Half-Duplex Decode-Partial-Forward

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***Abstract*—We propose a new transmission scheme, which we call Decode-Partial-Forward (DPF), for orthogonal half-duplex relay channels. We demonstrate that the DPF scheme achieves significantly higher rates than the conventional DF scheme that is based on repetition coding and it performs close to the capacity bounds of DF relaying. In DPF, the source superimposes two separate codewords and the relay decodes both codewords but it regenerates (forwards) "the more difficult to decode" codeword only. The destination node applies successive interference cancellation in order to decode the two codewords based on the received signals in both time slots. We provide a closed-form solution to find the optimal power and rate allocation for the two superimposed codewords in order to maximize the sum achievable rate. The DPF scheme provides a simple and practical alternative to the advanced cooperative coding techniques.**

***Index Terms*—Decode-and-forward; FD-AWGN relay channel; superposition coding; successive decoding**

I. Introduction

In this work, we consider the three-node relay channel in which a source node wants to send a message to a destination node with the assistance of a third node; the relay. In particular we are interested in the frequency-division additive white Gaussian noise "FD-AWGN" relay channel, which was studied in [1]. In this channel model, the source and the relay transmit their signals using orthogonal channels. This channel model is applicable for half-duplex relays and orthogonal relaying protocols in which the source node transmits in the first time slot while the relay and destination nodes listen. Then in the second time slot the relay node transmits and the destination node utilizes the two received messages in both time slots in order to decode the source message. The best known achievable schemes for the FD-AWGN relay channel are decode-and-forward (DF) and compress-and-forward (CF) [1]. However, the capacity is not known in general although DF achieves capacity in special cases. In this work, we consider decode and forward relaying schemes and we are particularly interested in the case when the direct link from the source node to the destination node is non-negligible, but a relay is used to enhance the transmission capacity over the channel.

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The information-theoretic achievable rates upper-bounds of DF schemes require using cooperative coding schemes in which the relay node generates a complementary part of the transmitted codeword by the source node in order to enable the destination node to decode the codeword reliably. However, approaching the capacity of DF in relay channels via cooperative coding is by no means a straightforward extension of capacity-achieving "direct communication" codes that are well-known in the literature, e.g. [2]. The cooperative coding problem introduces new fundamental problems regarding what the relay should forward in order to enable the destination node to decode the source message reliably. In principle, the relay codeword should take into consideration the channel strength (signal-to-noise ratio SNR) of both the relay-destination and the source-destination links. The design of cooperative coding schemes have been studied in the literature and some examples of the proposed schemes include cooperative coding based on bilayer Low-Density Parity-Check (LDPC) codes [3], rate compatible punctured convolutional codes [4], distributed convolutional codes [5], distributed turbo codes [6], threshold-based distributed turbo coding [7] and incremental redundancy [8]. Although some of the above mentioned papers were studying cooperation diversity schemes in which mobile users help each other, the extension to relay-assisted transmission in which the relays are not users themselves is straightforward.

In the communication theory literature, the term decode-forward (DF) is widely used to refer to the repetition coding scheme that is easier to implement than the cooperative coding scheme, though it is not optimal in general. In the decode-forward repetition coding scheme (DF-RC), the relay decodes the codeword of the source node and it regenerates the same codeword (noise-free) in order to provide the destination node with two copies of the source message. The receiver at the destination combines the two copies using maximal ratio combining

(MRC) [9] before decoding the message. Note that this scheme requires that the two time slots of the relaying protocol to have equal length since the same codeword is transmitted twice. This is a valid assumption in our work as well. The regenerative (i.e. repetition coding) DF scheme was proposed and studied in the literature in [10] and [11]. Although the cooperative coding scheme outperforms the regenerative DF scheme [12], the latter is preferred for practical implementation in relay-assisted networks for the next generation broadband systems such as in LTE-Advanced [13].

In this paper, we propose a novel scheme that outperforms the conventional repetition coding scheme and performs close to the capacity bounds of the decode-forward relaying without the need to apply cooperative coding. In our proposed scheme, which we call Decode-Partial-Forward (DPF), the relay node applies repetition coding, but instead of repeating the whole message of the source node, it regenerates only part of the total message. The DPF scheme involves superposition coding with successive interference cancellation at the receiver [14]. The scheme description is provided in Section II. We should mention here that a "partial repetition coding" scheme was proposed in [15]. However, our proposed scheme is different than the one in the aforementioned paper which does not involve superposition coding. Furthermore, our optimization and analysis is based on achievable rates and the work in [15] is based on outage probabilities.

