Minimum-energy packet forwarding over lossy networks under deadline and reliability constraints

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***Abstract*—This paper studies minimum-energy packet forwarding over multi-hop lossy networks under deadline and reliability constraints. We assume a routing topology in the form of a directed graph with packet loss processes on links described by finite-state Markov chains, and formulate the forwarding problem as a finite-horizon constrained Markov decision process. We show that the minimum energy forwarding policy under hard deadline and reliability constraint can be computed using dynamic programming, and that the optimal forwarding policy is a randomized policy over two history-independent and deterministic policies. Closed-form optimal policies are derived for some particular scenarios. Numerical examples show that the transmission energy cost of achieving reliabilities close to the maximum can be significant when links are bursty. In addition, transmission power adjustments can further reduce energy cost. Finally, we develop simple heuristic policies with a good balance between transmission energy cost and reliability.**

I. Introduction

The last decades' tremendous advances in wireless communications have been driven mainly by personal communications and radio resource allocation mechanisms for optimizing key metrics, such as average throughput and delay, for such traffic are by now rather well-developed. However, with the increased interest in wireless machine-to-machine communication, e.g. for industrial control or monitoring of large-scale infrastructures, new challenges emerge [1]. The performance of an estimator or closed-loop control system that operates over an unreliable wireless network depends on the full latency and loss distributions and not only on their averages. Hence, more suitable performance metrics are per-packet guarantees on latency and reliability (on-time delivery). In contrast to independent and saturated traffic sources often considered in personal communication, industrial networks are typically lightly loaded and traffic is transient (the majority of control design techniques rely on periodic sampling with sampling times longer than the minimal latencies) . Due to the limited range of low-power wireless, communication often occurs over multiple hops, and links are lossy with significant coherence times [2]. Accounting for channel burstiness becomes important for obtaining strong routing performance [3], [4]. Finally, energy-efficiency is critical as many devices are battery-powered with life-time targets in the order of years.

In our previous work [5], we developed optimal policies that maximize the probability that a single packet is delivered within its deadline over multi-hop networks with bursty links. These policies make full use of all available transmission opportunities, and do not account for the possible energy-inefficiency of always transmitting despite disadvantageous channel states. To address this problem, we extend our previous work by studying the problem of minimizing energy cost under both deadline and reliability constraints. The solution is immediately useful for co-design of wireless control systems with minimum packet-forwarding cost and guaranteed closed-loop control performance [6].

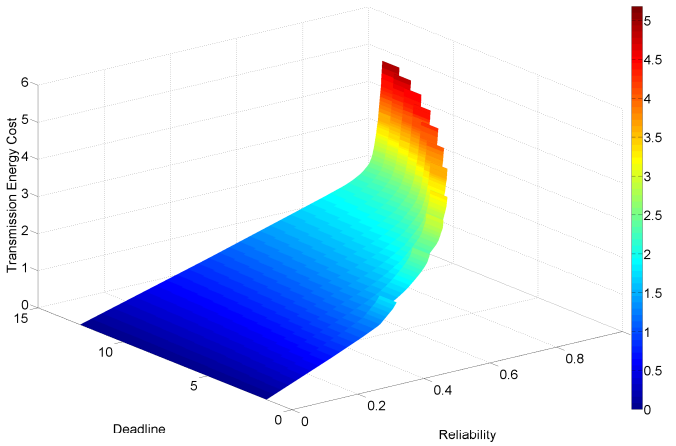


Fig. 1. Minimum forwarding energy vs deadline and reliability constraints.

The main contributions of this paper are:

- We derive the minimum-energy forwarding policy for a periodic unicast data flow with hard constraints on per-packet latency and reliability in a multi-hop network with bursty links. The routing topology of the network is represented by a directed graph with links modeled by finite-state Markov chains. The minimum energy problem is formulated as a finite-horizon constrained Markov decision process (CMDP). The solution allows us to trace out the Pareto frontier between reliability and energy cost for deadline-constrained traffic, as illustrated in Fig. 1.
- We show, using a Lagrangian approach, that the minimum energy problem can be solved by maximizing the weighted sum of reliability and forwarding energy subject to a hard deadline. Moreover, the minimum energy optimal policy is a random selection between two deterministic and computable forwarding policies, each of which can be found via dynamic programming.
- We study the structure of the optimal forwarding policies and develop closed-form expressions for the case where link losses are independent and for the case where packets are routed on a line with Gilbert-Elliot (GE) loss model.
- We study the latency-reliability-energy trade-off in nu-

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merical examples and discover a dramatic energy penalty of aiming for the maximum achievable reliability.
- We propose a heuristic policy that strikes a good balance between reliability and energy cost, and develop a dynamic programming-based algorithm to optimize its parameters and estimate its performance.

The rest of this paper is organized as follows. Section II reviews related work. Our model and assumptions are detailed in Section III and a CMDP problem formulation is presented in Section IV. A structured optimal policy is developed in Section V. Section VI studies optimal policies for two special cases, while Section VIII demonstrates the approach on numerical examples and the heuristic policy.

II. Related work

While average delay has always been an important metric in wireless communication, research on per-packet deadlines have appeared only recently. Shakkottai and Srikant [7] proposed a simple scheduling algorithm to minimize the deadline miss ratio for different data streams in a single-hop wireless network with time-varying and correlated link losses. However, the algorithm is optimal only for a specific class of arrival processes and link parameters. More recently, Hou and Kumar (see the survey paper [8] and the references therein) have proposed a tractable framework in which all packets arrive at the beginning of an interval and expire at the end of the interval. Moreover, the channel remains static in one interval. The timely throughput is defined as the number of packets that are delivered before their expiration. In this framework, Hou and Kumar study conditions for a feasible scheduling policy and find feasibility optimal policies when the timely throughput requirements are inelastic. They also study utility maximization problem when the timely throughput requirements are elastic. However, extensions to multi-hop network remain open and challenging, and energy consumption is not considered in their work.

