Fault tolerant control for induction motors using sliding mode observers

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***Abstract*— In this paper a fault tolerant control design based on a sliding mode observer for induction motors is proposed. First, a direct field oriented controller based on backstepping technique is designed in order to steer the flux and speed variables to their desired references and to compensate the load disturbance. Second, a sliding mode observer is designed in order to detect and reconstruct the faults and also to estimate the flux. Then, additional control laws based on the estimates of the faults are designed in order to compensate the faults. Numerical simulations show the effectiveness of the proposed control scheme.**

I. INTRODUCTION

Research on fault tolerant control systems has received a great attention due to significantly increasing demand for reliability, maintainability and survivability of physical plants ([1]-[3]). Fault tolerant control systems have abilities to detect the presence of a fault and eventually to isolate it (fault detection and isolation) and also to reconfigure the control law in order to compensate the effect of the fault and maintain the stability and the control performance (fault tolerance).

Induction Motors (IM) are subjected to rotor and stator faults, such as stator short circuits, broken bars or rings, eccentricity,...etc. As in ([4],[5]), this paper is concerned with the rotor asymmetries caused by broken bars or dynamic eccentricity and stator asymmetries caused by static eccentricity. The presence of rotor and stator asymmetries induces harmonic components in the stator currents with frequencies which are directly related to the kind of the fault (stator or rotor fault) and amplitude and phase which depend on the severity of the fault ([4],[5]).

There are many literatures concerning fault tolerant control of induction motors ([4]-[8]). This paper focuses on the implicit fault tolerant controller proposed in ([4],[5]), where the effects generated by the occurrence of the fault are assumed to be modeled as an exogenous signal given by an autonomous ”neutrally stable” system (exosystem). In ([4],[5]) the design of the fault tolerant controller is based on an internal model which requires to know or to estimate the vector of the frequencies characterizing the faults. This design method becomes very difficult in case of a large vector of frequencies.

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In this work the design of the fault tolerant controller for induction motors is based on the use of sliding mode observers for fault detection and reconstruction ([9]-[11]). The sliding mode observers are used in both linear and nonlinear systems with uncertainties. They permit to reconstruct explicitly the faults by analyzing the dynamic of the estimation error when the sliding mode occurs. In our method, the design of the fault tolerant controller does not require to know or to estimate the vector of frequencies. Moreover, the approach proposed here does not require to know the model of the faults effects and it can also compensate the faults caused by the parameters variations and additive faults in actuator. Whereas, the authors in ([4],[5]) are limited to stator and rotor asymmetries whose model is known. In this paper, it is shown that a Direct Field Oriented Controller based on the backstepping strategy permits to steer the flux and speed variables to their desired references and to compensate the load torque effect. A sliding mode observer is used to reconstruct the faults and to estimate the flux, then additional control laws based on the resulting faults estimates are designed in order to compensate the faults.

This paper is organized as follows: In Section 2, the induction motor oriented model is presented. Section 3 is devoted to the design of the backstepping controller in un-faulty mode, which is able to steer the flux and speed variables to their desired references and to compensate the load disturbance. Section 4 describes the induction motor model in presence of rotor and stator faults. Section 5 presents the faults reconstruction by using a sliding mode observer. Section 6 presents the design of the fault tolerant controller, while Section 7 gives the simulation results. In Section 8, some conclusion remarks on the proposed fault tolerant control are given.

II. INDUCTION MOTOR ORIENTED MODEL

In field oriented control, the flux vector is forced to align with the d-axis ($\phi_{qr} = \frac{d\phi_{qr}}{dt} = 0$). The resulting induction motor model in the $(d-q)$ reference frame is described by the following state equations [12]:

$$\begin{array}{rcl} \frac{di_{ds}}{dt} & = & -ai_{ds} + \omega_s i_{qs} + \frac{L_m}{\sigma L_s L_r \tau_r}\phi_{dr} + \frac{V_{ds}}{\sigma L_s} \\ \frac{di_{qs}}{dt} & = & -ai_{qs} - \omega_s i_{ds} - \frac{L_m}{\sigma L_s L_r} P\Omega\phi_{dr} + \frac{V_{qs}}{\sigma L_s} \\ \frac{d\phi_{dr}}{dt} & = & \frac{L_m}{\tau_r} i_{ds} - \frac{1}{\tau_r}\phi_{dr} \\ \frac{d\Omega}{dt} & = & \frac{PL_m}{L_r J} i_{qs}\phi_{dr} - \frac{f}{J}\Omega - \frac{T}{J} \end{array} \quad (1)$$

with:

$$\omega_s = P\Omega + \frac{L_m}{\tau_r \phi_{dr}} i_{qs} \quad (2)$$

$$a=(\frac{R_s}{\sigma L_s}+\frac{1-\sigma}{\sigma\tau_r})$$

Where σ is the coefficient of dispersion,given by:

$$\sigma=1-\frac{L_m^2}{L_sL_r}$$

L_s, L_r, L_m are stator, rotor and mutual inductance, respectively. R_s, R_r are respectively stator and rotor resistance. ω_s is the stator pulsation. τ_r is the rotor time constant($\frac{L_r}{R_r}$) . P is the number of pole pairs. V_{ds}, V_{qs} are stator voltage components. ϕ_{dr}, ϕ_{qr} are the rotor flux components. Ω is the mechanical speed. T is the load torque. i_{ds}, i_{qs} are stator current components. J is the moment of inertie of the motor. f is the friction coefficient.

III. BACKSTEPPING CONTROL DESIGN

This part deals with the speed and flux control by means of backstepping control. This nonlinear control technique can be applied efficiently to linearise a nonlinear system with the existence of uncertainties, it is usually incorporated with the nonlinear damping to enhance robustness ([13],[14]).

In this work in order to compensate the load disturbance the backstepping technique is used. The idea of backstepping design is to select recursively some appropriate functions of state variables as virtual control inputs for lower dimension subsystems of the overall system. At each step of the backstepping a new virtual control input is designed. When the procedure terminates, the actual control input results which achieves the original design objective by virtue of a final Lyapunov function, which is formed by summing up the Lyapunov functions associated with each individual design step.

A. *Step1:*Flux control

The objective is to steer the flux ϕ_{dr} to a desired reference ϕ_{dr}^* , let $e_\phi=\phi_{dr}-\phi_{dr}^*$ be the flux tracking error. The dynamic of e_ϕ is:

$$\dot{e}_\phi=\frac{L_m}{\tau_r}i_{ds}-\frac{1}{\tau_r}\phi_{dr}-\dot{\phi}_{dr}^* \tag{3}$$

A Lyapunov function is defined as:

$$V_\phi=\frac{1}{2}e_\phi^2 \tag{4}$$

By deriving (4) we obtain:

$$\dot{V}_\phi=e_\phi\dot{e}_\phi=e_\phi(\frac{L_m}{\tau_r}i_{ds}-\frac{1}{\tau_r}\phi_{dr}-\dot{\phi}_{dr}^*) \tag{5}$$

To make $\dot{V}_\phi$ negative definite, i_{ds} is chosen as virtual element of control for stabilizing the flux, its desired value i_{ds}^* is defined as:

$$i_{ds}^*=-\frac{\tau_r}{L_m}k_\phi e_\phi+\frac{\phi_{dr}}{L_m}+\frac{\tau_r}{L_m}\dot{\phi}_{dr}^* \tag{6}$$

where $k_\phi>0$ is a design parameter.

By setting $i_{ds}=i_{ds}^*$ in (5) we get :

$$\dot{V}_\phi=-k_\phi e_\phi^2<0 \tag{7}$$

This implies that $e_\phi\to 0$ or $\phi_{dr}\to\phi_{dr}^*$ asymptotically.