We discuss in Section III how our proposed DPF scheme can be optimized in order to maximize the achievable rate over the relay channel. The DPF scheme can be flexibly adapted according to the channel conditions of the three links of the relay channel (i.e. source-relay, source-destination, relay-destination). The scheme can be flexibly optimized using power control and code rate adaptation.

We demonstrate, through several numerical examples in Section IV, the capacity gains of DPF over DF-RC. Furthermore, we summarize the main conclusions of this work in Section V.

II. DPF Mode of Operation

The mode of operation is summarized in a diagram in Fig. 1. In the first time slot, the source node superimposes two separate messages (codewords) L_1 and L_2 with proper power and rate allocation. The relay node decodes L_1 treating L_2 as noise, removes L_1 after decoding it, then decodes L_2 (i.e. successive interference cancellation concept). The destination node is assumed to be unable to decode neither L_1 nor L_2 based on the received message from the source.

In the second time slot, the relay node transmits L_2 only using repetition coding (regenerates the same codeword transmitted by the source node). The destination node decodes L_2 first using the two copies from first and second time slots using maximal ratio combining (MRC), e.g.[9], treating L_1 received in the first slot as interference (i.e. noise), then it removes L_2 and decodes L_1 that was received in the first time slot.

A. General remarks on the DPF Scheme

The objective of this work is not to outperform the achievable rates by DF, but rather to provide a simple and practical scheme that operates close to the capacity bounds of DF. The advantages of the proposed DPF scheme are mainly from practical implementation perspectives since DPF involves repetition coding at the relay which is preferred in practice and it simplifies the decoder structure at the destination by applying MRC. Furthermore, the DPF scheme can be constructed using any capacity-achieving codes that are used in current communication standards.

Based on the DPF scheme, the relay assists the destination in decoding the transmitted messages from the source using two different strategies:

- Regenerative forwarding (repetition coding) of one part of the total message.
- Interference mitigation (cancellation) for the other part of the message, which enables the destination node to decode L_1 without the interference of L_2.

Note that the relay node cannot decode L_2 directly treating L_1 as interference. The power allocation done in the source node enables decoding L_1 first at the relay. Thus, the relay node has to decode all information sent by the source, although it forwards one part of the received messages. That's why this scheme is different than partial-decode-forward, e.g. [16], [17], because it is full-decode-partial-forward.

The design parameters of the DPF scheme are:

- The information rates (bits/sec) of L_1 and L_2. Note that in principle L_1 and L_2 can have different modulation constellation and different channel coding rate. Thus, their information rates can be different although they are transmitted within the same time slot.
- The power ratios of the total source power that is allocated to L_1 and L_2.

These parameters are subject to optimization in order to maximize the overall (sum) achievable rate.

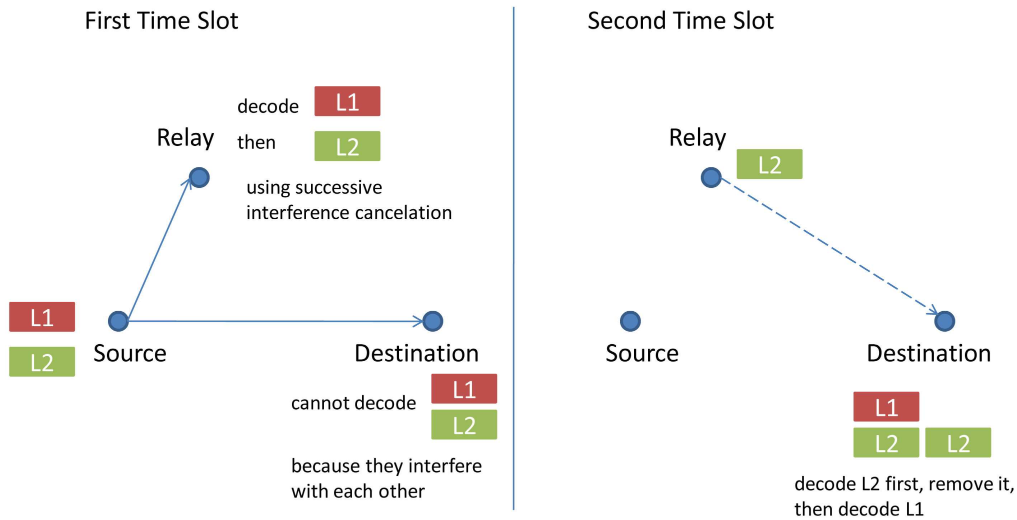

Fig. 1. System Diagram for the Decode-Partial-Forward Relaying Scheme

B. mathematical Notation and Upper and Lower Bounds on Achievable Rates

We use the notations (γ_{sd}, γ_{sr}, γ_{rd}) respectively to denote the signal-to-noise-ratio (SNR) of the (source-destination, source-relay, relay-destination) links. We use R_1, R_2 and R respectively to denote the achievable rates by codeword L_1 and codeword L_2 and the total (sum of both) achievable rate at the destination node. For simplicity, we assume the bandwidth to equal unity and hence we present the results of achievable rates as spectral efficiencies (bits/sec/Hz). Modifying the equations to include bandwidth is straightforward. Also, we use the notations of α_1 and α_2 for the ratio of the source node power that is allocated to codeword L_1 and L_2 respectively ($\alpha_1 + \alpha_2 = 1$). The total power is assumed to be fixed and not subject to optimization.