There is also a large body of literature on physical-layer techniques of rate adaptation for energy efficient transmission of deadline-constrained packets; see [9]–[12] and the references therein. In theory, rate adaptation techniques can provide error-free transmission for a given transmit power and channel conditions, and energy can be saved by transmitting with smaller rate. Optimal policies are derived for selecting the most energy-efficient transmission strategy that delivers all bits before the deadline under various assumptions of packet arrivals, deadline patterns and channel state information etc. Nevertheless, in wireless machine-to-machine communication, the transmission rate is usually fixed, and error-free transmission is not possible even with maximum transmission power. Hence, in this paper, we study the minimum energy problem with a fixed reliability requirement (smaller than 100%).

Another line of work, rooted in the real-time systems community, considers scheduling of periodic data flows in multi-hop WirlessHART networks [13]. The data packet is duplicated at the source, and each duplicated packet is treated as a separate data flow. That paper focuses on the schedulability analysis to support as many data flows as possible, hoping that one of them can successfully reach the sink. Priority-based scheduling to enhance deadline-constrained reliability is proposed in [14], [15]. However, this approach does not guarantee reliability, and is not energy-optimal.

In our previous work [5], we have developed optimal forwarding policies that maximize the probability that a single packet is delivered within a deadline for multi-hop networks with independent and bursty links. These policies, however, make full use of all transmissions opportunities and might be wasteful from an energy perspective. Some initial and partial results on energy-efficient packet forwarding are given in our technical report [16] and significantly expanded in this paper.

III. System model

To support reliable periodic communication, we consider a scenario where a single packet, generated by an arbitrary node at time $t = 0$, should be transmitted over a multi-hop wireless network to the sink node N within a deadline of D time slots. This single-period scenario then forms the building block for supporting periodic traffic with a packet generation period larger than D. Our aim is to minimize the transmission energy cost subject to a requirement on the minimum probability that the packet is delivered to the sink node before the deadline D.

Packets are routed along a directed graph $\mathcal{G} = (\mathcal{N}, \mathcal{L})$ where $\mathcal{N}$ is the set of nodes and $\mathcal{L}$ the set of links. The root node is labeled N. The presence of a directed link $(i, j) \in \mathcal{L}$ means that node i is able to transmit a packet to node j. We define $\mathcal{N}_i$ as the set of neighbors of node i.

Communication is slotted, and a single time slot allows the transmission of a packet and its acknowledgment. Communication links are unreliable and the communication channels are modeled by homogeneous finite-state Markov chains [17], [18]. Each Markov chain evolves in discrete time, and state transitions occur at transmission slot boundaries. The Markov chains for different links are independent, and each node can only access the state of its own outgoing links. Let ω denote the Markov state for a link and $r \in \Gamma$ denote the relative power (with respect to full power) used for a single transmission; we assume that Γ is a finite set. Similar to the approach in [19], a packet transmission on a link in state ω with relative power r is then successful with probability q_ω^r.

The optimal forwarding policy depends on what knowledge about the state of the underlying Markov chains that is available to nodes when they make a forwarding decision. In what follows, let $\boldsymbol{\omega}_i(t) = [\omega_{ij}(t)]_{j \in \mathcal{N}_i}$ represent the state of all links (i, j) outgoing from node i at time t. We assume that node i at time t knows the link states in the *previous* time slot $\boldsymbol{\omega}_i(t-1)$, and neglect the energy and time cost for acquiring such channel state information. A more realistic assumption would be that the channel state is acquired by transmitting a data packet on the channel or by probing the channel with an associated cost. Such extensions are deferred to future work.

IV. Constrained Markov decision process formulation

Our deadline-constrained packet forwarding problem can be formulated as a Markov decision process (MDP) [20] with a finite horizon of $D+1$ where D is the deadline of the packet. The decision is made at time $t \in \{0, 1, \ldots, D-1\}$. Let $x(t) \triangleq \big(m(t), \boldsymbol{\omega}_{m(t)}(t-1)\big)$ denote the state of the MDP where $m(t)$ is the packet location at time t and $\boldsymbol{\omega}_{m(t)}(t-1)$ are the channel states of node $m(t)$ at time $t-1$. The action $a(t) \triangleq \big(j(t), r(t)\big)$ chooses the next hop node $j(t)$ and the transmission power $r(t)$. The state transition probability $\Pr\big(x(t+1)|x(t), a(t)\big)$ is determined by link parameters. For ease of presentation, suppose the MDP state at time t is $x(t) \triangleq (i, \boldsymbol{\omega}_i(t-1))$ and the MDP state at time $t+1$ is $x(t+1) \triangleq (k, \boldsymbol{\omega}_k(t))$. Let $\Pr\big\{\boldsymbol{\omega}_i(t)|\boldsymbol{\omega}_i(t-1)\big\}$ be the channel state transition probability of node i's outgoing links, and $P(\boldsymbol{\omega}_i) = \prod_j \Pr(\boldsymbol{\omega}_{ij})$ be the stationary channel state distribution. The state transition probability function $\Pr\big(x(t+1)|x(t), a(t)\big)$ depends on the action: if the action is to hold the packet, i.e., $j(t) = i$, then

$$\Pr\big(x(t+1)|x(t), a(t)\big) = \begin{cases} \Pr\big(\boldsymbol{\omega}_k(t)|\boldsymbol{\omega}_i(t-1)\big) & \text{if } k = i, \\ 0 & \text{otherwise;} \end{cases}$$

if the action is to forward the packet, i.e., $j(t) \neq i$, then

$$\Pr\big(x(t+1)|x(t), a(t)\big) = \begin{cases} (1 - q^r_{\omega_{kj}(t)}) \Pr\big(\boldsymbol{\omega}_k(t)|\boldsymbol{\omega}_i(t-1)\big) & \text{if } k = i, \\ P\big(\boldsymbol{\omega}_k(t)\big) \sum\limits_{\boldsymbol{\omega}_i(t)} q^r_{\omega_{ij}(t)} \Pr\big(\boldsymbol{\omega}_i(t)|\boldsymbol{\omega}_i(t-1)\big) & \text{if } k = j(t), \\ 0 & \text{otherwise.} \end{cases}$$