B. *Step2:*Speed control

The objective is to steer the speed Ω to the desired reference Ω^*, let $e_\Omega=\Omega-\Omega^*$ be the speed tracking error. The error dynamic of the speed is:

$$\dot{e}_\Omega=\frac{PL_m}{L_rJ}i_{qs}\phi_{dr}-\frac{f}{J}\Omega-\frac{T}{J}-\dot{\Omega}^* \tag{8}$$

A Lyapunov function is defined as:

$$V_\Omega=\frac{1}{2}e_\Omega^2 \tag{9}$$

By deriving (9) we obtain:

$$\dot{V}_\Omega=e_\Omega\dot{e}_\Omega=e_\Omega(\frac{PL_m}{L_rJ}i_{qs}\phi_{dr}-\frac{f}{J}\Omega-\frac{T}{J}-\dot{\Omega}^*) \tag{10}$$

i_{qs} is chosen as virtual element of control for stabilizing the speed, its desired value i_{qs}^* is defined as:

$$i_{qs}^*=\frac{JL_r}{L_mP\phi_{dr}}(-k_\Omega e_\Omega-k_1\tanh\frac{k_1h}{\varepsilon_1}e_\Omega+\frac{f}{J}\Omega+\dot{\Omega}^*),\phi_{dr}\neq 0 \tag{11}$$

where: $h=0.2785$. $\varepsilon_1>0, k_1>0$ and $k_\Omega>0$ are design parameters.

By setting $i_{qs}=i_{qs}^*$ in (10) we get:

$$\dot{V}_\Omega=e_\Omega(-k_\Omega e_\Omega-k_1\tanh\frac{k_1h}{\varepsilon_1}e_\Omega-\frac{T}{J}) \tag{12}$$

For $k_1>|\frac{T}{J}|_{max}$, we obtain:

$$\dot{V}_\Omega\le -k_\Omega e_\Omega^2-k_1\tanh(\frac{k_1h}{\varepsilon_1}e_\Omega)e_\Omega+k_1|e_\Omega| \tag{13}$$

with:

$$|e_\Omega|=e_\Omega signe_\Omega \tag{14}$$

The derivative of the Lyapunov equation (13) becomes:

$$\dot{V}_\Omega\le -k_\Omega e_\Omega^2-k_1\tanh(\frac{k_1h}{\varepsilon_1}e_\Omega)e_\Omega+k_1e_\Omega signe_\Omega \tag{15}$$

We have (see [14]):

$$0\le k_1e_\Omega signe_\Omega-k_1\tanh(\frac{k_1h}{\varepsilon_1}e_\Omega)e_\Omega\le\varepsilon_1 \tag{16}$$

The derivative of the Lyapunov function (15) becomes:

$$\dot{V}_\Omega\le -k_\Omega e_\Omega^2+\varepsilon_1 \tag{17}$$

This implies that the variable e_Ω converges to a ball whose size depends on the parameter ε_1.

C. *Step3:* Currents control

The objective is to steer the currents i_{ds} and i_{qs} to their desired references i_{ds}^* and i_{qs}^*, respectively. Let $e_d=i_{ds}-i_{ds}^*$ and $e_q=i_{qs}-i_{qs}^*$ be the tracking errors of the currents, then the dynamic of the tracking errors are given by:

$$\begin{aligned}
\dot{e}_d&=-ai_{ds}+\omega_si_{qs}+\tfrac{L_m}{\sigma L_sL_r\tau_r}\phi_{dr}+\tfrac{V_{ds}}{\sigma L_s}-\tfrac{di_{ds}^*}{dt}\\
\dot{e}_q&=-ai_{qs}-\omega_si_{ds}-\tfrac{L_m}{\sigma L_sL_r}P\Omega\phi_{dr}+\tfrac{V_{qs}}{\sigma L_s}-\tfrac{di_{qs}^*}{dt}\\
\dot{e_\phi}&=\tfrac{L_m}{\tau_r}e_d+\tfrac{L_m}{\tau_r}i_{ds}^*-\tfrac{1}{\tau_r}\phi_{dr}-\dot{\phi}_{dr}^*\\
\dot{e}_\Omega&=\tfrac{PL_m}{L_rJ}e_q\phi_{dr}+\tfrac{PL_m}{L_rJ}i_{qs}^*\phi_{dr}-\tfrac{f}{J}\Omega-\tfrac{T}{J}-\dot{\Omega}^*
\end{aligned} \tag{18}$$

