Low Complexity Grouping for Massive Scheduling in 4G Networks

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Mobile Communication Dept., Eurecom, Email: firstname.name@eurecom.fr

***Abstract*—In this paper, we investigate user grouping for cooperative scheduling in a two-cell network. When the number of transmitters grows large, the complexity of the Hungarian algorithm optimum for user pairing becomes unaffordable in real-time systems. We consider user grouping algorithms maximizing the network sum rate in cells with a *massive* number of terminals and/or sensors. We provide a suboptimal user grouping algorithm which substantially reduces complexity compared to the optimum Hungarian algorithm with negligible capacity degradation. Surprisingly, the proposed algorithm outperforms the greedy algorithm with a considerable lower complexity.**

I. Introduction

Cellular networks operating with universal frequency reuse, such as the 3GPP UTRAN Long Term Evolution Advanced (LTE-A), can potentially support very high throughput but suffer from considerable inter-cell interference levels. Then, interference management is fundamental in the design of high throughput and energy efficient networks when operating with universal frequency reuse. In the literature, cooperative scheduling schemes have been proposed as a key approach for inter-cell interference coordination (ICIC). The benefits of joint scheduling and power control schemes for downlink scenarios with multiple cells and single resource block (RB) have been analyzed in [1]. Important properties, such as the upper bound and lower bound of global system capacity have been derived in the asymptotic conditions when the number of users grows large and the system can widely benefit from user diversity.

LTE-A adopts this scheduling policy and proposes three regular scheduling schemes: persistent, semi-persistent, and dynamic scheduling. The latter two approaches are cooperative [2] and their scheduling procedures aim at optimizing a given global metric, e.g. total capacity, jointly over all cells by making use of a limited amount of exchanged information among evolved nodeBs (eNBs). For uplink, eNBs exchange received signal strengths (RSS) and interference signal strengths (ISS) and the supported cooperative scheduling can achieve better global capacity, fairness, and QoS than non-cooperative scheduling. In [3], a multiuser multiple transmit multiple receive antennas (MU-MIMO) single-cell system is considered and simultaneous two-user transmissions are optimally scheduled based on the Hungarian algorithm [4]. In [5], [6], an LTE single-cell frequency-selective fading channel in uplink is considered. Two simultaneous SC-FDMA transmissions are possible and equalization is performed at the receiver side. Substantial gains are shown (e.g. by up to 6 dB) by joint user grouping and frequency-domain resource allocation compared to random user grouping and frequency-domain resource allocation [6]. A user pairing algorithm satisfying proportional fairness quality of service constraints and based on the Edmond's algorithm has been proposed in [7] for single-cell system. A centralized proportional fair uplink scheduling scheme for a multi-cell LTE network with single RB has been proposed in [8]. Two frequency and power sub-optimal allocation schemes for the scenario with multiple cells and multiple RBs in LTE uplink are proposed in [9].

Optimization problems for cooperative scheduling are in general discrete and often have non-polynomial (NP) complexity. This makes cooperative scheduling not feasible on a real system when the number of active UEs becomes very large as in the case of sensors and machine-type communications [10], shortly referred as user equipment (UE) communications. In this contribution, we design algorithms for cooperative scheduling in the uplink of a two-cell LTE-A system when a *massive* number of UEs populates the cells and the UEs are uniformly distributed in the cell. We utilize the knowledge of the statistical distribution of the RSS and ISS at both eNBs to *reduce the complexity* of the scheduling algorithm. We propose two scheduling algorithms that achieve near-optimum performance with a lower complexity than the Hungarian algorithm. The key idea of the proposed algorithms consists in partitioning the UEs of each cell in subsets and establishing a bijective mapping between subsets of different cells. Then, an assignment algorithm is applied to pair UEs belonging to the subsystems generated by partitioning and bijection. The initial partition step allows to keep the dimension of the assignment problem reasonably low and reduce drastically the scheduling complexity. The algorithm dubbed *partitioning and greedy grouping*, surprisingly, achieves higher performance than the greedy algorithm with a significantly lower complexity. Our analysis shows that, for the proposed system model, coordinated scheduling does not benefit from user diversity, the average performance does not increase with the number of UEs in the system, and the greedy algorithm does not appear to be asymptotically optimum. Technically, this is due to the fact that the random variables that define our problem are strongly correlated.

