Satisfying demands in heterogeneous networks

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***Abstract*—In this paper, we consider a heterogeneous network (eg. Macrocells overlaid with small cells), where users associated with their respective base stations (BS) demand certain rates. These users interfere with each other and one needs an algorithm that satisfies the demands irrespective of the number of interferers and the amount of interference (whenever the demands are within the achievable limits). In our previous paper, we proposed one such iterative power allocation algorithm called UPAMCN (Universal power allocation algorithm for multicell networks), when all the agents update their power profiles at the same rate. Using two time scale stochastic approximation analysis, we analyze the same (UPAMCN) algorithm, when the heterogeneous agents update their power profiles at different rates. We obtain partial analysis of the algorithm using an ODE framework and demonstrate the convergence of the proposed algorithm via numerical examples and simulations.**

I. Introduction

Mobile broadband users demand certain rates depending on the end application and QOS requirements. The base station serving these users has to allocate power to satisfy user demands operating within its own total power budget. Intra-cell and inter-cell interference diminish the available rates in multicell networks. Neighboring base stations can co-operate to exchange some form of channel state information depending on back haul capacity and processing power to alleviate interference and thus enhance achievable rates. Further, system specific components like modulation, coding, rate allocation, channel estimation and synchronization impact the achievable rates and hence the power allocation. In this context, it would be desirable to have an universal power allocation algorithm which runs at each BS and can satisfy user demand rates in a variety of system configurations.

In a recent work [1], we proposed such an universal algorithm (UPAMCN) [1]. The stochastic approximation based algorithm runs at each BS, independently and simultaneously to meet the user demands as long as the demands are achievable. The algorithm setting was in a homogeneous network, where in all the base stations update their power profiles at the same rate.

In this paper, we consider a heterogeneous network. An example of such a network is macro cells overlaid with small cells (pico, femto). In such a deployment, the various agents (macro cells and small cells) can operate at different speeds. However they still interfere with each other. We would like to know if the same UPAMCN algorithm can satisfy demands even with heterogeneous agents. But, the analysis with heterogeneous agents is not straight forward. We propose to study the behavior of the algorithm in this heterogeneous scenario using the the two time scale stochastic approximation approach. We show that despite the disparities in the update rate, the UPAMCN algorithm converges to the same power profile, that satisfies the demand rate.

[1]For a historical perspective on power allocation algorithms, refer [2] and the discussion in [1]

II. System model

We consider a multicell MIMO network (figure 1) with K single-antenna users in a cell. Each BS has M transmit antennas. Every user experiences both intra-cell (transmissions from parent BS) and inter-cell (transmissions from neighboring BS) interference. Each user in a cell demands a certain rate and all these rates have to be satisfied by the BS (present in the cell) while operating within a total power constraint.

The received signal vector (of length K) received by users in cell j is given by,

$$\mathbf{y}_j = \mathbf{H}_{j,j}\mathbf{x}_j + \sum_{l=1,l\neq j}^{N} \gamma_l \mathbf{H}_{j,l}\mathbf{x}_l + \mathbf{n}_j \text{ for all } j \leq N. \quad (1)$$

In the above, $\mathbf{H}_{j,l}$ represents the $K \times M$ channel matrix (users in cell j receive signals from BS of cell l and we assume its elements are zero-mean unit-variance i.i.d. complex Gaussian), $\mathbf{n}_j$ represents the additive white Gaussian noise, $\mathbf{x}_j$ represents the M length transmit vector in cell j and $\gamma_l \in [0,1]$ is the interference factor (representative of the level of interference from cell l). In (1) the first term represents the useful signal part plus the intra-cell interference while the second term is the inter-cell interference.