With:

$$\begin{aligned}
i_{ds}^{*} =& -\frac{\tau_r}{L_m}k_\phi e_\phi+\frac{\phi_{dr}}{L_m}+\frac{\tau_r}{L_m}\dot{\phi}_{dr}^{*}\\
i_{qs}^{*} =& \frac{JL_r}{L_mP\phi_{dr}}(-k_\Omega e_\Omega-k_1\tanh\frac{k_1h}{\varepsilon_1}e_\Omega+\frac{f}{J}\Omega+\dot{\Omega}^{*})\\
\frac{di_{ds}^{*}}{dt} =& (\frac{1-\tau_rk_\phi}{L_m})(\frac{L_m}{\tau_r}i_{ds}-\frac{\phi_{dr}}{\tau_r})+\frac{\tau_rk_\phi}{L_m}\dot{\phi}_{dr}^{*}+\frac{\tau_r}{L_m}\ddot{\phi}_{dr}^{*}\\
\frac{di_{qs}^{*}}{dt} =& \frac{JL_r}{L_mP\phi_{dr}}F_1(e_\Omega)\left(\frac{PL_m}{L_rJ}i_{qs}\phi_{dr}-\frac{f}{J}\Omega\right)\\
&+F_2(e_\Omega,\Omega,\phi_{dr})+\frac{JL_r}{L_mP\phi_{dr}}\left(\frac{f}{J}-F_1(e_\Omega)\right)\dot{\Omega}^{*}\\
&+\frac{JL_r}{L_mP\phi_{dr}}\ddot{\Omega}^{*}-\frac{L_rF_1(e_\Omega)}{PL_m\phi_{dr}}T
\end{aligned} \tag{19}$$

Where:

$$F_1(e_\Omega)=-k_\Omega-\frac{k_1^2h}{\varepsilon_1}\left(1-tanh(\frac{k_1h}{\varepsilon_1}e_\Omega)^2\right)+\frac{f}{J}$$

$$\begin{aligned}
F_2(e_\Omega,\Omega,\phi_{dr}) =& \frac{JL_r}{PL_m\phi_{dr}^2}\dot{\phi_{dr}}\\
&\left(k_\Omega e_\Omega+k_1tanh(\frac{k_1h}{\varepsilon_1}e_\Omega)-\frac{f}{J}\Omega-\dot{\Omega}^{*}\right)
\end{aligned}$$

By substituting i_{ds}^{*}, i_{qs}^{*},$\frac{di_{ds}^{*}}{dt}$ and $\frac{di_{qs}^{*}}{dt}$ by their expressions, the system of the tracking errors (18) becomes:

$$\begin{aligned}
\dot{e}_d =& -ai_{ds}+\omega_si_{qs}+\frac{L_m}{\sigma L_sL_r\tau_r}\phi_{dr}+\frac{V_{ds}}{\sigma L_s}\\
&-\left(\frac{1-\tau_rk_\phi}{L_m}\right)\left(\frac{L_m}{\tau_r}i_{ds}-\frac{\phi_{dr}}{\tau_r}\right)-\frac{\tau_rk_\phi}{L_m}\dot{\phi}_{dr}^{*}\\
&-\frac{\tau_r}{L_m}\ddot{\phi}_{dr}^{*}\\
\dot{e}_q =& -ai_{qs}-\omega_si_{ds}-\frac{L_m}{\sigma L_sL_r}P\Omega\phi_{dr}+\frac{V_{qs}}{\sigma L_s}\\
&-\frac{JL_r}{L_mP\phi_{dr}}F_1(e_\Omega)(\frac{PL_m}{L_rJ}i_{qs}\phi_{dr}-\frac{f}{J}\Omega)\\
&-\frac{JL_r}{L_mP\phi_{dr}}(\frac{f}{J}-F_1(e_\Omega))\dot{\Omega}^{*}-\frac{JL_r}{L_mP\phi_{dr}}\ddot{\Omega}^{*}\\
&+\frac{L_rF_1(e_\Omega)}{PL_m\phi_{dr}}T-F_2(e_\Omega,\Omega,\phi_{dr})\\
\dot{e}_\phi =& -k_\phi e_\phi+\frac{L_m}{\tau_r}e_d\\
\dot{e}_\Omega =& \frac{PL_m}{L_rJ}e_q\phi_{dr}-k_\Omega e_\Omega-k_1tanh(\frac{k_1h}{\varepsilon_1}e_\Omega)-\frac{T}{J}
\end{aligned} \tag{20}$$