The reminder of the paper is organized as follows. Section II presents the system model and problem statement. In section III, scheduling statistical patterns based on Hungarian algorithms is derived and two heuristic cooperative scheduling algorithms are proposed. Section VI presents the numerical results of the proposed algorithms and compares them with exiting algorithms. Section V provides concluding remarks and future directions.

II. System Model and Problem Description

In this paper, we consider two adjacent eNBs with a massive number of UEs in uplink communications. Each UE communicates only with the nearest eNB, forming a star topology within hexagonal cells. When a UE transmits to its eNB, the overhearing adjacent eNB receives this signal as inter-cell interference.

We assume that each cell is populated by uniformly distributed UEs. The channel states for all UEs to both eNBs are static. The two eNBs are denoted by eNB_K and eNB_L, while the UEs in the two cells are denoted by UE_K^i, for $i = 1, \ldots, N_K$, and UE_L^j, for $j = 1, \ldots, N_L$, respectively. The channel attenuation from UE_K^i to eNB_K and eNB_L are denoted by g_K^i and h_K^i. Similarly, channel attenuation from UE_L^j to eNB_L and eNB_K are denoted by g_L^j and h_L^j. The attenuation coefficients g_x^i, h_x^i, with $x \in \{K, L\}$, follow a log distance pathloss model with no fading, i.e., if d is the distance between a UE and a eNB, the corresponding attenuation coefficient is given by $d^{-\alpha}$, where α is the pathloss exponent ranging typically from 2 to 4. Two UEs, one in each cell, may transmit simultaneously to the respective eNBs. If UE_K^i and UE_L^j transmit simultaneously, their contributed spectral efficiency is given by $r_{ij}^K = \log(1 + \frac{Pg_K^i}{Ph_L^j + \sigma^2})$ and $r_{ji}^L = \log(1 + \frac{Pg_L^j}{Ph_K^i + \sigma^2})$, respectively, where P is the transmitting power and σ^2 is the variance of the additive white Gaussian noise (AWGN).

Let $N = \min(N_K, N_L)$. We divide the available time resource in $\max(N_K, N_L)$ equal time slots and consider a cooperative scheduling that assigns a slot to each of $|N_K - N_L|$ UEs in the cell with higher number of transmitters. For the remaining UEs, the scheduler assigns a single slot to two UEs, one in each cell. We assume that the selection of the $|N_K - N_L|$ UEs transmitting alone is arbitrarily done by the eNB according to priority or quality of service criteria. Therefore, in the following, we focus on the user grouping problem of two sets of N UEs over N time slots. Let us denote by π a permutation of the set $\mathcal{N} = \{1, ..., N\}$ and by π_i its ith elements. Without loss of generality, a scheduling can be represented by a permutation π and the time slot i is assigned to UE_i^K and $UE_{\pi_i}^L$, simultaneously. The average spectral efficiency of the two cell network corresponding to permutation π is

$$\gamma(\pi) = \frac{1}{N} \sum_{i=1,\ldots,N} r_{i,\pi_i}^K + r_{\pi_i,i}^L. \tag{1}$$

Let Π be the set of all possible permutations, an optimal cooperative scheduling allocation maximizes the average spectral efficiency of the system, i.e., it is the permutation π^* that maximizes $\gamma(\pi)$.

This optimization problem reduces to the classical problem of assignment that can be solved optimally by the Hungarian algorithm [4] with polynomial complexity in the case of two-cell system. More specifically, it can be solved with a complexity order $O(N^3)$. A lower complexity algorithm is provided by the greedy algorithm, which has a complexity in the order of $O(N^2 \log N)$. Although approaches with polynomial complexities are available, their application to cells with massive number of active UEs becomes rapidly unaffordable. This motivates a design of low complexity algorithms tailored to the peculiarities of the system at hand.

III. Cell Partitioning and Grouping Scheme

To unveil statistical properties of optimum scheduling in systems with a massive number of UEs we applied the Hungarian method [4] to a two-cell system with a massive number of UEs randomly generated. Fig. 1 shows the obtained pairing for two-cell system, each of them populated by 1000 UEs, under the assumptions of a log distance pathloss model without fading for the channel attenuations, and equal transmit powers for all the UEs. A UE of a certain color in cell L is paired with some user of the same color in cell K. Fig. 1 is plotted such that UEs in cell L have gradually varying colors from center to edge. It can be seen that users with higher RSS in cell K, i.e. closer to eNB_K and with greater g_k^i, tend to be paired with users with small ISS in cell L, i.e. farther from eNB_L and with lower h_L^j. A mirroring figure can be obtained when the coloring is driven by users in cell K. This shows that the statistical pattern is symmetric between the two cells. Additionally, if we only consider the capacity of UEs in cell L, the statistic pattern is close to the one shown in Fig. 1, except some blur for edge UEs.