If $\bar{P}_j$ represents the total power constraint in cell j, then $\mathbf{tr}(\mathbf{E}[\mathbf{x}_j\mathbf{x}_j^H]) \leq \bar{P}_j$ to satisfy the power constraint. As an example if the BS in cell j uses power levels specified by $\mathbf{p}_j$ and a *precoding matrix* $\mathbf{G}_j$ *(of size* $M \times K$*),* then the transmit vector is given by $\mathbf{x}_j = \mathbf{G}_j(\sqrt{\mathbf{p}_j}.\mathbf{s}_j)$ ($\mathbf{s_j}$ is a K length independent symbol vector of zero mean and unit variance components). In this case the power constraint leads to,

$$\mathbf{tr}(\mathbf{E}[\mathbf{x}_j\mathbf{x}_j^H]) \leq \mathbf{tr}(\mathbf{E}[\mathbf{G}_j\sqrt{\mathbf{p}_j}(\mathbf{G}_j\sqrt{\mathbf{p}_j})^H]) \leq \bar{P}_j \text{ for any } j.$$

Given a precoding scheme, this constraint can equivalently be represented by (for a possibly different $\bar{P}_j$) $\sum_k p_{k,j} \leq \bar{P}_j$. The symbol, $y_{k,j}$, received by the user k of cell j is ,

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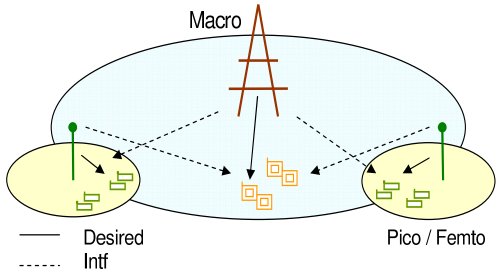

Fig. 1. A heterogeneous network

$$
\begin{aligned}
y_{k,j} &= \mathbf{h}^{\mathsf{H}}_{k,j,j}\mathbf{x}_j + \sum_{i=1, i\neq k}^{K} \mathbf{h}^{\mathsf{H}}_{i,j,j}\mathbf{x}_j \\
&+ \sum_{l=1, l\neq j}^{N} \sum_{i=1}^{K} \gamma_l \mathbf{h}^{\mathsf{H}}_{i,j,l}\mathbf{x}_l + n_{k,j} \\
&= u_{k,j} + i_{k,j,j} + \sum_{l\neq j} i_{k,j,l} + n_{k,j} \qquad (2)
\end{aligned}
$$

where $\mathbf{h}_{k,j,l}$, is the k^{th} row of matrix $\mathbf{H_{j,l}}$. In the above, $u_{k,j}$, $i_{j,j,k}$ and $i_{k,j,l}$ respectively represent the useful, intra-cell interference and inter-cell interference signal, respectively.

Every BS has to meet its users demands, for example BS j has to meet its users demand rates represented by $\mathbf{r}_j := \{r_{k,j}, k \leq K\}$. It has to tune its power levels $\mathbf{p}_j$ to achieve this. But the rates achieved will also depend upon the powers used by the other base stations. We define the following.

Power profile, $\mathcal{P} := \{p_{k,j}\}_{k\leq K,\ \ j\leq N}$, represents the vector of the powers used at all the base stations and for all the users, with $p_{k,j}$ represents the power used by the BS of cell j for user k in cell j.

Channel State (CS), $\mathcal{H} := \{\mathbf{H}_{1,1}, \mathbf{H}_{1,2}, \cdots, \mathbf{H}_{N,N}\}$, a $KN \times MN$ matrix, represents the entire CS.

Rate for a given power profile and system, $R^{sys}_{k,j}(\mathcal{P}, \mathcal{H})$, represents the transmission rates allocated, to the user k by the base station j, in system represented by sys (refer table 2 and Appendix of [1] for description of example systems) when the base stations use powers $\mathcal{P}$ and when the CS is $\mathcal{H}$.

Average Rate for a given system and power profile, is the rate that is achieved on average when a given system uses the power profile $\mathcal{P}$: $R^{sys}_{avg,k,j}(\mathcal{P}) = \mathbf{E}_{\mathcal{H}}[R^{sys}_{k,j}(\mathcal{P}, \mathcal{H})]$. Let $R^{sys}_{avg} := \{R^{sys}_{avg,k,j}\}_{k,j}$.

Power constraint $(\mathcal{P} \leq \bar{\mathcal{P}})$ We use $\leq$ in a special manner to define the power constraint. We say a power profile $\mathcal{P}$ is "less that or equal to" and hence satisfies the constraint defined in terms of another power profile $\bar{\mathcal{P}}$ if the two profiles satisfy the constraints for each BS as: $\sum_k p_{k,j} \leq \sum_k \bar{p}_{k,j}$ for all $j \leq N$.