In a mathematical context, the upper bound of the achievable rate by using DF relaying can be represented as [1]

$$R^{\text{up}} = \min \left\{ \begin{array}{l} \frac{1}{2}\log\left(1+\gamma_{sr}\right), \\ \frac{1}{2}\log\left(1+\gamma_{sd}\right) + \frac{1}{2}\log\left(1+\gamma_{rd}\right) \end{array} \right. \quad (1)$$

Furthermore, the repetition-coding decode-forward scheme achieves a lower bound that can be represented as [11]

$$R^{\text{RC}} = \min \left\{ \begin{array}{l} \frac{1}{2}\log\left(1+\gamma_{sr}\right), \\ \frac{1}{2}\log\left(1+\gamma_{sd}+\gamma_{rd}\right) \end{array} \right. \quad (2)$$

It is clear that DF-RC achieves the upper bound of DF only when the rate is bounded by the source-relay link (i.e. $\gamma_{sr} < \gamma_{sd} + \gamma_{rd}$). Otherwise, DF-RC is not optimal.

III. Optimization of DPF

Our objective is to optimize the rates and power allocation of L_1 and L_2 in order to maximize the sum achievable rate over the channel. The optimization is based on the knowledge of the channel gains γ_{sr}, γ_{rd} and γ_{sd} which are assumed to be constant, i.e. additive white Gaussian noise (AWGN) channels. Note that DF-RC is a special case of DPF with $\alpha_1 = 0$ and $R_1 = 0$. Thus, the achievable rates using the optimized DPF scheme does always outperform or at least equal the achievable rates using the DF-RC scheme. Before formulating the optimization problem and solving it, we start by characterizing the achievable rates bounds for L_1 and L_2 at both the relay node and the destination node.

A. Achievable Rates Characterizations

At the relay node, L_1 is decoded first then L_2. Thus, the achievable rates can be expressed as

$$R_1 \le \frac{1}{2}\log\left(1+\frac{\gamma_{sr}\alpha_1}{1+\gamma_{sr}\alpha_2}\right) \quad (3)$$

$$R_2 \le \frac{1}{2}\log\left(1+\gamma_{sr}\alpha_2\right) \quad (4)$$

The destination node decodes L_2 then L_1. In order to decode L_2, it applies maximal Ratio Combining (MRC) of the two received messages in the two time slots while treating the received L_1 in first time slot as noise. Thus, the achievable rates can be expressed as

$$R_2 \le \frac{1}{2}\log\left(1+\gamma_{rd}+\frac{\gamma_{sd}\alpha_2}{1+\gamma_{sd}\alpha_1}\right) \quad (5)$$

$$R_1 \leq \frac{1}{2}\log\left(1+\gamma_{sd}\alpha_1\right) \tag{6}$$

B. Optimization Problem Formulation – Maximizing Achievable Rate

We have four optimization variables α_1, α_2, R_1 and R_2 and we want to maximize the sum achievable rate $R = R_1 + R_2$. Thus, We can write the optimization problem as

$$\max_{\alpha_1,\alpha_2,R_1,R_2} R_1 + R_2 \tag{7a}$$

$$\text{subject to} \quad (3),\ (4),\ (5),\ (6),$$

$$\alpha_1 \geq 0,\ \alpha_2 \geq 0,\ \alpha_1+\alpha_2 = 1. \tag{7b}$$

Alternatively, we can write the optimization problem in a simpler form as

$$\max_{\alpha_1} R_1 + R_2 \tag{8a}$$

$$\text{subject to} \quad 0 \leq \alpha_1 \leq 1, \tag{8b}$$

$$\text{where,} \quad \alpha_2 = 1-\alpha_1, \tag{8c}$$

$$R_1 = \min\left\{\begin{array}{l}\frac{1}{2}\log\left(1+\frac{\gamma_{sr}\alpha_1}{1+\gamma_{sr}\alpha_2}\right),\\ \frac{1}{2}\log\left(1+\gamma_{sd}\alpha_1\right)\end{array}\right. \tag{8d}$$

$$R_2 = \min\left\{\begin{array}{l}\frac{1}{2}\log\left(1+\gamma_{sr}\alpha_2\right),\\ \frac{1}{2}\log\left(1+\gamma_{rd}+\frac{\gamma_{sd}\alpha_2}{1+\gamma_{sd}\alpha_1}\right)\end{array}\right. \tag{8e}$$

Note that (8d) is obtained by combining (3) and (6). Similarly, (8e) is obtained from (4) and (5).