The initial state is $x(0)$ with the packet at the source node and the unconditional channel states distribution,

$$\Pr\big(x(0)\big) = \begin{cases} P\big(\boldsymbol{\omega}_{m(0)}\big) & \text{if } m(0) = \text{src}, \\ 0 & \text{otherwise.} \end{cases}$$

There is a terminal reward $\mu\big(x(D)\big)$ if the packet arrives at the sink node n at the last time slot D,

$$\mu\big(x(D)\big) = \begin{cases} 1 & \text{if } m(D) = N, \\ 0 & \text{otherwise.} \end{cases}$$

The cost $c\big(x(t), a(t)\big)$ is incurred when the packet is transmitted to neighbors,

$$c\big(x(t), a(t)\big) = \begin{cases} r(t) & \text{if } j(t) \neq m(t), \\ 0 & \text{otherwise.} \end{cases}$$

Note that a unit of transmission energy cost corresponds to a transmission with full power.

Let history $h(t)$ be a sequence of previous states and actions, i.e., $h(t) \triangleq \big(x(0), a(0), \ldots, x(t-1), a(t-1), x(t)\big)$, and $H(t)$ be the set of all possible histories. The decision rule is a function $d(t) : H(t) \to \mathcal{P}\big(A(t)\big)$ that maps $H(t)$ into a set of probability distributions on the action space $A(t)$ of all possible actions. A policy $\pi \triangleq \big(d(0), d(1), \ldots, d(D-1)\big)$ is a sequence of decision rules, indexed by time.

Under a policy π, the expected reward (deadline-constrained packet reliability) is

$$R^\pi \triangleq \sum_{x(0)} \Pr\{x(0)\} E^\pi_{x(0)}\big\{\mu\big(x(D)\big)\big\},$$

and the expected energy cost is

$$C^\pi \triangleq \sum_{x(0)} \Pr\{x(0)\} E^\pi_{x(0)}\Big\{ \sum_{t=0}^{t=D-1} c\big(x(t), a(t)\big)\Big\}.$$

In this notation, the minimum energy forwarding problem subject to a reliability constraint R_{req} can be formulated as

$$\begin{array}{ll} \underset{\pi}{\text{minimize}} & C^\pi \\ \text{subject to} & R^\pi \geq R_{\text{req}}. \end{array} \tag{1}$$

We have solved the reliability maximization problem with optimal policy $\pi_{\max}$ in [5]. One can verify that if $R_{\text{req}} = R^{\pi_{\max}}$, the minimum energy is $C^{\pi_{\max}}$. Thus, in this paper, we only consider the case where $R_{\text{req}} < R_{\max}$.

This problem falls into the category of constrained MDP (CMDP). The Lagrangian approach was proposed in [21] to convert it to a non-constrained weighted sum problem. In this paper, we will show that studying the problem of maximizing the weighted sum of reliability and energy allows us to construct an optimal policy which is a randomization of two history-independent and deterministic policies.

V. The minimum energy and a structured optimal policy

The Lagrange dual of the minimum energy problem (1) is

$$\begin{array}{ll} \underset{\lambda}{\text{maximize}} & \min\limits_{\pi}\big(C^\pi + \lambda(R_{\text{req}} - R^\pi)\big) \\ \text{subject to} & \lambda \geq 0 \end{array}$$

which is equivalent to

$$\begin{array}{ll} \underset{\delta}{\text{minimize}} & 1/\delta \cdot \max\limits_{\pi}\{R^\pi - \delta \cdot C^\pi\} - 1/\delta \cdot R_{\text{req}} \\ \text{subject to} & \delta \geq 0 \end{array} \tag{2}$$

where $\delta = 1/\lambda$. Our finite-horizon CMDP can be cast into infinite horizon case with total cost criterion. The Markov state x is extended to include the time from $t = 0$ to $t = D$. It goes to the next state with time $t+1$ only if the current state's time is t. We define a termination state to which all the states with time $D+1$ are directed. This is an absorbing state with no reward and cost. All other parameters including rewards, costs and state transition probabilities remain the same. It can be shown that this is a contracting MDP defined in [21, Def. 2.4]. Hence, by [21, Thm. 4.8 ii], the duality gap is zero.

To solve the problem (2), we hence need to solve the weighted sum maximization of reliability and energy,

$$\max_{\pi}\{R^\pi - \delta \cdot C^\pi\} \tag{3}$$

for a given $\delta \geq 0$. Next, we develop a dynamic programming solution to the weighted sum maximization problem and show how a structured optimal policy for our original problem (1) can be constructed.

A. Dynamic programming for weighted sum maximization

By treating the weighted energy cost $\delta \cdot C^{\pi}$ as a negative reward scaled by δ in the MDP formulation, a history-independent and deterministic optimal policy can be found by dynamic programming (DP) [20]. The MDP state is composed of the packet location and the channel state of the outgoing links at the previous time slot. Thus at time t and node i, the maximum conditional and unconditional utility are

$$U_i^\star\big(t|\boldsymbol{\omega}_i(t-1)\big) = R_i^\star\big(t|\boldsymbol{\omega}_i(t-1)\big) - \delta C_i^\star\big(t|\boldsymbol{\omega}_i(t-1)\big),$$
$$U_i^\star(t) = \sum_{\boldsymbol{\omega}_i(t-1)} \Pr\{\boldsymbol{\omega}_i(t-1)\} U_i^\star\big(t|\boldsymbol{\omega}_i(t-1)\big).$$