Proposition 1: Let:

$$\begin{aligned}
V_{ds} =& \sigma L_s\left(-k_de_d+ai_{ds}-\frac{L_m}{\tau_r}e_\phi-\omega_si_{qs}-\frac{L_m}{\sigma L_sL_r\tau_r}\phi_{dr}\right.\\
&\left.+(\frac{1-\tau_rk_\phi}{L_m})(\frac{L_m}{\tau_r}i_{ds}-\frac{\phi_{dr}}{\tau_r})+\frac{\tau_rk_\phi}{L_m}\dot{\phi}_{dr}^{*}+\frac{\tau_r}{L_m}\ddot{\phi}_{dr}^{*}\right)
\end{aligned} \tag{21}$$

$$\begin{aligned}
V_{qs} =& \sigma L_s\left(-k_qe_q-k_2tanh(\frac{k_2h}{\varepsilon_2}e_q)+ai_{qs}+\omega_si_{ds}\right.\\
&+\frac{L_m}{\sigma L_sL_r}P\Omega\phi_{dr}-\frac{PL_m}{JL_r}e_\Omega\phi_{dr}+\frac{JL_r}{L_mP\phi_{dr}}F_1(e_\Omega)\\
&(\frac{PL_m}{L_rJ}i_{qs}\phi_{dr}-\frac{f}{J}\Omega)+F_2(e_\Omega,\Omega,\phi_{dr})\\
&\left.+\frac{JL_r}{L_mP\phi_{dr}}(\frac{f}{J}-F_1(e_\Omega))\dot{\Omega}^{*}+\frac{JL_r}{L_mP\phi_{dr}}\ddot{\Omega}^{*}\right)
\end{aligned} \tag{22}$$

be the actual control inputs, where : $k_d>0$, $k_q>0$,$k_2>0$ and $\varepsilon_2>0$ are design parameters. Then, if $k_2>\left|\frac{L_rF_1(e_\Omega)}{PL_m\phi_{dr}}T\right|_{max}$, the error variables e_ϕ,e_Ω,e_d and e_q are globally uniformly bounded.

Proof. By substituting the control laws (21) and (22) in (20) we get:

$$\begin{aligned}
\dot{e_d} =& -k_de_d-\frac{L_m}{\tau_r}e_\phi\\
\dot{e_q} =& -k_qe_q-k_2tanh(\frac{k_2h}{\varepsilon_2}e_q)-\frac{PL_m}{JL_r}e_\Omega\phi_{dr}+\frac{L_rF_1(e_\Omega)}{PL_m\phi_{dr}}T\\
\dot{e_\phi} =& -k_\phi e_\phi+\frac{L_m}{\tau_r}e_d\\
\dot{e_\Omega} =& \frac{PL_m}{L_rJ}e_q\phi_{dr}-k_\Omega e_\Omega-k_1tanh(\frac{k_1h}{\varepsilon_1}e_\Omega)-\frac{T}{J}
\end{aligned} \tag{23}$$

Consider the following Lyapunov function:

$$V=\frac{1}{2}(e_\phi^2+e_\Omega^2+e_d^2+e_q^2) \tag{24}$$

The derivative of V with respect to time is:

$$\begin{aligned}
\dot{V} =& e_\phi\left(-k_\phi e_\phi+\frac{L_m}{\tau_r}e_d\right)\\
&+e_\Omega\left(\frac{PL_m}{L_rJ}e_q\phi_{dr}-k_\Omega e_\Omega-k_1tanh(\frac{k_1h}{\varepsilon_1}e_\Omega)-\frac{T}{J}\right)\\
&+e_d\left(-k_de_d-\frac{L_m}{\tau_r}e_\phi\right)\\
&e_q\left(-k_qe_q-k_2tanh(\frac{k_2h}{\varepsilon_2}e_q)-\frac{PL_m}{JL_r}e_\Omega\phi_{dr}+\frac{L_rF_1(e_\Omega)}{PL_m\phi_{dr}}T\right)
\end{aligned} \tag{25}$$