III. Analysis of UPAMCN algorithm with heterogeneous agents

We briefly describe the universal power allocation algorithm for multicell networks (UPAMCN), before we proceed on with the analysis of the same in heterogeneous scenarios.

A. *Universal Algorithm (UPAMCN): Brief summary*

Each BS j in every time slot knows the rates at which data is transmitted to its users, $\{R^{sys}_{k,j}(\mathcal{P}, \mathcal{H})\}_k$. An iterative algorithm can find the average value of it. One can then update the power vectors to force this average towards the demands $\{r_{k,j}\}$.

Let $d^{t+1}_{k,j}$ represent the number of bytes of data transmitted successfully in time slot $t+1$ by the j^{th} base station to its user k divided by the duration of the time slot. This ratio depends upon the power profile of the entire system in the previous slot ($\mathcal{P}^t$) and the entire CS in the current slot ($\mathcal{H}^{t+1}$), but ($\mathcal{P}^t$, $\mathcal{H}^{t+1}$) are only partially known at the base stations. However $d^{t+1}_{k,j}$ is still known at base station j as it is the source that pumps out the data and it precisely equals $d^{t+1}_{k,j} = R^{sys}_{k,j}(\mathcal{P}^t, \mathcal{H}^{t+1})$ (see [1]).

With $\Pi_{\mathbb{A}}$ representing the projection in to set $\mathbb{A}$, the UPAMCN algorithm is ($\{\mu^t\}$ are the step sizes)

$$
\begin{aligned}
p^{t+1}_{k,j} &= \Pi_{\mathbb{A}_j}\left[p^t_{k,j} - \mu^t\left(d^t_{k,j} - r_{k,j}\right)\right] \text{ with} \\
\mathbb{A}_j &:= \left\{\mathbf{p} \in \mathcal{R}^K : \sum_k p_k \leq \bar{P}_j\right\}; \\
\mathbb{A} &:= \mathbb{A}_1 \times \mathbb{A}_2 \cdots \times \mathbb{A}_N. \qquad (3)
\end{aligned}
$$

Via UPAMCN algorithm (3), all the agents (e.g., BS j) update their own components (e.g., $\mathbf{p}^t_j = \{p^t_{k,j}\}_k$) in a decentralized manner, i.e., without the requirement of the other agents updates (e.g., without $\mathcal{P}^t_{-j} = \{\mathbf{p}^t_{j'}; j' \neq j\}$) and this is true for all time t. Basically the agents (e.g., agent j) observe the estimates required for their own iteration (e.g., $\{d^t_{k,j}\}$) directly, avoiding the need for the knowledge of the other's parameter (e.g., $\mathcal{P}^t_{-j}$) thus leading to a distributed algorithm. The analysis of (3) is available in [1].

B. *UPAMCN with heterogeneous agents*

In a heterogeneous network, various agents need not be synchronized and may update their components at different rates, i.e., μ^t in (3) may depend upon the agent j. For example, the macro cells can operate at a much slower rate than the small cells or vice versa. That is, there can be some index N_1 such the agents with $j < N_1$ update at every time step:

$$p_{k,j}^{t+1} = \Pi_{\mathbb{A}_j}\left[p_{k,j}^t - \frac{\mu}{t}\left(d_{k,j}^t - r_{k,j}\right)\right] \text{ for all } j \leq N_1,$$

while the agents with $j \geq N_1$ update only once in κ time steps:

$$p_{k,j}^{t+1} = \begin{cases} \Pi_{\mathbb{A}_j}\left[p_{k,j}^t - \frac{\epsilon}{n}\left(d_{k,j}^t - r_{k,j}\right)\right] \\ \qquad \text{if } t = \kappa n \text{ for some } n \in \{1, 2, \cdots\} \\ p_{k,j}^t \\ \qquad \text{else.} \end{cases}$$