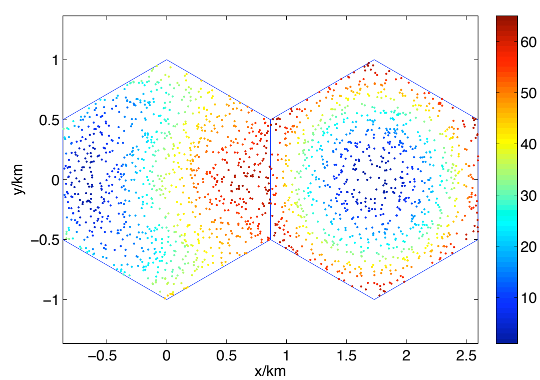

Fig. 1: Statistical pattern using Hungarian algorithm.

These observations suggest a low complexity algorithm based on the ordering of RSS and ISS within a cell. Before detailing our cell partitioning and grouping schemes, we show an interesting property of user grouping based on the ordering of RSS and ISS within a cell. Let us focus on cell K. We call *quick pairing* in cell K the algorithm that sorts UEs in cell K in descending order of g_K^i and UEs in cell L in ascending order of h_L^j and, then, pairs UEs with identical ordering index. This algorithm provides the optimum pairing when the objective function to be maximized is the spectral efficiency of cell K as shown in the following theorem.

Theorem 1. *The quick pairing algorithm maximizes the objective function* $\frac{1}{N}\sum_{i=1}^{N} r^K_{i,\pi_i}$ *over all possible permutations* $\pi \in \Pi$.

Proof: Let us observe that, for any two slots i and j, if $g^i_K \geq g^j_K$ and $h^i_L \leq h^j_L$, it results

$$\begin{aligned}
&(Ph^i_L + \sigma^2 + Pg^i_K)(Ph^j_L + \sigma^2 + Pg^j_K) \\
&\quad =(Ph^i_L + \sigma^2 + Pg^j_K + Pg^i_K - Pg^j_K) \\
&\qquad \times (Ph^j_L + \sigma^2 + Pg^i_K + Pg^j_K - Pg^i_K) \\
&\quad =(Ph^i_L + \sigma^2 + Pg^j_K)(Ph^j_L + \sigma^2 + Pg^i_K) \\
&\qquad + (Ph^i_L + \sigma^2 + Pg^j_K)(Pg^j_K - Pg^i_K) \\
&\qquad + (Ph^j_L + \sigma^2 + Pg^i_K)(Pg^i_K - Pg^j_K) \\
&\qquad + (Pg^j_K - Pg^i_K)(Pg^i_K - Pg^j_K) \\
&\quad =(Ph^i_L + \sigma^2 + Pg^j_K)(Ph^j_L + \sigma^2 + Pg^i_K) \\
&\qquad + P^2(g^i_K - g^j_K)(h^j_L - h^i_L) \\
&\quad \geq(Ph^i_L + \sigma^2 + Pg^j_K)(Ph^j_L + \sigma^2 + Pg^i_K). \qquad (2)
\end{aligned}$$

Let us assume that the UEs in cell K are ordered in decreasing order of g^i_K. Let π be any permutation of UEs in cell L. We focus on any arbitrary pair i, j of time slots with $i < j$ and $h^i_L \leq h^j_L$. Then, the total capacity corresponding to the scheduling induced by permutation π and the total capacity corresponding to the permutation π^+ obtained from π switching UE^i_L and UE^j_L are related by

$$\begin{aligned}
r_\pi =&\sum_{n=1}^{N} r^K_{n,\pi_n} \\
=&\sum_{n=1}^{N} \log(1 + \frac{Pg^n_K}{Ph^{\pi_n}_L + \sigma^2}) \\
=&\sum_{\substack{n=1,\dots,N\\ n\neq i,j}} \log(1 + \frac{Pg^n_K}{Ph^{\pi_n}_L + \sigma^2})+ \\
&\log\left[\frac{(Ph^{\pi_i}_L + \sigma^2 + Pg^i_K)(Ph^{\pi_j}_L + \sigma^2 + Pg^j_K)}{(Ph^{\pi_i}_L + \sigma^2)(Ph^{\pi_j}_L + \sigma^2)}\right] \\
\leq&\sum_{\substack{n=1,\dots,N\\ n\neq i,j}} \log(1 + \frac{Pg^n_K}{Ph^{\pi_n}_L + \sigma^2})+ \\
&\log\left[\frac{(Ph^{\pi_i}_L + \sigma^2 + Pg^j_K)(Ph^{\pi_j}_L + \sigma^2 + Pg^i_K)}{(Ph^{\pi_i}_L + \sigma^2)(Ph^{\pi_j}_L + \sigma^2)}\right] \\
=&r_{\pi^+},
\end{aligned}$$