Another alternative form is

$$\max_{\alpha_1} \quad \min\left\{R_1^r+R_2^r,\ R_1^r+R_2^d,\ R_1^d+R_2^r,\ R_1^d+R_2^d\right\} \tag{9a}$$

$$\text{subject to} \quad 0 \leq \alpha_1 \leq 1, \tag{9b}$$

$$\text{where,} \quad \alpha_2 = 1-\alpha_1, \tag{9c}$$

$$R_1^r = \frac{1}{2}\log\left(1+\frac{\gamma_{sr}\alpha_1}{1+\gamma_{sr}\alpha_2}\right), \tag{9d}$$

$$R_1^d = \frac{1}{2}\log\left(1+\gamma_{sd}\alpha_1\right), \tag{9e}$$

$$R_2^r = \frac{1}{2}\log\left(1+\gamma_{sr}\alpha_2\right), \tag{9f}$$

$$R_2^d = \frac{1}{2}\log\left(1+\gamma_{rd}+\frac{\gamma_{sd}\alpha_2}{1+\gamma_{sd}\alpha_1}\right). \tag{9g}$$

C. Closed Form Optimal Solution

The optimization problem in Section III-B can in principle be solved by a one dimensional exhaustive search over $0 \leq \alpha_1 \leq 1$. Although this problem is not a convex optimization problem, we provide a simple closed-form analytic solution.

We solve the optimization problem under a set of assumptions which are relevant from practical considerations. The set of assumptions includes

$$\gamma_{sr} > \gamma_{sd}, \tag{10a}$$

$$\gamma_{rd} > \gamma_{sd}, \tag{10b}$$

$$\gamma_{sr} > \gamma_{sd} + \gamma_{rd} \tag{10c}$$

Note that if (10a) or (10b) are not valid, DF cannot achieve a higher rate than direct transmission using both time slots. Also, if (10c) is not valid, repetition coding becomes optimal. Furthermore, note that under all possible values for $\gamma_{sr} > 0$, $\gamma_{rd} > 0$ and $\gamma_{sd} > 0$, the following two conditions are always true $\frac{d}{d\alpha_1}R_1 > 0$ and $\frac{d}{d\alpha_1}R_2 < 0$ where R_1 and R_2 are shown in (8d) and (8e) respectively. On the other hand, we cannot judge on the sign of the derivative of the sum rate (i.e. $\frac{d}{d\alpha_1}[R_1+R_2]$). It can in principle be positive or negative, and hence we may have multiple local peaks. The problem is obviously non-convex. Note that at $\alpha_1 = 0$, R_2 in (8e) becomes similar to the achievable rate of DF-RC shown in (2).

Lemma 1 (Optimal Solution): Under the set of assumptions in (10), the optimal solution can be achieved at the point where the two terms of (8e) equal each other, i.e.

$$\gamma_{sr}\alpha_2 = \gamma_{rd} + \frac{\gamma_{sd}\alpha_2}{1+\gamma_{sd}\alpha_1}. \tag{11}$$

If the two terms do not intersect within $0 \leq \alpha_1 \leq 1$, then DF-RC is the optimal solution (i.e. $\alpha_1 = 0$), which means that only the codeword L_2 is transmitted.

Equation (11) can be written as a quadratic equation and by simple manipulations we can formulate a closed form procedure to obtain the optimal solution. The optimization steps are:

- Based on γ_{sr}, γ_{sd} and γ_{rd}, obtain

$$A = 1 - \frac{\gamma_{rd}}{\gamma_{sr}} - \frac{1}{\gamma_{sd}} + \frac{1}{\gamma_{sr}} \tag{12a}$$

$$B = \frac{1}{\gamma_{sd}}\left(1-\frac{\gamma_{rd}}{\gamma_{sr}}\right) - \frac{1}{\gamma_{sr}} \tag{12b}$$

- Find optimal α_1 according to:
  If $A^2 + 4B > 0$, then

$$\alpha_1 = \max\left\{0, \frac{1}{2}A + \frac{1}{2}\sqrt{A^2+4B}\right\}, \tag{13}$$

  otherwise, $\alpha_1 = 0$.
- Obtain α_2, R_1 and R_2 using (8c), (8d) and (8e) respectively by substituting the optimal value for α_1.