C. Two time scale algorithms: Brief Summary and comparison

Stochastic approximation algorithms can be described in a general setting by:

$$\begin{aligned} p_j^{t+1} &= p_j^t + \mu_j^t H_j(\mathcal{P}^t, M_j^t), \qquad (4) \\ \mathcal{P}^t &= [p_1^t \cdots, p_N^t] \text{ for all } j \leq N, \end{aligned}$$

where $\{M_j^t\}$ is a random process that can for example, represent some form of estimation error. These algorithms can be approximated by the solution of the ODE (see for e.g., [3]):

$$\begin{aligned} \dot{p}_j &= h_j(\mathcal{P}) \text{ with} \\ h_j(\mathcal{P}) &:= E[H_j(\mathcal{P}, M)] \text{ for all } j \qquad (5) \end{aligned}$$

and are shown to converge to a zero of the average function $[h_1, \cdots, h_N]$, under appropriate conditions whenever $\mu_j^t = \mu^t$ for all j. There are situations in which different agents can update at different rates, i.e., μ_j^t need not be the same for all j. For example one might have in (4) for some $N_1 < N$

$$\begin{aligned} \mu_j^t &= \mu_1^t \text{ for all } j \leq N_1 \text{ and} \\ \mu_j^t &= \mu_2^t \text{ for all } j > N_1 \text{ with } \mu_1^t \neq \mu_2^t. \qquad (6) \end{aligned}$$

Such situations are analyzed via the Two time scale based stochastic approximation results (see for example [3], [4]) when say $\mu_1^t = o(\mu_2^t)$. These analysis primarily assume that, for any given fixed value of the slower component ($\mathcal{P}_s := \{p_j\}_{j>N_1}$), the ODE corresponding to the faster components ($\mathcal{P}_f := \{p_j\}_{j \leq N_1}$)

$$\begin{aligned} \dot{\mathcal{P}}_f(t) &= \mathbf{h}_f(\mathcal{P}_f(t), \mathcal{P}_s) \text{ with} \qquad (7) \\ \mathbf{h}_f &:= [h_1, \cdots, h_{N_1-1}] \end{aligned}$$

has a unique globally stable attractor $A^*(\mathcal{P}_s)$. Under some more assumptions, it is shown that the trajectory of the slower components is approximated by the ODE (see [3, Theorem 6.1, pp 287], [4, Theorem 1.1])

$$\begin{aligned} \dot{\mathcal{P}}_s(t) &= \mathbf{h}_s(\mathcal{P}_s(t), A^*(\mathcal{P}_s(t))) \text{ with} \qquad (8) \\ \mathbf{h}_s &:= [h_{N_1}, \cdots, h_N] \end{aligned}$$

and the slower components converge towards the limit set of the above ODE.

So we have two sets of ODEs, equations (5) for single time scale algorithms and the equations (7)-(8) for two time scale algorithms. And the comparison of the limits of any algorithm (e.g., UPAMCN), with or without the same update rate by all the agents, can be done by comparing the zeros of the right hand sides of these ODEs. One can notice[2] that the two limit points have to be the same in the sense:

a) if $\mathcal{P}_s^*$ is an attractor (so a zero of RHS) of ODE (8) then $(\mathcal{P}_s^*, A^*(\mathcal{P}_s^*))$ is an attractor of ODE (5);
b) if $(\mathcal{P}_s^*, \mathcal{P}_f^*)$ is an attractor of the joint ODE (5) then necessarily $\mathcal{P}_f^* = A^*(\mathcal{P}_s^*)$ (because ODE (7) has a unique attractor for any given $\mathcal{P}_s$) and hence $\mathcal{P}_s^*$ is an attractor of the ODE (8).

Remark: *This implies that the algorithm converges to the same set of limit points, irrespective of the disparities in the update rates.*

D. Analysis of UPAMCN with Heterogeneous agents

The algorithm that we would like to study (4) is slightly different from the two time scale algorithms discussed just above. Here, we need the result when say $\mu_1^t > 0$ for all t and $\mu_2^t > 0$ only when $t = \kappa m$ for some integer m. We approximate this algorithm with an algorithm in which the slower component is also updated every time slot but with a smaller update co-efficient, that is when $\mu_2^t = \mu_1^t/\kappa_k$ (and in the limit $\kappa_k \to \infty$). This system can now be analyzed using the two time scale ODEs (7)-(8).