From the step 2 we have $k_1 > |\frac{T}{J}|_{max}$, then the derivative of the Lyapunov function (25) becomes:

$$\begin{array}{rcl} \dot{V} & \leq & -k_\phi e_\phi^2 - k_\Omega e_\Omega^2 + \varepsilon_1 - k_d e_d^2 - k_q e_q^2 \\ & & -k_2 tanh(\frac{k_2 h}{\varepsilon_2} e_q) e_q + \frac{L_r F_1(e_\Omega)}{P L_m \phi_{dr}} T e_q \end{array} \quad (26)$$

For $k_2 > \left|\frac{L_r F_1(e_\Omega)}{P L_m \phi_{dr}} T\right|_{max}$, the derivative of the Lyapunov function (26) becomes:

$$\dot{V} \leq -k_\phi e_\phi^2 - k_\Omega e_\Omega^2 - k_d e_d^2 - k_q e_q^2 + \varepsilon_1 + \varepsilon_2 \quad (27)$$

This implies that the error variables e_ϕ, e_Ω, e_d and e_q are globally uniformly bounded.

IV. IM model in presence of fault

The presence of faults caused by rotor asymmetries (broken bars or dynamic eccentricity) and stator asymmetries (static eccentricity) induces sinusoidal components in the stator currents (for more detail see ([4],[5])), i.e:

$$\begin{aligned} i_{ds} =& i_{ds} + A sin(\omega_1 t + \varphi) + \sum_{i=1}^{N} [A_i sin(\omega_{2,i} t + \varphi_i) \\ & + A_{-i} sin(\omega_{2,-i} t + \varphi_{-i})] \\ i_{qs} =& i_{qs} + A cos(\omega_1 t + \varphi) + \sum_{i=1}^{N} [A_i cos(\omega_{2,i} t + \varphi_i) \\ & + A_{-i} cos(\omega_{2,-i} t + \varphi_{-i})] \end{aligned} \quad (28)$$

Where i_{ds} and i_{qs} denote the stator currents in the $(d-q)$ reference frame. The pulsations of the $2N+1$ harmonic components depend on the kind of fault (ω_1 is due to the stator asymmetries, while $\omega_{2,\pm i}, i = 1,..,N$ are due to the rotor asymmetries). The amplitudes $A, A_{\pm i}$ and the phases $\varphi, \varphi_{\pm i}$ are unknown, they depend on the entity of the stator or rotor asymmetries.

The sinusoidal components generated by the presence of the rotor and stator faults can be modeled by the following exosystem ([4],[5]):

$$\dot{w} = S(\varpi) w, w \in \Re^{4N+2} \quad (29)$$

With: $\varpi = \begin{pmatrix} \omega_1 & \omega_{2,1} & \omega_{2,-1} & . & . & \omega_{2,N} & \omega_{2,-N} \end{pmatrix}$ is the vector of the pulsations.

$$S(\varpi) = \begin{pmatrix} S_s & 0 \\ 0 & S_r \end{pmatrix}$$

$$S_s = \begin{pmatrix} 0 & \omega_1 \\ -\omega_1 & 0 \end{pmatrix}, S_r = diag \ (S_{r,1}, \quad . \quad . \quad . \quad S_{r,N})$$

$$S_{r,i} = diag\left(\begin{pmatrix} 0 & \omega_{2,i} \\ -\omega_{2,i} & 0 \end{pmatrix}, \begin{pmatrix} 0 & \omega_{2,-i} \\ -\omega_{2,-i} & 0 \end{pmatrix}\right)$$

Where ω_1 is the pulsation of the harmonic generated by the stator faults and $\omega_{2,\pm i}, i = 1,...,N$ are the pulsations of the harmonics generated by the rotor faults. The amplitudes and the phases of the harmonics are unknown, they depend on the initial state $w(0)$ of the exosystem. Then, the additive sinusoidal terms in (28) can be as a suitable combination of the exosystem state, i.e:

$$\begin{aligned} i_{ds} = i_{ds} + Q_d w \\ i_{qs} = i_{qs} + Q_q w \end{aligned} \quad (30)$$