The maximum utility $U_i^\star(0)$ and the optimal policy can be computed backwards by DP from the initial condition

$$R_i^\star\big(D|\boldsymbol{\omega}_i(D-1)\big) = \begin{cases} 1 & \text{if } i = N, \\ 0 & \text{if } i \neq N; \end{cases}$$
$$C_i^\star\big(D|\boldsymbol{\omega}_i(D-1)\big) = 0.$$

At each step with $t < D$, the maximum conditional utility is

$$U_i^\star\big(t|\boldsymbol{\omega}_i(t-1)\big) = \max\Big\{ \max_{j,r} U_i^{jr}\big(t|\boldsymbol{\omega}_i(t-1)\big),\ U_i^{i}\big(t|\boldsymbol{\omega}_i(t-1)\big)\Big\}, \quad (4)$$

where $U_i^{jr}\big(t|\boldsymbol{\omega}_i(t-1)\big)$ is the utility of forwarding to neighbor j with power r, and $U_i^{i}\big(t|\boldsymbol{\omega}_i(t-1)\big)$ is the utility of withholding the packet at node i, respectively. These utilities are computed as

$$U_i^{jr}\big(t|\boldsymbol{\omega}_i(t-1)\big) = \sum_{\boldsymbol{\omega}_i(t)} \Pr\{\boldsymbol{\omega}_i(t)|\boldsymbol{\omega}_i(t-1)\}\Big(\underbrace{q^r_{\omega_{ij}(t)} U_j^\star(t+1)}_{\text{Success forward}}$$
$$+ \underbrace{(1-q^r_{\omega_{ij}(t)}) U_i^\star\big(t+1|\boldsymbol{\omega}_i(t)\big)}_{\text{Fail forward}} - \underbrace{\delta r}_{\text{Tx Cost}} \Big); \quad (5)$$

$$U_i^{i}\big(t|\boldsymbol{\omega}_i(t-1)\big) = \underbrace{\sum_{\boldsymbol{\omega}_i(t)} \Pr\{\boldsymbol{\omega}_i(t)|\boldsymbol{\omega}_i(t-1)\} U_i^\star\big(t+1|\boldsymbol{\omega}_i(t)\big)}_{\text{Staying at node } i}. \quad (6)$$

Intuitively, transmitting to neighbors can increase the probability of successful delivery before deadline, but it also costs more energy. The parameter δ, that balances the energy cost and the deadline constrained reliability, may refrain a node from transmitting i.e., enforce Eq. (5) $\leq$ Eq. (6).

Note that the above dynamic programming is optimal given that each node can only access the state of its own outgoing links. At each time t, the update in Eq. (5) requires the maximum *unconditional* utility $U_j^\star(t+1)$ computed by next hop node j at time $t+1$. The unconditional utility is used because node i does not know the state of node j's outgoing links. Hence, the dynamic programming update can be implemented in a *distributed* manner at each node with message passing only among neighbors (see [16] for a detailed discussion).

The optimal policy at time t forwards the packet to the node $j_i^\star\big(t|\boldsymbol{\omega}_i(t-1)\big)$ with the power $r_i^\star\big(t|\boldsymbol{\omega}_i(t-1)\big)$ that maximizes Eq. (4). For ease of notation, define $j_i^\star \triangleq j_i^\star\big(t|\boldsymbol{\omega}_i(t-1)\big)$ and $r_i^\star \triangleq r_i^\star\big(t|\boldsymbol{\omega}_i(t-1)\big)$. Thus, we have

$$\big(j_i^\star, r_i^\star\big) = \begin{cases} (i,0) & \text{if (6)} \geq \text{(5)}\ \forall j \in \mathcal{N}_i, r \in \Gamma; \\ \arg\max_{j\in\mathcal{N}_i, r\in\Gamma} (5) & \text{otherwise.} \end{cases}$$

Note that withholding the packet does not consume energy, and hence has a higher priority than forwarding the packet. We break ties arbitrarily among forwarding decisions. The corresponding reliability and energy cost can be readily computed from the optimal policy. A complete description of this DP framework, the complexity, and various extensions can be found in [16].

B. Structure and construction of the energy-optimal policy

In this section, we show that the optimal policy for the minimum energy problem (1) is a randomization of two deterministic policies found by the weighted sum problem (3) with different values of δ. Similar results for infinite-horizon CMDP with total reward and total cost constraint appeared in [22]. In this paper, we clearly state these two policies and specify the probabilities at which they are selected in the optimal randomized policy. To this end, let $R^\star(\delta)$, $C^\star(\delta)$ and $\pi^\star(\delta)$ be the reliability, the energy and the optimal policy in the weighted sum problem (3) for a given δ respectively. Define $\mathcal{R} \triangleq \{R^\star(\delta), \text{ for all } \delta\}$ and $\Delta_R \triangleq \{\delta : R^\star(\delta) = R\}$ for a given $R \in \mathcal{R}$. We have the following results:

Lemma 5.1: $\mathcal{R}$ is a finite set. For a given $R \in \mathcal{R}$, $C^\star(\delta)$ is unique for all $\delta \in \Delta_R$.

Proof: See Appendix A. ∎

Theorem 5.2: Let $R^{(1)} = \max\{R \in \mathcal{R} : R \leq R_{\text{req}}\}$ and $R^{(2)} = \min\{R \in \mathcal{R} : R > R_{\text{req}}\}$ with the associated unique energy costs $C^{(1)}$ and $C^{(2)}$. The optimal value of the minimum energy forwarding problem (1) is then

$$C^\star = C^{(1)} + \frac{R_{\text{req}} - R^{(1)}}{R^{(2)} - R^{(1)}}(C^{(2)} - C^{(1)}). \quad (7)$$

Suppose that the optimal policies that attain $(R^{(1)}, C^{(1)})$ and $(R^{(2)}, C^{(2)})$ are $\pi^{(1)}$ and $\pi^{(2)}$ respectively. An optimal policy $\pi^\star$ for the minimum energy problem is obtained by random selection of policies $\pi^{(1)}$ and $\pi^{(2)}$ with probabilities

$$\theta^{(1)} = \frac{R^{(2)} - R_{\text{req}}}{R^{(2)} - R^{(1)}}; \quad \theta^{(2)} = \frac{R_{\text{req}} - R^{(1)}}{R^{(2)} - R^{(1)}}.$$