where r_π and r_{π^+} denote the total capacities of cell K corresponding to the scheduling π and π^+ respectively, and the inequality is a straightforward consequence of (2).

Then, by applying repeatedly this swapping with the above mentioned criterion, we obtain the quick pairing as optimum scheduling policy. ■

Although the quick pairing is optimum when the objective function to be optimized is the spectral efficiency of a single cell, it is not an optimal solution for maximizing the total capacity of the two cells. Therefore, based on Theorem 1, we propose the following heuristic scheme to partition each cell into n subsets of equal size and pair them:

- Sort the UEs in cell K in a descending order of RSS and the UEs in cell L in an ascending order of ISS;
- Partition the N users in each cell into n subsets such that the i^{th} subset contains users in the order positions from $\frac{(i-1)N}{n} + 1$ to $\frac{(i-1)N}{n} + \frac{N}{n}$;
- Consider the subsystem consisting of the ith subsets of cell K and cell L, and apply any standard pairing algorithm to it.

The proposed partitioning allows to keep the dimension of the assignment problem reasonably low hence reduce drastically the scheduling complexity. We propose the application of the Hungarian or the greedy algorithm to each subsystem. Hence, we obtain two algorithms that we dub *partitioning and Hungarian grouping* and *partitioning and greedy grouping*, respectively. The preliminary partitioning and subsequent grouping decreases the time complexity of the algorithms from $O(N^3)$ to $O(\frac{N^3}{n^2})$ when Hungarian grouping is performed, and from $O(N^2 \log N)$ to $O(\frac{N^2}{n} \log \frac{N}{n})$ when greedy grouping is applied.

Other grouping algorithms can be applied within a subsystem as long as their objective function is the total capacity of the two-cell system.

IV. Numerical Performance Analysis

In this section we assess the performance of the proposed algorithms in terms of both attained average total capacity and complexity by numerical simulations. The heuristic partitioning and grouping algorithms are compared to cooperative scheduling based on Hungarian algorithm and greedy algorithm.

Since the class of "greedy/myopic" algorithms is a large group of suboptimum approaches aiming at solving combinational optimization problems based on the idea to select the locally best choice in each decision stage without considering overall global optimality we briefly describe the greedy algorithm adopted in our simulations. Let us introduce a matrix $\boldsymbol{R}^{(N)}$ with components $R^{(N)}_{ij} = r^K_{ij} + r^L_{ji}$. The greedy algorithm consists of N steps. At step ℓ, it selects the maximum entry from the matrix $\boldsymbol{R}^{(N-\ell+1)}$ and constructs a matrix $\boldsymbol{R}^{(N-\ell)}$ obtained from $\boldsymbol{R}^{(N-\ell+1)}$ by removing the column and the row corresponding to the selected entries.

A first group of simulations is performed assuming $\alpha = 2$, generating randomly, according to a uniform distribution, a number of users N varying from 100 to 500, and fixing the number of subsets in the partition equal to 10. The results are averaged over 100 experiments. Fig. 2 shows the average capacity[1] of the two-cell system as a function of UEs in each cell for the Hungarian and greedy algorithms, for the proposed algorithms and for the quick pairing. Random pairing

[1]Note that the optimization does not change if we consider average capacity instead of total capacity. However, the former metrics enables an insightful graphical comparison of systems with different number of UEs.

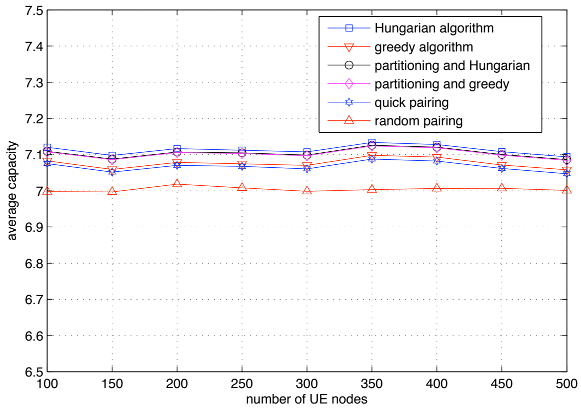

Fig. 2: Comparison of optimal and suboptimal algorithms: average system capacity versus number of UEs per cell.