With:

$$Q_d = \begin{pmatrix} 1 & 0 & 1 & 0 & . & . & . & 1 & 0 \end{pmatrix}$$

$$Q_q = \begin{pmatrix} 0 & 1 & 0 & 1 & . & . & . & 0 & 1 \end{pmatrix}$$

By deriving (30) we get:

$$\begin{aligned} \frac{di_{ds}}{dt} =& -a i_{ds} + \omega_s i_{qs} + \frac{L_m}{\sigma L_s L_r \tau_r} \phi_{dr} + \frac{V_{ds}}{\sigma L_s} \\ & + a Q_d w + Q_d S w - \omega_s Q_q w \\ \frac{di_{qs}}{dt} =& -a i_{qs} - \omega_s i_{ds} - \frac{L_m}{\sigma L_s L_r} P \Omega \phi_{dr} + \frac{V_{qs}}{\sigma L_s} \\ & + a Q_q w + Q_d S w + \omega_s Q_d w \end{aligned} \quad (31)$$

Then, the IM model in presence of faults becomes:

$$\begin{aligned} \frac{di_{ds}}{dt} =& -a i_{ds} + \omega_s i_{qs} + \frac{L_m}{\sigma L_s L_r \tau_r} \phi_{dr} + \frac{V_{ds}}{\sigma L_s} + \Gamma_d(w) \\ \frac{di_{qs}}{dt} =& -a i_{qs} - \omega_s i_{ds} - \frac{L_m}{\sigma L_s L_r} P \Omega \phi_{dr} + \frac{V_{qs}}{\sigma L_s} + \Gamma_q(w) \\ \frac{d\phi_{dr}}{dt} =& \frac{L_m}{\tau_r} i_{ds} - \frac{1}{\tau_r} \phi_{dr} \\ \frac{d\Omega}{dt} =& \frac{P L_m}{L_r J} i_{qs} \phi_{dr} - \frac{f}{J} \Omega - \frac{T}{J} \end{aligned} \quad (32)$$

With:

$$\Gamma_d(w) = a Q_d w + Q_d S w - \omega_s Q_q w$$

$$\Gamma_q(w) = a Q_q w + Q_q S w + \omega_s Q_d w$$

In this work the pulsations $\omega_1, \omega_{2,\pm i}, i = 1,...,N$ are assumed to be unknown. In order to reconstruct the fault effects $\Gamma_d(w)$ and $\Gamma_q(w)$ a sliding mode observer is used.

V. Fault reconstruction

Consider the system (32), where the currents i_{ds}, i_{qs} and speed Ω are assumed to be measured. In order to estimate the fault effects $\Gamma_d(w)$, $\Gamma_q(w)$ and the flux, a sliding mode observer is defined as:

$$
\begin{aligned}
&\tfrac{d\hat{i}_{ds}}{dt} = -a\hat{i}_{ds} + \hat{\omega}_s i_{qs} + \tfrac{L_m}{\sigma L_s L_r \tau_r}\hat{\phi}_{dr} + \tfrac{V_{ds}}{\sigma L_s} - u_d signs_d \\
&\tfrac{d\hat{i}_{qs}}{dt} = -a\hat{i}_{qs} - \hat{\omega}_s i_{ds} - \tfrac{L_m}{\sigma L_s L_r} P\Omega\hat{\phi}_{dr} + \tfrac{V_{qs}}{\sigma L_s} - u_q signs_q \\
&\tfrac{d\hat{\phi}_{dr}}{dt} = \tfrac{L_m}{\tau_r} i_{ds} - \tfrac{1}{\tau_r}\hat{\phi}_{dr} \\
&\hat{\omega}_s = P\Omega + \tfrac{L_m}{\tau_r \hat{\phi}_{dr}} i_{qs}
\end{aligned}
\tag{33}
$$