Example Scenario, Low SNR regime: We consider Low SNR regime and a single cell updating fast in comparison with the rest, i.e., as in equation (4) with $N_1 = 1$. Using [3, Theorem 6.1, pp 287] we show that the UPAMCN converges to the same demand satisfying power profile, as it would have done in case all the users were updating at the same rate (i.e., when $\kappa = 1$).

We consider the system **C-D-ZF** (a system with complete CS information, discrete transmission rates and Zero forcing Precoder, see Table 2 of [1] for more details) under low SNR regime (with small x, $log(1+x) \approx x$)

$$R_{k,j}^{\mathbf{C-D-ZF}}(\mathcal{P}, \mathcal{H}) \approx \frac{p_{k,j}^t}{\sum_{l=1, l\neq j}^{N} \gamma_l \frac{\mathrm{tr}\left(\mathbf{H}_{j,l}^t Q_l \mathbf{H}_{j,l}^{t\ \mathsf{H}}\right)}{K} + \sigma_{k,j}^2}. \qquad (9)$$

Let us say without loss of generality that the BS 1 updates fast, i.e., it updates its power profile every time slot, while the rest update once in κ time slots where κ is very large. We approximate it with a system in which the rest of the components are updated every time slot, albeit with a smaller

[2]the precise analysis would require some extra conditions and here we are just pointing out the general outline. We obtain the exact analysis for an example system in the coming subsection.

coefficient μ^t/κ_t with $\kappa_t \to \infty$, say as below (for all k, j):

$$p_{k,j}^{t+1} = \Pi_{\mathbb{A}_j}\left[p_{k,j}^t - \frac{\mu}{t}\left(d_{k,j}^t - r_{k,j}\right)\right], \text{ when } j = 1 \quad (10)$$

$$p_{k,j}^{t+1} = \Pi_{\mathbb{A}_j}\left[p_{k,j}^t - \frac{\mu}{t\kappa_t}\left(d_{k,j}^t - r_{k,j}\right)\right], \text{ when } j > 1.$$

We study the above heterogeneous UPAMCN using the two interlaced ODEs (7)-(8). The fast ODE (7) for UPAMCN, at slower components $\mathcal{P}_{-1}$, equals:

$$\dot{p}_{k,1} = r_{k,1} - p_{k,1}\alpha_{k,1}(\mathcal{P}_{-1}), \text{ with}$$

$$\alpha_{k,j}(\mathcal{P}_{-1}) := \mathbf{E}_{\mathcal{H}}\left[\frac{1}{\sum_{l=1,l\neq j}^{N}\gamma_l \frac{\mathrm{tr}\left(\mathbf{H}_{j,l}Q_l\mathbf{H}_{j,l}^{\mathsf{H}}\right)}{K} + \sigma_{k,j}^2}\right]$$

while the ODE (8) corresponding to the slower components (i.e., BS j with $j > 1$)
for any k and $j > 1$,

$$\dot{p}_{k,j} = r_{k,j} - p_{k,j}\psi_{k,j}(\mathcal{P}_{-1}) + z_{k,j}. \quad (11)$$

$$\psi_{k,j}(\mathcal{P}_{-1}) := \alpha_{k,j}\left(\left[\mathbf{p}_1^*(\mathcal{P}_{-1}), \mathcal{P}_{-1}\right]_{-j}\right),$$

$$\mathbf{p}_1^*(\mathcal{P}_{-1}) := \left[p_{1,1}^*(\mathcal{P}_{-1}), \cdots, p_{K,1}^*(\mathcal{P}_{-1})\right] \text{ and}$$

$$p_{k,1}^* := \frac{r_{k,1}}{\alpha_{k,1}(\mathcal{P}_{-1})}.$$

Note in the above that $\mathbf{p}_1^*(\mathcal{P}_{-1})$ represents the attractor of the faster ODE when the slower components are fixed at $\mathcal{P}_{-1}$. We obtain the following (Proof in Appendix):