Note that in general the optimal solution of the optimization problem in Section III-B might not be unique for some ranges of γ_{sr}, γ_{sd} and γ_{rd}. However, using the simple procedure described above achieves optimality at one of the possible optimal values of α_1.

We provide below the proof for Lemma 1.

Proof: With the notation $R_1^d(\alpha_1) = \frac{1}{2}\log(1+\gamma_{sd}\alpha_1)$ and $R_1^r(\alpha_1) = \frac{1}{2}\log\left(1+\frac{\gamma_{sr}\alpha_1}{1+\gamma_{sr}(1-\alpha_1)}\right) = \frac{1}{2}\log\left(\frac{1+\gamma_{sr}}{1+\gamma_{sr}(1-\alpha_1)}\right)$, we can write $R_1(\alpha_1) = \min\left\{R_1^d(\alpha_1), R_1^r(\alpha_1)\right\}$.

Note that R_1^r is monotonically increasing and convex, while R_1^d is monotonically increasing and concave. Furthermore, $R_1^r(0) = R_1^d(0) = 0$ and $R_1^r(1) = \frac{1}{2}\log(1+\gamma_{sr}) > R_1^d(1) = \frac{1}{2}\log(1+\gamma_{sd})$, under the assumption (10a). Furthermore, $\frac{dR_1^d}{d\alpha_1}(0) = \frac{\gamma_{sd}}{2}$ and $\frac{dR_1^r}{d\alpha_1}(0) = \frac{\gamma_{sr}}{2(1+\gamma_{sr})}$. Thus, if $\gamma_{sd} < \frac{\gamma_{sr}}{1+\gamma_{sr}}$, then $R_1^d \leq R_1^r$ with equality achieved at $\alpha_1 = 0$ only. Hence, $R_1 = R_1^d$ in this case. On the other hand, if $\gamma_{sd} > \frac{\gamma_{sr}}{1+\gamma_{sr}}$, then R_1^r and R_1^d intersects once within $\alpha_1 \in (0,1)$ at $\alpha_1 = 1 - \frac{1}{\gamma_{sd}} + \frac{1}{\gamma_{sr}}$. Thus, we can write R_1 in the following form

$$R_1 = \begin{cases} R_1^r = \frac{1}{2}\log\left(1+\frac{\gamma_{sr}\alpha_1}{1+\gamma_{sr}\alpha_2}\right) & : 0 \leq \alpha_1 \leq \kappa \\ R_1^d = \frac{1}{2}\log\left(1+\gamma_{sd}\alpha_1\right) & : \kappa \leq \alpha_1 \leq 1 \end{cases} \tag{14}$$

where κ is defined as

$$\kappa = \max\left\{1 - \frac{1}{\gamma_{sd}} + \frac{1}{\gamma_{sr}}, 0\right\} \tag{15}$$

Similarly, with the notation $R_2^r(\alpha_1) = \frac{1}{2}\log\left(1+\gamma_{sr}(1-\alpha_1)\right)$ and $R_2^d(\alpha_1) = \frac{1}{2}\log\left(1+\gamma_{rd}+\frac{\gamma_{sd}(1-\alpha_1)}{1+\gamma_{sd}\alpha_1}\right) = \frac{1}{2}\log\left(\gamma_{rd}+\frac{1+\gamma_{sd}}{1+\gamma_{sd}\alpha_1}\right)$, we can write $R_2(\alpha_1) = \min\left\{R_2^d(\alpha_1), R_2^r(\alpha_1)\right\}$.

Note that R_2^r is monotonically decreasing and concave, while R_2^d is monotonically decreasing and convex. Furthermore, $R_2^d(0) = \frac{1}{2}\log(1+\gamma_{sd}+\gamma_{rd}) < R_2^r(0) = \frac{1}{2}\log(1+\gamma_{sr})$, under the assumption (10c). Also, $R_2^d(1) = \frac{1}{2}\log(1+\gamma_{rd}) > R_2^r(1) = 0$. Thus, R_2^r and R_2^d intersects once within $\alpha_1 \in (0,1)$ at the point the satisfies (11). This gives a quadratic equation with one positive root and one negative root. We are interested in the positive solution sine $\alpha_1 \geq 0$. We can write R_2 in the following form

$$R_2 = \begin{cases} \frac{1}{2}\log\left(1+\gamma_{rd}+\frac{\gamma_{sd}\alpha_2}{1+\gamma_{sd}\alpha_1}\right) & : 0 \leq \alpha_1 \leq \epsilon \\ \frac{1}{2}\log\left(1+\gamma_{sr}\alpha_2\right) & : \epsilon \leq \alpha_1 \leq 1 \end{cases} \tag{16}$$

where ϵ is defined as

$$\epsilon = \max\left\{\frac{1}{2}A + \frac{1}{2}\sqrt{A^2+4B}, 0\right\} \tag{17}$$