Proof: Lemma 5.1 shows the existence of $R^{(1)}$ and $R^{(2)}$ and the uniqueness of $C^{(1)}$ and $C^{(2)}$. The rest of the proof is in Appendix A. ∎

The theorem states that the optimal forwarding policy is to make a random selection between two history-independent and deterministic policies, each found by dynamic programming. A naive implementation would be to randomly select one of the deterministic policies when the packet is created, mark the packet accordingly, and let intermediate nodes forward according to the chosen policy. Moreover, the minimum energy for any $R_{\text{req}} \in [R^{(1)}, R^{(2)})$ can be computed by Eq. (7) and the Pareto frontier of achievable reliability and energy cost

can be traced out by linearly interpolating the closest pairs of reliability and energy values obtained from the DP framework.

VI. Simplified policies in some special scenarios

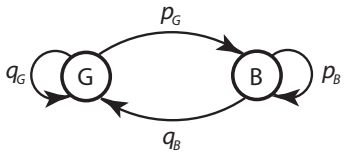


Fig. 2. Two-state Markov chain link model.

In this section, we study the structure of the optimal policies without transmission power adjustments under a special two-state Markov chain link model with "good" (G) and "bad" (B) state illustrated in Fig. 2. We let $\omega_{ij}(t) = G$ if the link is in good state, and $\omega_{ij}(t) = B$ if the link is in bad state. Since the channel state in the previous time slot is known, the probability of successful packet transmission at time slot t given that the Markov chain was in good state during time slot $t-1$ equals q_G, and the conditional probability of successful transmission at time t given that the Markov chain was in bad state during time slot $t-1$ is q_B. The average (unconditional) packet loss probability is

$$\Pi_B = \frac{1-q_G}{1-q_G+q_B}.$$

This model describes a basic variation of the Gilbert-Elliot model [23], [24] for correlated link losses, and it further reduces to the Bernoulli model of independent losses when $p_B = p_G = p$ and $q_B = q_G = q = 1-p$. Despite its simplicity, this model is able to capture real packet loss behavior and its parameters q_G and q_B can be readily estimated from loss traces, see e.g. [2].

A. Optimal policy under Bernoulli loss model

We first consider the optimal forwarding policy when events on links are uncorrelated in time described by Bernoulli loss model. We let h_i be the minimum hop count to the sink. Intuitively, the optimal forwarding policy does not forward the packet at a node if the remaining time to deadline is smaller than the node's the minimum hop count. The next theorem shows that with a transmission energy cost, the optimal forwarding policy may stop forwarding even when the time to deadline is higher than the minimum hop count h_i. Since the optimal policy and the optimal utility for Bernoulli model are independent from channel states, we let $U_i^j(t)$ be the utility of forwarding to node j with full transmission power.

Theorem 6.1: Under Bernoulli link losses,

$$j_i^\star(t) = \begin{cases} \arg\max\limits_{j\in\mathcal{N}_i}\{U_i^j(t)\} & \text{for } 0 \le t \le D-\overline{h}_i, \\ i & \text{for } t > D-\overline{h}_i, \end{cases}$$

where $\overline{h}_i = \arg\min\limits_{h\in[h_i,D-1]}\Big\{\max\limits_{j\in\mathcal{N}_i} q_{ij}U_j^\star(D-h+1) > \delta\Big\}$.

Proof: See [25, Appendix B]. ∎

The optimal forwarding policy has an interesting structure. There exists a positive number $\overline{h}_i \ge h_i$ such that if $t > D-\overline{h}_i$, the optimal policy does not attempt to forward the packet. The value $\overline{h}_i$ is the "effective minimum hop count" for node i considering transmission energy cost.

B. Optimal policy under line topology and GE loss model

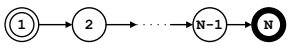


Fig. 3. Line topology.

We also study the optimal forwarding policy with GE loss model in the line topology shown in Fig. 3. Suppose node j is the next hop for node i. Let the optimal conditional utility with previous slot in good (G) state and bad (B) state be respectively

$$U_i^\star(t|G) = \max\big\{q_G U_j^\star(t+1) + p_G U_i^\star(t+1|B) - \delta, q_G U_i^\star(t+1|G) + p_G U_i^\star(t+1|B)\big\}, \quad (8)$$

$$U_i^\star(t|B) = \max\big\{q_B U_j^\star(t+1) + p_B U_i^\star(t+1|B) - \delta, q_B U_i^\star(t+1|G) + p_B U_i^\star(t+1|B)\big\}. \quad (9)$$

By comparing the two terms in (8) and (9), the optimal forwarding decisions are

$$j_i^\star\big(t|G\big) = \begin{cases} i & \text{if } \zeta_t \le \frac{\delta}{q_G}, \\ j & \text{if } \zeta_t > \frac{\delta}{q_G}; \end{cases} \quad j_i^\star\big(t|B\big) = \begin{cases} i & \text{if } \zeta_t \le \frac{\delta}{q_B}, \\ j & \text{if } \zeta_t > \frac{\delta}{q_B}, \end{cases}$$

where $\zeta_t \triangleq U_j^\star(t+1) - U_i^\star(t+1|G)$.

Similar to the policy under Bernoulli model, the optimal forwarding policy with good state stops forwarding the packet when the time to deadline is small and this time can be larger than the minimum hop count h_i. The following theorem states the optimal forwarding policy with good channel state.

Theorem 6.2: When packet losses on links are described by the two-state Gilbert-Elliot model

$$j_i^\star(t|G) = \begin{cases} j & \text{for } 0 \le t \le D-\tilde{h}_i, \\ i & \text{for } t > D-\tilde{h}_i \end{cases}$$

where $\tilde{h}_i = \arg\min\limits_{h\in[h_i,D-1]} \zeta_{D-h_i} > \frac{\delta}{q_G}$.