Theorem 1: Under the assumptions **A.**1-4 (Section IV B of [1]) and for the system (10), whenever the interference level $\gamma_{max} := \max \gamma_l$ is within a limit the below result is true. For every $\delta > 0$, the fraction of time the tail of the slower components of the UPAMCN algorithm ($\{\mathcal{P}_{-1}^\tau\}_{\tau > t}$ with initialization $\mathcal{P}^t < \bar{\mathcal{P}}$) spends in the δ-neighborhood of the limit set, $\mathbb{L}^{slow}$, of the slower ODE (11) tends to one (in probability) as $t \to \infty$. Further, when $\mathbb{L}^{sys}$ the set of the demand satisfying power profiles is inside the capacity region $\mathbb{C}^{sys}(\bar{\mathcal{P}})$, it is a part of the limit set $\mathbb{L}^{slow}$ in the sense:

$$\mathbb{L}^{sys} \in \left\{\left[\mathbf{p}_1^*(\mathcal{P}_{-1}), \mathcal{P}_{-1}\right]; \quad \mathcal{P}_{-1} \in \mathbb{L}^{slow}\right\}. \qquad \diamond$$

The above theorem also characterizes the limit set of the ODEs. We show that all the demand satisfying power profiles are indeed the attractors (for this low SNR example) and hence constitute the Limit set (see the proof in Appendix). In fact, the result about the limit set is correct even when we consider all agents updating at the same rate. So the main conclusion is that the UPAMCN is not effected by the variation of the update rates at different agents. The example considered in this section is a restrictive example, and we have partial theoretical justification in this case. However, in the next sub section, via some numerical examples, we indeed show that UPAMCN is unaffected by disparities in the update rates for many general cases.

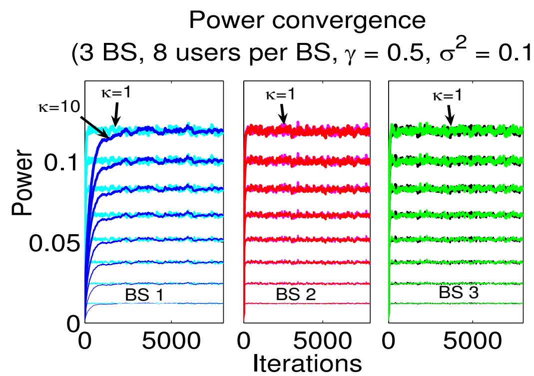

Fig. 2. Power convergence with heterogeneous users

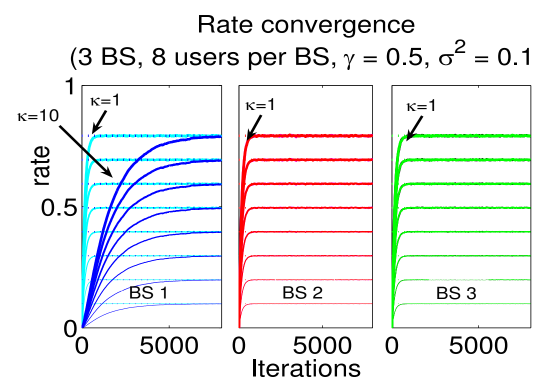

Fig. 3. Rate convergence with heterogeneous users

IV. Simulations and Numerical Examples

We consider a network with 3 base stations as in figure II. This network has a macro BS (BS1) and two small cell base stations (BS2 and BS3), both of which update at a faster rate in comparison with the macro BS. Each of them support 8 users. Further, we assume system configuration **C-D-ZF** (refer Table 2 and Appendix of [1]). Note that this is a system which supports only finite number of transmission rates and hence is a more practical example in comparison with the one studied using the two time scale analysis in the previous subsection. To keep it simple, we assume that the demand rates of users in each cell is the same and is given by $\mathbf{r_i} = [.099 \ \ .198 \ \ .297 \ \ .396 \ \ .495 \ \ .594 \ \ .693 \ \ .792]$ for $i = 1, 2, 3$. The interference factor $\gamma = 0.5$. We show via this case study the working of the algorithm and claim that due to the inherent nature of the algorithm, UPAMCN can converge and track in a variety of scenarios.