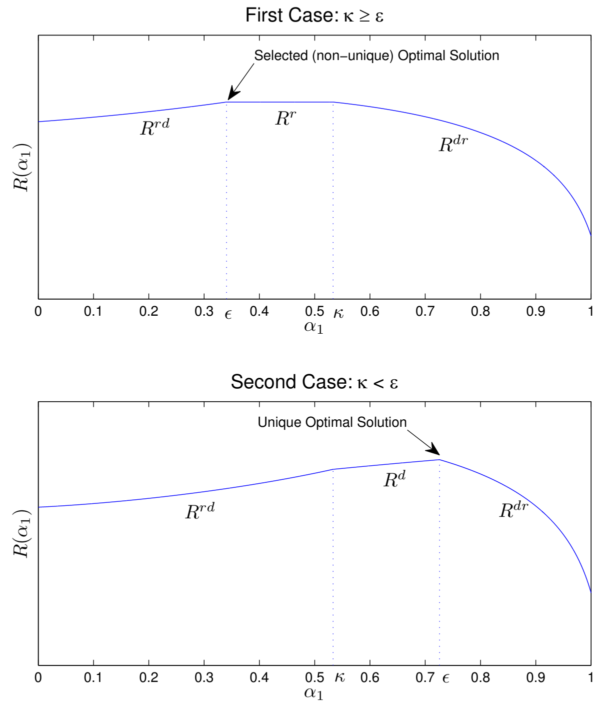

Fig. 2. Two cases for the optimization problem solution.

where A and B are defined in (12a) and (12b) respectively.

From (14) and (16), we can distinguish between two possible cases: $\kappa \geq \epsilon$ and $\kappa < \epsilon$. These two cases are shown in Fig. 2.

In the first case,

$$R = R_1 + R_2 = \begin{cases} R_1^r + R_2^d & : 0 \leq \alpha_1 \leq \epsilon \\ R_1^r + R_2^r & : \epsilon \leq \alpha_1 \leq \kappa \\ R_1^d + R_2^r & : \kappa \leq \alpha_1 \leq 1 \end{cases} \tag{18}$$

In this case, the optimal solution is obviously $\alpha_1 \in [\epsilon, \kappa]$ since we achieve the upper bound for the rate $\frac{1}{2}\log(1+\gamma_{sr})$.

In the second case,

$$R = R_1 + R_2 = \begin{cases} R_1^r + R_2^d & : 0 \leq \alpha_1 \leq \kappa \\ R_1^d + R_2^d & : \kappa \leq \alpha_1 \leq \epsilon \\ R_1^d + R_2^r & : \epsilon \leq \alpha_1 \leq 1 \end{cases} \tag{19}$$

Let's characterize these functions. $R^{rd} = R_1^r + R_2^d = \frac{1}{2}\log\left(\frac{1+\gamma_{sr}}{1+\gamma_{sr}\alpha_2}\left(\gamma_{rd}+\frac{1+\gamma_{sd}}{1+\gamma_{sd}\alpha_1}\right)\right)$ is convex, but it could be increasing or decreasing. Furthermore, it can be shown that $R^{rd}(\kappa) > R^{rd}(0)$.

$R^d = R_1^d + R_2^d = \frac{1}{2}\log\left((1+\gamma_{sd}\alpha_1)\left(\gamma_{rd}+\frac{1+\gamma_{sd}}{1+\gamma_{sd}\alpha_1}\right)\right)$, which can be written as $R^d = \frac{1}{2}\log\left(1+\gamma_{sd}+\gamma_{rd}+\gamma_{sd}\gamma_{rd}\alpha_1\right)$. This is monotonically increasing and concave as a

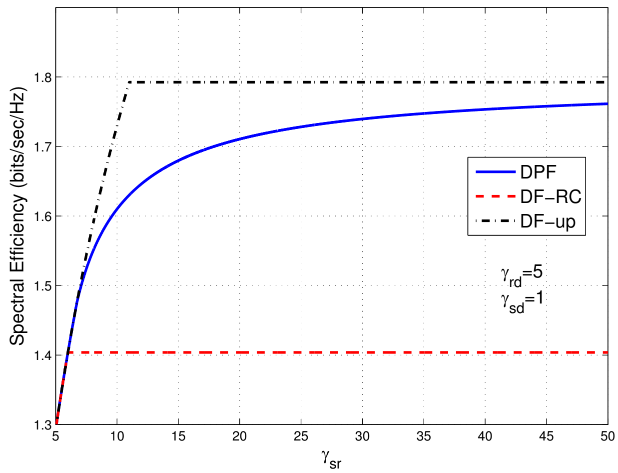

Fig. 3. The achievable rate versus the SNR of the source-relay link (i.e. γ_{sr}) where γ_{rd} and γ_{sd} are fixed.

