Computer-aided cryptographic proofs

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Provable security [6] is at the heart of modern cryptography. It advocates a mathematical approach in which the security of new cryptographic constructions is defined rigorously, and provably reduced to one or several assumptions, such as the hardness of a computational problem, or the existence of an ideal functionality. A typical provable security statement is of the form: for all adversary $\mathcal{A}$ against the cryptographic construction $\mathcal{S}$, there exists an adversary $\mathcal{B}$ against a security assumption $\mathcal{H}$, such that if $\mathcal{A}$ has a high probability of breaking the scheme $\mathcal{S}$ in time t , then $\mathcal{B}$ has a high probability of breaking the assumption $\mathcal{H}$ in time t' (defined as a function of t).

EasyCrypt [1] is a framework for building and verifying machine-checked security proofs for cryptographic constructions in the computational model. Following the code-based approach [4], **EasyCrypt** uses probabilistic programs with adversarial computations to formulate unambiguously reductionist arguments. In **EasyCrypt**, cryptographic constructions are modelled as probabilistic programs, and their security is given by the probability of an event in an experiment, where an adversary interacts with the construction; similarly, security assumptions are stated in terms of the probability of an event in a probabilistic experiment. The key novelty of **EasyCrypt** (and its predecessor **CertiCrypt** [2]) is to provide programming languages tools to capture common reasoning patterns in cryptographic proofs. In particular, **EasyCrypt** provides support for a probabilistic relational Hoare Logic (pRHL) [2], whose judgments $\models c_1 \sim c_2 : \Psi \Rightarrow \Phi$ relate two probabilistic programs c_1 and c_2 (that typically involve adversarial code) relative to a pre-condition Ψ and a post-condition Φ , both defined as relations over program states. Informally, a judgment is valid iff for every initial memories that are related by the pre-condition, the sub-distributions of final memories are related by the lifting of the post-condition to distributions; the definition of the lifting operator $\mathcal{L}$ is adopted from probabilistic process algebra [7], and has close connections with the Kantorovich metric, and with flow networks [5]. As security properties are typically expressed in terms of probability of events rather than pRHL judgments, **EasyCrypt** implements mechanisms to derive from valid judgments probability claims, i.e. inequalities between expressions of the form $\Pr[c, m : S]$ that denote the probability of the event S in the sub-distribution $\llbracket c \rrbracket m$.

To automate reasoning in pRHL, **EasyCrypt** implements an automated procedure that given a logical judgment involving loop-free closed programs, computes a set of sufficient conditions for its validity, known as verification conditions. In the presence of loops or adversarial code, we require the user to provide the necessary annotations. The outstanding feature of this procedure, and the key to its

effectiveness, is that verification conditions are expressed as first-order formulae, without any mention of probability, and thus can be discharged automatically using off-the-shelf SMT solvers and theorem provers.

To date, **EasyCrypt** (and its predecessor **CertiCrypt**) have been used to verify prominent examples of cryptographic constructions, including the OAEP padding scheme, the Cramer-Shoup encryption scheme, the Full Domain Hash signature scheme, the Merkle-Damgård hash function design, and zero-knowledge proofs. Moreover, **CertiCrypt** and **EasyCrypt** have been extended to reason about differentially private computations [3]. More recently, **EasyCrypt** has been used for the first time to prove the security of a novel cryptographic construction. Specifically, we have used **EasyCrypt** to prove the IND-CCA security of ZAEP, a redundancy-free public-key encryption scheme based on the Rabin function and RSA with exponent 3.

More information about the project can be found at:

<http://easycrypt.gforge.inria.fr>

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