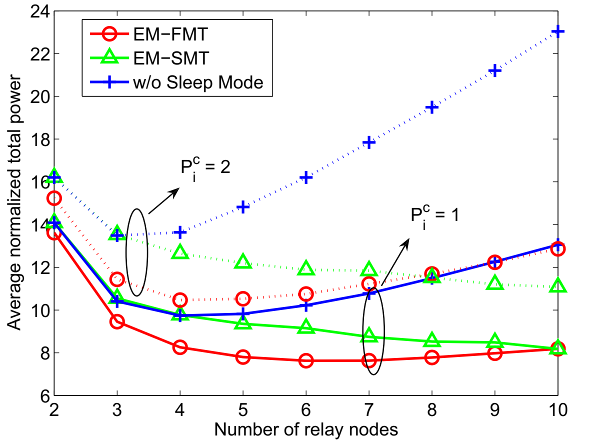

Fig. 8. Comparison between fast mode and slow mode transition with varying number of RNs.

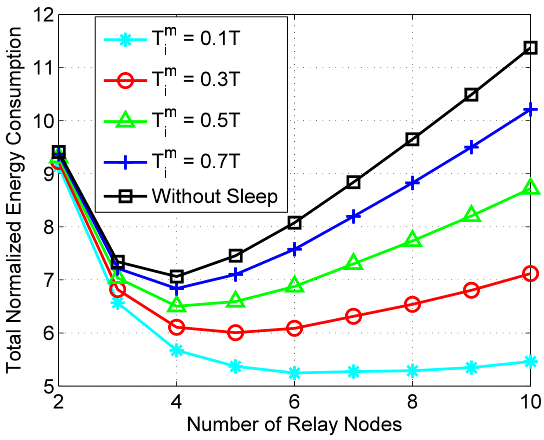

Fig. 7. The effects of mode transition time T_i^m. Clustered deployment wit cluster position (-0.5,0), and circuit power P_i^c is 1.

forming is not much. This is because the channel gain variance of different RNs are quite large due to the path loss, and most of the RNS do not contribute in the cooperation.

In Fig. 7, we investigate the effects of the mode transition time T_i^m. It shows that when the length of mode transition time is small, e.g. $T_i^m = 0.1T$, significant energy can be saved especially when the number of RNs is large. However, when the length of transition time becomes larger, the circuit energy consumption of the RNs will become a burden for energy saving. For example, when $T_i^m = 0.7T$, there is little benefit for mode transition.

B. Slow mode transition

In Fig. 8, we compare the slow mode transition, fast mode transition and the extreme case without mode transition. The uniform RN deployment scenario is considered. We use the RBA algorithm for the EM-FMT problem, and use the ACG algorithm, which proposed in Section IV-B for the EM-SMT problem. For the SMT case, some RNs will always be in sleep mode, and do not participate in the communication process, even if the channel condition is favorable. Thus, the cooperative diversity is smaller than in the FMT case. When the number of RNs is small, the transmission energy consumption dominates, thus the performance of SMT is worse than that of FMT. Since the RNs which are not selected by the SMT algorithm are usually in the worse positions, SMT still performs better then the extreme case without mode transition. When the number of RNs becomes larger, the circuit energy consumption dominates and the SMT can even perform better than the FMT.

VI. Conclusion

In this paper, we have considered the energy minimization problem in cooperative relay networks with sleep modes. Two different time scales of mode transition were considered. In the case of fast transition, we showed that the problem is a supermodular minimization problem. A local greedy search algorithm and an efficient relaxation based algorithm with provable performance bounds were proposed, and the proposed algorithms perform close to optimal in the simulation. In the case of slow mode transition, we showed that the problem is NP-hard, and developed algorithms based on the supermodular structure. Our simulations study is used to understand the performance of our proposed solutions, and illustrates the performance gain of our solutions over cases that do not exploit the sleep modes.

Appendix A
Proof of Proposition 2

Proof: If we have two RNs r_i and r_j which have $0 < \mu_i, \mu_j < 1$, then by Eq. (11), we have

$$\frac{\omega_d |h_i|^2}{E_i^c} = \frac{\omega_d |h_|^2}{E_j^c}. \tag{17}$$

Since $\mathbf{h} = \{|h_i|^2\}, \mathbf{E}^c = \{E_i^c\}$ are real numbers, the above equation occurs with the probability of 0. Thus there is at most one RN r_i which satisfy $0 < \mu_i < 1$. This means that for any RN r_j before r_i, $\mu_j = 1$, and for any RN r_k after r_i, $\mu_k = 0$.

Assume r_k is the last RN that has non-zero μ_k. Since the optimal solution to (8) is a lower bound to (7), we have

$$OPT(\mathbf{h}, \mathbf{E}^c) \geq \frac{\omega_d}{\sum_{i=1}^{k-1} |h_i|^2 + \mu_k |h_k|^2} + \sum_{i=1}^{k-1} E_i^c + \mu_k E_k^c. \tag{18}$$

The optimal solution of RBA algorithm is to select either the first $k-1$ RNs or k RNs, depending on which one has the less power consumption. Let $E^*(k)$ denote the power consumption when first k RNs is selected by RBA algorithm, which is given by

$$E^*(k) = \frac{\omega_d}{\sum_{i=1}^{k} |h_i|^2} + \sum_{i=1}^{k} E_i^c.$$

Then

$$RBA(\mathbf{h}, \mathbf{E}^c) = \min\{E^*(k-1), E^*(k)\}.$$

Thus

$$\begin{aligned} \frac{RBA(\mathbf{h}, \mathbf{E}^c)}{OPT(\mathbf{h}, \mathbf{E}^c)} &\leq \frac{\frac{\omega_d}{\sum_{i=1}^{k-1} |h_i|^2} + \sum_{i=1}^{k-1} E_i^c}{\frac{\omega_d}{\sum_{i=1}^{k-1} |h_i|^2 + \mu_k |h_i|^2} + \sum_{i=1}^{k-1} E_i^c + \mu_k E_k^c} \\ &\leq \frac{\sum_{i=1}^{k-1} |h_i|^2 + \mu_k |h_i|^2}{\sum_{i=1}^{k-1} |h_i|^2} \\ &\leq 1 + \frac{\mu_k \delta_h}{k-1} \leq 1 + \delta_h. \end{aligned}$$

Similarly, we can have

$$\frac{RBA(\mathbf{h}, \mathbf{E}^c)}{OPT(\mathbf{h}, \mathbf{E}^c)} \leq 1 + \delta_p. \tag{19}$$

Hence the proposition holds. ■

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