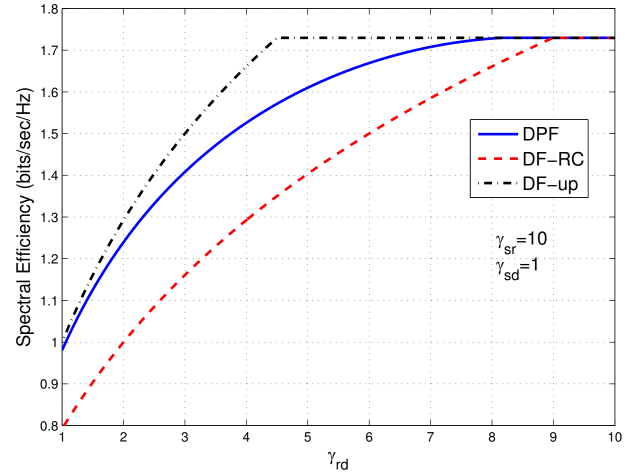

Fig. 4. The achievable rate versus the SNR of the relay-destination link (i.e. γ_{rd}) where γ_{sr} and γ_{sd} are fixed.

function of α_1.

$R^{dr} = R_1^d + R_2^r = \frac{1}{2}\log(1+\gamma_{sd}\alpha_1) + \frac{1}{2}\log(1+\gamma_{sr}\alpha_2)$ is concave and it can be shown that it is monotonically decreasing over $\alpha_1 > \epsilon$.

Thus, we conclude that in this case the function $R(\alpha_1)$ is quasi-concave over the range $[\kappa, 1]$ with a maximum value at $\alpha_1 = \epsilon$. Furthermore, the optimal solution cannot belong to $[0, \kappa]$ since $R(\kappa) > R(0)$.

By combining the solution for the two cases (i.e. $\kappa \geq \epsilon$ and $\kappa < \epsilon$), we find that we do always have the optimal solution at $\alpha_1 = \epsilon$ although it may not be the unique solution in the first case.

■

IV. Numerical Results

We provide several numerical examples for the achievable rate using the optimized DPF scheme (using the procedure described in Section III-C) and we compare it with respect to the achievable rate using the conventional repetition coding DF scheme shown in (2) as well as the information-theoretic upper bound for the achievable rate for decode-forward relaying shown in (1).

Fig. 3 shows the case when we assume that γ_{rd} and γ_{sd} are fixed and the achievable rates are plotted versus γ_{sr}. When γ_{sr} is high, which means that the source-relay link is very reliable, the achievable rate of the DF relaying schemes becomes bounded by the relay and source links to the destination node. In this case, DF-RC gets bounded at $\frac{1}{2}\log(1+\gamma_{sd}+\gamma_{rd})$ while the upper bound is $\frac{1}{2}\log(1+\gamma_{sd}) + \frac{1}{2}\log(1+\gamma_{rd})$. Fig. 3 demonstrates that the DPF scheme bridges the gap between DF-RC and DF upper bound and it can exploit the enhancement

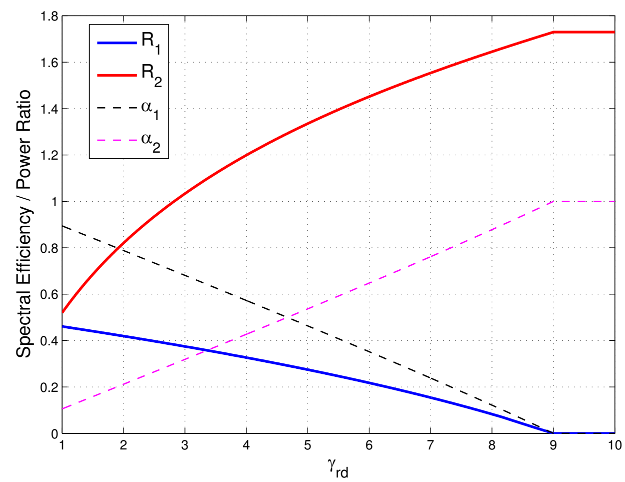

Fig. 5. The achievable rates and power ratios versus the SNR of the relay-destination link (i.e. γ_{rd}) where γ_{sr} and γ_{sd} are fixed.

is γ_{sr} to approach the upper bound closer. Note that when γ_{sr} is small, the relaying schemes become bounded by the source-relay link and thus all of them become equal.