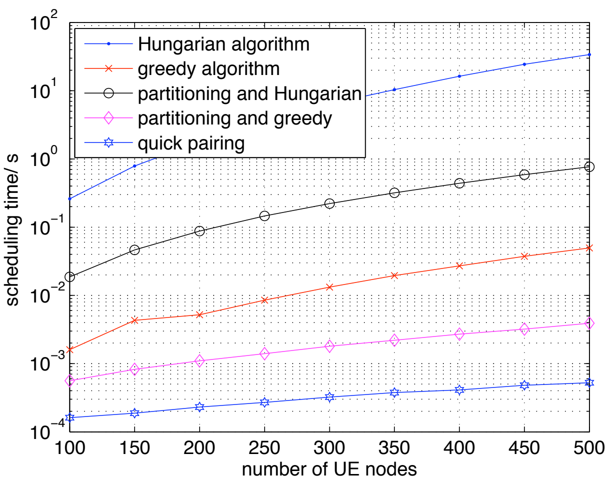

Fig. 3: Comparison of optimal and suboptimal algorithms: processing time versus number of UEs per cell.

is adopted as benchmark to assess the benefits of pairing. Interestingly, the average capacity for a sufficiently large number of UEs stays constant and the user diversity does not enable any capacity gain. We will further discuss this aspect in some conclusive remarks in this section. Fig. 3 shows the complexity of the investigated algorithms in terms of processing time for increasing number of users. The comparison among the analyzed algorithms shows that any pairing algorithm outperforms significantly a random pairing. Additionally, the two proposed algorithms attain almost the same average system capacity. They outperform the greedy algorithm and the quick pairing and achieve almost the same performance as the optimum Hungarian algorithm. The comparison in terms of complexity changes substantially: the partitioning and greedy grouping algorithm has a complexity significantly lower than the greedy algorithm and the partitioning and Hungarian grouping algorithm while it attains substantially better performance than the former and the same performance as the latter. As expected, it has higher complexity than the significantly suboptimal quick pairing.

Let us observe that the partitioning and greedy grouping algorithm reduces to the greedy algorithm and to the quick pairing as the subsets in the partition is equal to 1 and to N, respectively. Similarly, the partitioning and Hungarian grouping boils down to the Hungarian and quick pairing algorithms in analogous situations. Then, it is interesting to investigate performance and complexity of the proposed algorithms as the number of partition subsets varies. The numerical analysis is presented in Figures 4 and 5 in terms of average capacity and processing time, respectively. In contrast to the fact that performance and complexity of the partitioning and Hungarian grouping algorithm decreases as the number of subsets increases, the performance of the partitioning and greedy grouping algorithm has an optimum number of partition subsets both for the average capacity and the time processing. The two optima are very close each other and quite high: between 30 and 50 subsets. Surprisingly, there is a region where the time processing decreases while the performance increases. Then, for practical implementation the number of partition subsets needs to be optimized. An analytical performance analysis exceeds the scope of this contribution. In fact, the state of art on the theoretical analysis of random assignment problems is still at its infancy to be applied to the complex setting of the problem at hand. However, some qualitative insights on the system are still possible. Let us consider the random variables $r^K_{i,\pi_i} + r^L_{\pi_i,i}$. Their marginal distribution matches very accurately a gamma distribution. Figure 6 shows this matching for a probability density function (p.d.f) obtained in a regular hexagonal cell with edge of length 1 and $\alpha = 2$. The marginal random variable fits a Gamma distribution with $\Gamma(k,\theta) = \Gamma(10.8724, 0.6508)$. If all the random variables had been independent and identically distributed, the performance of the system could have been analytically determined by applying the results in [11] with the following conclusions:

- The asymptotic performance of optimal grouping are given by $N\theta \log N + Nk\theta \log\log N + O(N)$, i.e. the average utility scales as $\log N$ and increases with the system size. As an example, the average performance of a random assignment problem with $\Gamma(10.8724, 0.6508)$ is around 13.7 and 16.86 for $N = 100$ and $N = 500$. This implies that the optimization enables pairing of users with performance in the tail of the distribution.
- The greedy algorithm is asymptotically near-optimal, i.e. it converges asymptotically to the optimal algorithm. The same could have been shown for the proposed algorithms.