Proof: See [25, Appendix C]. ∎

On the other hand, the optimal forwarding decision conditioned on a previously bad channel state is different and it allows the packet to wait. Intuitively, it waits for good state when there are plenty of transmission opportunities. The policy eventually transmits the packet when the remaining time is small. However, there does not exist a single threshold type optimal policy with bad state observation. In order to illustrate the waiting strategy, let us consider the optimal forwarding policy with a single link where node j is the sink and $U_j^\star(t) = 1$ for all t. The optimal forwarding policy is

$$j_i^\star\big(t|B\big) = \begin{cases} i & \text{if } q_B - \delta \le q_B U_i^\star(t+1|G), \\ j & \text{if } q_B - \delta > q_B U_i^\star(t+1|G). \end{cases}$$

Since $U_i^\star(t|G)$ decreases as t becomes larger (larger (See [25, Appendix C]), the optimal forwarding decision at node i conditioned on a previously bad channel state may change with the number of remaining time slots. For instance, a large $U_1^\star(t+1|G)$ may induce node i to withhold the transmission allowing the channel to turn good. As the deadline approaches, however, $U_1^\star(t+1|G)$ becomes smaller and smaller to the point that it is optimal to transmit despite a bad channel observation. The parameter δ trades between energy and reliability by affecting the moment when the optimal policy with bad channel state switches from waiting to transmitting.

VII. Numerical examples

In this section, we illustrate our techniques with numerical examples. Let us consider the network in Fig. 4 where a source (node 1) sends packets to a sink (node 6), and analyze the end-to-end reliability and transmission energy cost for different deadline constraints. We consider both the Bernoulli link loss model and the GE link loss model shown in Fig. 2. For the Bernoulli link loss model, the loss probability Π_B is randomly generated in $[0.2, 0.9]$. For the GE link loss model, the links are homogeneous with unconditional loss probability $\Pi_B = 0.5$, and the burstiness parameter p_B of each link is uniformly chosen in the range $[0.75, 0.95]$.

A. Reliability and energy tradeoff without power adjustments

We first show the energy-reliability tradeoff without power adjustments (PA) in Fig. 5. We note that when link losses are correlated in time (GE model), the energy penalty of aiming for the maximum reliability is substantial. For instance, for $D = 12$, the final 3% of reliability demands approximately double the energy. We further observe that higher energy gains typically occur with larger deadlines since the energy-optimal forwarding policy then can wait the appropriate time when links are in bad state.

For the Bernoulli loss model, on the other hand, the expected transmission energy increases linearly with the reliability, and there is no longer any threshold value after which the energy cost for additional reliability increases dramatically.

B. Reliability and energy tradeoff with power adjustments

We also evaluate the energy-reliability tradeoff with power adjustments. The success probability in good state now depends on the transmission power r, and is denoted by q_G^r. The success probability in bad state is always zero. We use values of success probability in GE good state q_G^r in Table I that mimic what can be expected on an IEEE 802.15.4 platform and show a waterfall type relation between reliability and transmission power (see, e.g., [26, Fig.2.1]). The link reliability does not decrease linearly with the reduced transmission power in the beginning. A drop of 10% transmission power, for example, results in only a 2% decrease in the reliability. Nevertheless, the reliability falls sharply when the transmission power is lower than 80%.

Fig. 6 and Fig. 7 show that power adjustments always allow to reduce energy cost for a given target reliability for both Bernoulli and GE model. According to Theorem 5.2, the optimal policy without power adjustments forwards the packet on a link with probability, e.g., α, to achieve α portion of reliability and energy. However, with power adjustments, the same energy saving can be achieved by transmitting with relative power α, but with a smaller decrease in reliability.

TABLE I
An example of Power versus Reliability in the good state of Gilbert-Elliot model.

r	100%	90%	85%	83%	80%	78%	76%	75%
q_G^r	100%	98%	94%	90%	80%	70%	60%	50%

In order to illustrate the benefits of power adjustments, Fig. 8 shows the achievable reliability-energy pairs and the Pareto frontier with Bernoulli model under all deterministic policies from the DP solutions for $D = 2$. There are more pairs of achievable reliability and energy due to power adjustments. The Pareto frontier is the convex hull of these points, and a better Pareto frontier is obtained by linearly interpolating the appropriately chosen pairs of reliability and energy.

C. A heuristic policy for correlated link losses

The energy cost under correlated losses increases dramatically when reliability requirement exceeds a certain value, see Fig. 5. The main reason for this behavior is that the final reliability gains are obtained by transmitting, even when the channel is likely to be in a bad state. In this section, we develop a heuristic policy that attempts to operate at the "knee" of the energy-reliability trade-off curve by avoiding, to the extent possible, to transmit on channels that were in bad state during the last period. This policy is computed in two steps. First, we compute the maximum deadline-constrained reliability that can be achieved by policies that only transmit if the channel state in the previous time slot was good. We call this value R_{knee}. Then, we apply the technique in Section V to compute the energy-optimal policy that achieves this specific reliability value.