The power profile at the macro cell is updated once in κ time slots while the small cells update every time slot. We plot the UPAMCN power updates for two values of κ (1 and 10) in

κ	Demand satisfying NE (converged power)							
1	.0118	.0238	.0368	.0505	.0655	.0813	.0983	.1168
2	.0121	.0242	.0373	.0513	.0664	.0824	.0998	.1183
5	.0118	.0240	.0368	.0508	.0656	.0817	.0987	.1170
10	.0120	.0241	.0371	.0510	.0660	.0821	.0993	.1178
25	.0120	.0242	.0375	.0515	.0666	.0828	.1001	.1186
50	.0120	.0242	.0374	.0514	.0665	.0827	.0999	.1184
100	.0120	.0243	.0375	.0516	.0667	.0829	.1002	.1189

TABLE I
CONVERGED POWER PROFILE AT BS1 FOR DIFFERENT κ

κ	User 8 Small cell (BS 2)	User 8 Macro cell (BS1)
1	506	507
2	505	957
5	501	2321
10	506	4540
25	507	11269
50	506	22907
100	506	45014

TABLE II
NO. OF ITERATIONS TO CONVERGE

figures 2-3. We see from the figures that the demands are met, power profile converges to the same point irrespective of κ, albeit with different speeds. We further observe from the two figures that the convergence speed of the slower components is proportional to κ, which is quite intuitive (the effective number of updates before convergence remain the same). More surprisingly the convergence speed of the faster components does not depend much upon κ. The reason for this being the following: the attractors of the faster components $\mathcal{P}_{-1}^*(\mathbf{p}_1)$ are (Lipschitz) continuous in the slower component $\mathbf{p}_1$ and hence will vary little with small changes in the slower component and hence it appears to have converged faster.

We further illustrate the robustness of UPAMCN algorithm against update rate disparities in Table I. Here, we tabulate the converged power profile of all the eight users of the macro cell with different κ. We do not tabulate the quantities corresponding to the small cells as they do not change much with κ. We observe that UPAMCN converges to the same power profile for all the values of κ. In Table II, we also tabulate the number of time steps needed to converge to a ball which is within 5% of the demand rates for all the users of the macro cell. We see that the rough number of time steps for the convergence is proportional to κ.

V. CONCLUSIONS

Cellular networks with heterogeneous agents (e.g macro cells overlaid with small cells) can update their power profile at different rates. We analyzed the previously proposed universal power allocation algorithm to satisfy demand rates in multicell networks (UPAMCN) in this setting using the two time scale stochastic approximation approach and showed that the proposed algorithm works irrespective of the disparities in the update rates. This paper illustrates one procedure for analyzing heterogeneous agents.

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APPENDIX: PROOFS RELATED TO TWO TIME SCALE ALGORITHMS

Proof of Theorem 1: We obtain this proof via the [3, Theorem 6.1, pp 287]. The [3, Assumptions A6.0, A6.1, A6.2, A6.3 and A63.5, pp 287] hold for this example. Thus it remains to prove the [3, Assumption A6.4], in order to apply [3, Theorem 6.1, pp 287]. The ODE corresponding to the faster component (BS 1) for a given value of the other user's parameters $\mathcal{P}_{-1}$ equals (for any k and see [3])

$$\begin{aligned}
\dot{p}_{k,1} &= r_{k,1} - R_{avg,k,j}(p_{k,1}, \mathcal{P}_{-1}) \\
&= r_{k,j} - p_{k,j}\alpha_{k,1}(\mathcal{P}_{-1}), \text{ where} \\
\alpha_{k,j}(\mathcal{P}_{-j}) &:= \mathbb{E}_{\mathcal{H}}\left[\frac{1}{\sum_{l=1,l\neq j}^{N}\gamma_l \frac{\mathrm{tr}\left(\mathbf{H}_{j,l}Q_l\mathbf{H}_{j,l}{}^{\mathsf{H}}\right)}{K} + \sigma_{k,j}^2}\right].
\end{aligned}$$