Due to a strong correlation of the random variables that define the assignment problem, the average performance of the system at hand does not scale with the number of UEs and, also for large number of users, the average performance is close to the mean of the marginal distribution with a considerable performance loss compared with a hypothetical system defined by i.i.d. random variables. Additionally, the greedy algorithm is suboptimal also in asymptotic conditions.

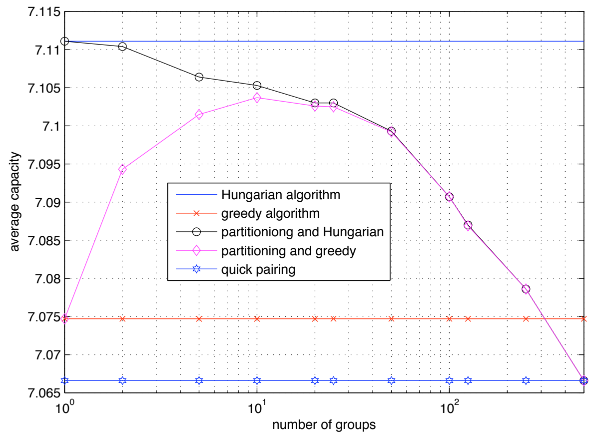

Fig. 4: Comparison of optimal and suboptimal algorithms: average system capacity versus number of partition subsets.

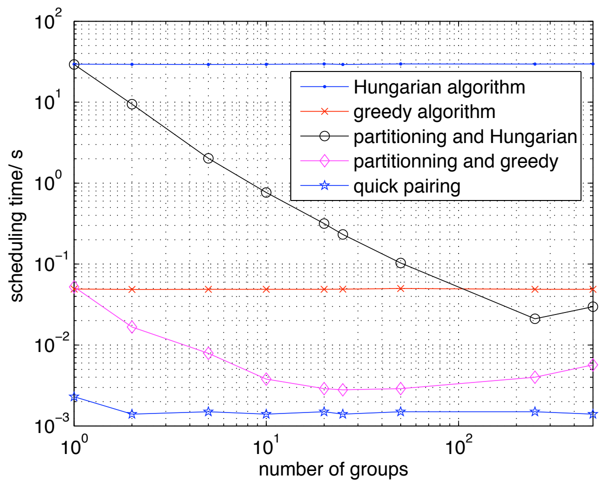

Fig. 5: Comparison of optimal and suboptimal algorithms: processing time versus number of partition subsets.

V. Conclusion and Future Experiments

Two heuristic cooperative scheduling algorithms for dense cells are provided based on the statistical scheduling pattern for cells with a massive number of UEs. The partitioning and greedy grouping algorithm can significantly decrease the processing time with negligible performance degradation in terms of capacity. Surprisingly, the proposed algorithm has complexity significantly lower than the greedy algorithm with significantly higher performance.

In future studies, the optimal number of partition subsets for a fixed number of UEs in the cell should also be investigated. In this contribution, the analysis is based on the assumption of log distance pathloss model for the channel. More complex models should be considered to determine if partitioning and greedy grouping is always efficient and applicable.

Finally, a theoretical performance analysis of the random assignment problem for correlated utilities is highly desirable for a deep understanding of the problem at hand.

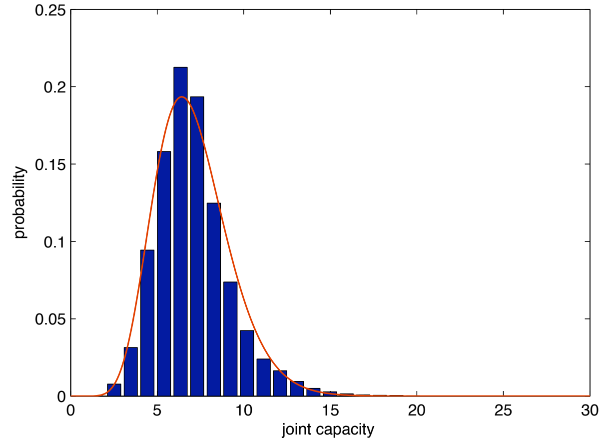

Fig. 6: histogram and fitted probability density function of joint capacity

Acknowledgement

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