A variation of the dynamic programming framework in Section V-A can be applied to compute R_{knee}. We let $\delta = 0$, and $R^\star(\cdot)$ coincides with $U^\star(\cdot)$ in the dynamic programming framework. At each time t, a negative penalty -1 is induced if the packet is forwarded on the link in bad state. The dynamic programming procedure remains the same otherwise. It chooses the optimal decisions that maximize the reliability $R_i^\star(t)$ at each step t, and returns the reliability value $R_i^\star(0)$, i.e., the estimated reliability R_{knee} of the knee location. More specifically, at each time t, the maximum conditional reliability $R_i^\star\big(t|\boldsymbol{\omega}_i(t-1)\big)$ is computed similar to Eq. (4), and the computation of the reliability of staying $R_i^i\big(t|\boldsymbol{\omega}_i(t-1)\big)$ is the same as the Eq. (6). However, the reliability of forwarding $R_i^j\big(t|\boldsymbol{\omega}_i(t-1)\big)$ have two cases conditioned on the channel state. If $\omega_{ij}(t-1) = G$ (good state), then

$$R_i^j\big(t|\boldsymbol{\omega}_i(t-1)\big) = \sum_{\boldsymbol{\omega}_i(t)} \Pr\{\boldsymbol{\omega}_i(t)|\boldsymbol{\omega}_i(t-1)\} \cdot \Big(q_{\omega_{ij}(t)} R_j^\star(t+1) + (1 - q_{\omega_{ij}(t)}) R_i^\star\big(t+1|\boldsymbol{\omega}_i(t)\big)\Big).$$

If $\omega_{ij}(t-1) = B$ (bad state), then $R_i^j\big(t|\boldsymbol{\omega}_i(t-1)\big) = -1$.

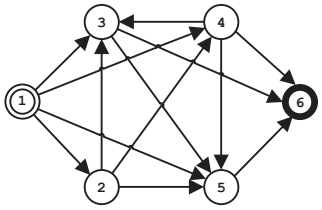


Fig. 4. Example network topology.

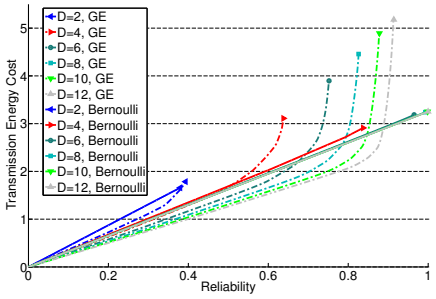


Fig. 5. Reliability-Energy curves w/o power adjustments (PA) under Bernoulli and GE model.

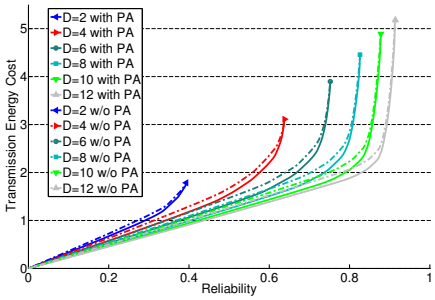


Fig. 6. Reliability-Energy curves with and w/o power adjustments (PA) under GE model.

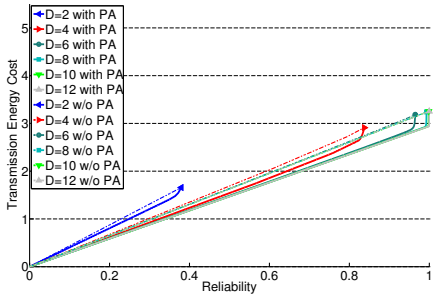


Fig. 7. Reliability-Energy curves with and w/o power adjustments (PA) under Bernoulli model.

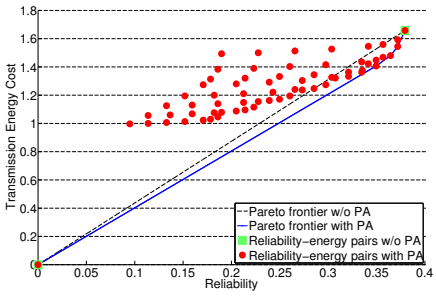


Fig. 8. Illustration of the Pareto frontier under Bernoulli model and deadline $D = 2$.

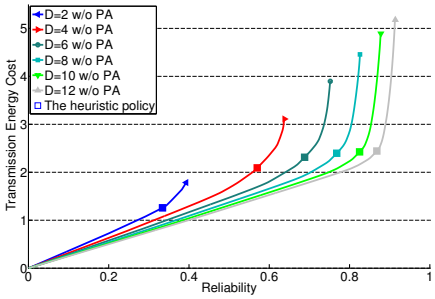


Fig. 9. Reliability and energy values for the heuristic policy under GE loss model.

We evaluate the heuristic policy on the network in Fig. 4 with GE link loss model. Fig. 9 shows that such policies strike a nice balance between energy and reliability.

VIII. Conclusions and Future Work

We studied the optimal minimum-energy packet forwarding policies for a periodic data-flow under per-packet deadline and reliability constraints. The packet deadline is smaller than the packet generation period, which allows us to study the one-period scenario with one packet on the network. The packet is routed on a multi-hop network represented by a directed graph, and the link losses are modeled by finite-state Markov chains. We showed that the minimum energy forwarding problem under reliability and deadline constraints can be formulated as a constrained Markov decision process. By a Lagrange multiplier approach, we converted the constrained problem to a weighted sum maximization problem and proved that the optimal policy is random selection between two deterministic policies found by dynamic programming. A simple threshold-type optimal policy was derived for Bernoulli link loss model, and for GE link loss model with good channel state observation in a line topology. Numerical examples of the energy-reliability tradeoff show that the energy cost of achieving reliabilities close to the maximum is dramatic when links are bursty. The energy cost can be reduced by transmission power adjustments. Finally, a heuristic policy that strikes a good balance between energy and reliability was proposed.

There are a number of interesting directions for future work. Firstly, the structure of optimal forwarding polices would change if the time and energy cost for probing channel states were considered. Secondly, the performance can be improved if each node can also acquire the one time slot delayed channel states from its immediate downstream nodes. Thirdly, it will be interesting to study the scenario with multiple streams where interference and queueing delays also need to be addressed.

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APPENDIX

A. Proof of Theorem 5.2

Lemma A.1: $R^\star(\delta)$ and $C^\star(\delta)$ are non-increasing functions for $\delta \geq 0$.