This fast ODE has unique solution,

$$\begin{aligned}
p_{k,1}(t) &= p_{k,1}^*(\mathcal{P}_{-1}) - e^{-\alpha_{k,1}(\mathcal{P}_{-1})t}, \text{ with} \\
p_{k,1}^*(\mathcal{P}_{-1}) &:= \frac{r_{k,1}}{\alpha_{k,1}(\mathcal{P}_{-1})} \text{ for all } k
\end{aligned}$$

and $\mathbf{p}_1^*(\mathcal{P}_{-1})$ is its unique globally stable attractor. Further it is easy to see that there exists a constant $\nu < \infty$ such that,

$$\begin{aligned}
|\alpha_{k,1}(\tilde{\mathcal{P}}_{-1}) - \alpha_{k,1}(\mathcal{P}_{-1})| &\leq \frac{\nu\gamma_{max}}{\sigma_{k,1}^4}\left|\tilde{\mathcal{P}}_{-1} - \mathcal{P}_{-1}\right|, \text{ with} \\
\gamma_{max} &:= \max \gamma_l \text{ for all } k,
\end{aligned}$$

and thus the function $\mathbf{p}_1^*$ is locally Lipschitz, satisfying [3, Assumption A6.4]. By [3, Theorem 6.1, pp 287], the tail of the trajectory of UPAMCN spends its time mainly in the limit set of the mean ODE (see [3]) and with

$$\psi_{k,j}(\mathcal{P}_{-1}) := \alpha_{k,j}\left(\left[\mathbf{p}_1^*(\mathcal{P}_{-1}), \mathcal{P}_{-1}\right]_{-j}\right)$$

$$\dot{p}_{k,j} = r_{k,j} - p_{k,j}\psi_{k,j}(\mathcal{P}_{-1}) + z_{k,j} \text{ for any } k \text{ and } j > 1.$$

We now characterize this limit set, denoted by $\mathbb{L}^{slow}$. We will show that every internal (which is not on the boundary of the constraint set) demand satisfying power profile is a part of $\mathbb{L}^{slow}$: i) it is easy to see that $\left[\mathbf{p}_1^*(\mathcal{P}_{-1}^*), \mathcal{P}_{-1}^*\right]$ is an internal demand satisfying power profile if and only if $\mathcal{P}_{-1}^*$ is an internal zero of the RHS of the above ODE; ii) below, we will show that every internal zero of the RHS of the above mean ODE, will be an asymptotically stable attractor and hence is in $\mathbb{L}^{slow}$.

Let $\mathcal{P}_{-1}^*$ be any internal zero of this ODE and let $e_{k,j} := p_{k,j} - p_{k,j}^*$ for every k and $j > 1$. Consider a neighborhood of $\mathcal{P}_{-1}^*$ which is contained inside the constraint set (hence the projection term would be zero) and in this neighborhood we have (note $p_{k,j}^* \psi_{k,j}(\mathcal{P}_{-1}^*) = r_{k,j}$)

$$
\begin{aligned}
\dot{e}_{k,j} &= p_{k,j}^* \psi_{k,j}(\mathcal{P}_{-1}^*) - p_{k,j} \psi_{k,j}(\mathcal{P}_{-1}) \\
&= -e_{k,j} \psi_{k,j}(\mathcal{P}_{-1}^*) + p_{k,j} \left(\psi_{k,j}(\mathcal{P}_{-1}^*) - \psi_{k,j}(\mathcal{P}_{-1}) \right).
\end{aligned}
$$

Let $\mathcal{E}$ represent the vector $\{e_{k,j}\}_{\{k,j>1\}}$ and there exists a constant c depending upon the r and $\mathcal{P}_{-1}^*$ such that for all $\mathcal{E}$ with $|\mathcal{E}| \leq r$,

$$
<\dot{\mathcal{E}}, \mathcal{E}> \; < \sum_{k,j>1} \left(-\psi_{k,j}(\mathcal{P}_{-1}^*) + c\nu_{max} \right) |\mathcal{E}|^2.
$$

Thus by [5, Global existence theorem, pp 169-170],

$$
|\mathcal{E}|(t) \leq e^{\sum_{k,j>1} \left(-\psi_{k,j}(\mathcal{P}_{-1}^*) + c\nu_{max} \right) t}
$$

when initial condition$\mathcal{E}(0) \in \{\mathcal{E} : |\mathcal{E}| < r\}$.

Hence as the interference reduces, i.e., as $\nu_{max} \to 0$ the exponent term becomes negative (in which case, the error tends to zero asymptotically). For these small values of interference, every internal zero is an asymptotically stable attractor. $\diamond$