Fig. 4 shows a different scenario where γ_{sr} and γ_{sd} are fixed and the changes in the achievable rates are plotted versus γ_{rd}. In this case, all schemes get bounded by the source-relay channel achievable rate (i.e. $\frac{1}{2}\log(1+\gamma_{sr})$) when γ_{rd} is high. However, when γ_{rd} is not sufficiently high, there is a gap between the repetition coding scheme and the upper bound for the achievable rate. It is shown in Fig. 4 how the DPF scheme outperforms the DF-RC scheme in this case and achieves a performance that is close to the upper bounds.

Fig. 5 shows the optimal power and rate allocation for

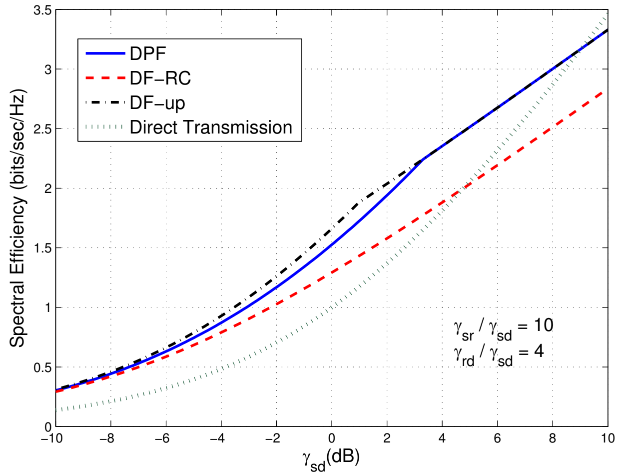

Fig. 6. The achievable rate versus the SNR of the source-destination link (i.e. γ_{sd}) where the ratios of γ_{sr} and γ_{rd} with respect to γ_{sd} are fixed.

L_1 and L_2 of the DPF scheme for the same numerical example of Fig. 4. We observe that when γ_{rd} is high, DF-RC becomes optimal and only L_2 gets transmitted. Otherwise, both L_1 and L_2 are transmitted and their power ratios and rates are dependent on the channels SNR of the three links of the relay channel.

Fig. 6 shows the achievable rate versus γ_{sd} under the assumption that the ratios of γ_{sr} and γ_{rd} with respect to γ_{sd} are fixed. The figure includes the case of direct transmission between the source node and the destination node without any role for the relay node under the assumption that the source transmits over the whole available bandwidth, i.e. both time slots ($R^{\mathrm{DT}} = \log(1 + \gamma_{sd})$). Fig. 6 demonstrates that DPF is better than DF-RC and performs closer to the upper bounds of DF and it can for some ranges of γ_{sd} achieves the upper bound for DF. We note also that when γ_{sd} is high (10 dB), direct transmission achieves better than the DF relaying schemes. Furthermore, when the direct link is very weak, the repetition coding scheme becomes almost optimal. Thus, the advantages of our proposed DPF scheme are more evident for the ranges of γ_{sd} between -5 and 8 dB.

As a final remark in this section, we can intuitively predict that optimized DPF can outperform the conventional DF-RC scheme under any other performance measure such as error probabilities because the gains in achievable rates (i.e. channel capacity) can be also exploited as gains in error rate (outage) performance.

V. Conclusions

In this paper, we have proposed a novel relaying scheme based on half-duplex decode-and-forward. The new scheme is named Decode-Partial-Forward (DPF), and as its name suggests, it is based on regenerating only part of the source message at the relay node. The destination node decodes the regenerated part using maximal ratio combining techniques while it decodes the remaining part of the message based on what it received from the direct link with the source after removing the interference effect of the regenerated part of the message. We believe that our proposed scheme is advantageous because it is demonstrated to achieve higher transmission rates than the conventional decode-forward relaying scheme that is based on repetition coding. The Decode-Partial-Forward scheme can be used as an alternative to the cooperative coding schemes as well because it performs close or at the upper bounds that these cooperative coding schemes can achieve. Additionally, it is easy to adapt flexibly, based on the channel conditions of the three links of the relay channel, by adjusting the power ratios and information rates of the two superimposed codewords at the source node. This feature might not be easy to achieve using the cooperative coding schemes. Furthermore, the relay node transceiver is simpler than in the cooperative coding scheme because it applies repetition coding. Additionally, the DPF scheme can be optimized using simple closed-form analytic formulas.

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