Proof: Since $\delta \geq 0$, increasing δ results in a smaller utility of forwarding to neighbors in Eq. (5), and the optimal policy will not use more transmissions or higher transmit power, hence the optimal reliability and the optimal energy cost cannot become larger. ∎

Proof of Lemma 5.1:

Proof: At each step of the DP, the number of choices are limited by the number of neighboring nodes and channel states. We have finite number policies that leads to a finite number of optimal reliabilities. Thus, $\mathcal{R}$ is a finite set.

For any given $\delta_1, \delta_2 \in \Delta_R$, we have $R^\star(\delta_1) = R^\star(\delta_2)$ with optimal polices $\pi^\star(\delta_1)$ and $\pi^\star(\delta_2)$, respectively. Suppose that $C^\star(\delta_1) \neq C^\star(\delta_2)$. Without loss of generality, we let $\delta_1 < \delta_2$. According to Lemma A.1, we have $C^\star(\delta_1) \geq C^\star(\delta_2)$ and since, by assumption, $C^\star(\delta_1) \neq C^\star(\delta_2)$, we have $C^\star(\delta_1) > C^\star(\delta_2)$. The optimal utility with δ_1 is then $R^\star(\delta_1) - \delta_1 \cdot C^\star(\delta_1)$. However if we apply policy $\pi^\star(\delta_2)$, the utility with δ_1 is $R^\star(\delta_2) - \delta_1 \cdot C^\star(\delta_2) > R^\star(\delta_1) - \delta_1 \cdot C^\star(\delta_1)$, which contradicts the optimality of $\pi^\star(\delta_1)$. Hence, for all $R \in \mathcal{R}$, $C^\star(\delta)$ is unique for all $\delta \in \Delta_R$. ∎

Proof of Theorem 5.2:

Proof: Let $g(\delta) \triangleq 1/\delta \cdot \max_\pi\{R^\pi - \delta \cdot C^\pi\} - 1/\delta \cdot R_{\text{req}}$ and $h(\delta) \triangleq \max_\pi\{R^\pi - \delta \cdot C^\pi\}$. The proposed dynamic programming framework computes $h(\delta)$ for a given value of δ and returns an optimal history-independent and deterministic policy. According to Puterman [20], $h(\delta)$ can be formulated as a linear program whose objective function coefficients depend on δ, and it can be shown that $h(\delta)$ is a continuous function over δ. Hence, $g(\delta)$ is also a continuous function over δ because $g(\delta) = 1/\delta \cdot h(\delta) - 1/\delta \cdot R_{\text{req}}$.

For a given $R^{(m)} \in \mathcal{R}$, by Lemma A.1 and the fact $\mathcal{R}$ is a finite set from Lemma 5.1, we have that $\Delta_{R^{(m)}}$ is an interval. Moreover, $h(\delta) = R^\star(\delta) - \delta C^\star(\delta)$ is a continuous function over δ and $C^\star(\delta)$ is unique for $\delta \in \Delta_{R^{(m)}}$. Hence, this interval is closed; let us denote it $\Delta_{R^{(m)}} = [\delta_{m-}, \delta_{m+}]$. Then, the function $g(\delta)$ is

$$g(\delta) = 1/\delta(R^{(m)} - R_{\text{req}}) - C^{(m)},\ \delta \in [\delta_{m-}, \delta_{m+}]. \quad (10)$$

Now let $R^{(1)} = \max\{R \in \mathcal{R} : R \leq R_{\text{req}}\}$ and $R^{(2)} = \min\{R \in \mathcal{R} : R > R_{\text{req}}\}$ with associated energy costs $C^{(1)}$ and $C^{(2)}$. Note that $R^{(1)} \leq R_{\text{req}} < R^{(2)}$. Their associated δ range is $\Delta_{R^{(1)}} = [\delta_{1-}, \delta_{1+}]$ and $\Delta_{R^{(2)}} = [\delta_{2-}, \delta_{2+}]$. Furthermore, we have $\delta^\star \triangleq \delta_{2+} = \delta_{1-}$ because $h(\delta)$ is a continuous function. Since $R^\star(\delta)$ is a non-increasing function over δ from Lemma A.1, we have $R^\star(\delta) \leq R^{(1)} \leq R_{\text{req}}$ for $\delta \geq \delta_{1+}$ and $R^\star(\delta) \geq R^{(2)} > R_{\text{req}}$ for $\delta \leq \delta_{2-}$. Thus, we have $R^\star(\delta) \leq R_{\text{req}}$ for $\delta \geq \delta_{1-}$ and $R^\star(\delta) > R_{\text{req}}$ for $\delta \leq \delta_{2+}$. Furthermore, by Eq. (10), $g(\delta)$ is a decreasing function for $\delta \leq \delta_{2+}$ and a non-decreasing function for $\delta \geq \delta_{1-}$, so the minimum value of $g(\delta)$ is obtained for $\delta = \delta^\star = \delta_{2+} = \delta_{1-}$. The optimal $\delta^\star$ can be found from

$$1/\delta^\star(R^{(2)} - R_{\text{req}}) - C^{(2)} = 1/\delta^\star(R^{(1)} - R_{\text{req}}) - C^{(1)},$$

and the minimal energy cost is

$$C^\star = C^{(1)} + \frac{R_{\text{req}} - R^{(1)}}{R^{(2)} - R^{(1)}}(C^{(2)} - C^{(1)}).$$

Suppose the optimal policies to obtain $(R^{(1)}, C^{(1)})$ and $(R^{(2)}, C^{(2)})$ are $\pi^{(1)}$ and $\pi^{(2)}$ respectively. The policy $\pi^\star$ that randomizes between $\pi^{(1)}$ and $\pi^{(2)}$ with probabilities

$$\theta^{(1)} = \frac{R^{(2)} - R_{\text{req}}}{R^{(2)} - R^{(1)}} \ \text{ and } \ \theta^{(2)} = \frac{R_{\text{req}} - R^{(1)}}{R^{(2)} - R^{(1)}}$$

achieves this minimum energy. Thus, it is an optimal policy, which concludes the proof. ∎