Where: $\hat{i}_{ds}, \hat{i}_{qs}$ are the observed stator currents and $\hat{\phi}_{dr}$ is the flux estimate, $u_d > 0$ and $u_q > 0$ are design parameters. s_d and s_q are the sliding surfaces defined as:

$$
\begin{aligned}
s_d &= \hat{i}_{ds} - i_{ds} \\
s_q &= \hat{i}_{qs} - i_{qs}
\end{aligned}
\tag{34}
$$

The currents and flux estimation errors are defined as: $\varepsilon_d = s_d = \hat{i}_{ds} - i_{ds}$, $\varepsilon_q = s_q = \hat{i}_{qs} - i_{qs}$ and $\varepsilon_\phi = \hat{\phi}_{dr} - \phi_{dr}$, then the errors dynamics are given by:

$$
\begin{aligned}
\frac{d\varepsilon_d}{dt} =& - a\varepsilon_d + (\hat{\omega}_s - \omega_s) i_{qs} + \frac{L_m}{\sigma L_s L_r \tau_r}\varepsilon_\phi - \Gamma_d(w) \\
& - u_d sign\varepsilon_d \\
\frac{d\varepsilon_q}{dt} =& - a\varepsilon_q - (\hat{\omega}_s - \omega_s) i_{ds} - \frac{L_m}{\sigma L_s L_r} P\Omega\varepsilon_\phi - \Gamma_q(w) \\
& - u_q sign\varepsilon_q \\
\frac{d\varepsilon_\phi}{dt} =& - \frac{1}{\tau_r}\varepsilon_\phi
\end{aligned}
\tag{35}
$$

Proposition 2: For:

$$
u_d > \left| -a\varepsilon_d + (\hat{\omega}_s - \omega_s) i_{qs} + \frac{L_m}{\sigma L_s L_r \tau_r}\varepsilon_\phi - \Gamma_d(w) \right|_{max} \tag{36}
$$

$$
u_q > \left| -a\varepsilon_q - (\hat{\omega}_s - \omega_s) i_{ds} - \frac{L_m}{\sigma L_s L_r} P\Omega\varepsilon_\phi - \Gamma_q(w) \right|_{max} \tag{37}
$$

the sliding mode will occur, i.e: $\varepsilon_d = \dot{\varepsilon}_d = 0$ and $\varepsilon_q = \dot{\varepsilon}_q = 0$, then the faults can be estimated.

Proof. Consider the following Lyapunov function:

$$
V = \frac{1}{2}\varepsilon_d^2 + \frac{1}{2}\varepsilon_q^2 \tag{38}
$$

The derivative of (38) with respect to time is:

$$
\begin{aligned}
\dot{V} =& \varepsilon_d(-a\varepsilon_d + (\hat{\omega}_s - \omega_s) i_{qs} + \frac{L_m}{\sigma L_s L_r \tau_r}\varepsilon_\phi - \Gamma_d(w) \\
& - u_d sign\varepsilon_d) + \varepsilon_q(-a\varepsilon_q - (\hat{\omega}_s - \omega_s) i_{ds} \\
& - \frac{L_m}{\sigma L_s L_r} P\Omega\varepsilon_\phi - \Gamma_q(w) - u_q sign\varepsilon_q)
\end{aligned}
\tag{39}
$$

By choosing:

$$
u_d > \left| -a\varepsilon_d + (\hat{\omega}_s - \omega_s) i_{qs} + \frac{L_m}{\sigma L_s L_r \tau_r}\varepsilon_\phi - \Gamma_d(w) \right|_{max}
$$

$$
u_q > \left| -a\varepsilon_q - (\hat{\omega}_s - \omega_s) i_{ds} - \frac{L_m}{\sigma L_s L_r} P\Omega\varepsilon_\phi - \Gamma_q(w) \right|_{max}
$$

the sliding mode occurs, i.e: $\varepsilon_d = \dot{\varepsilon}_d = 0$ and $\varepsilon_q = \dot{\varepsilon}_q = 0$. Therefore the equations (35) become:

$$
(\hat{\omega}_s - \omega_s) i_{qs} + \frac{L_m}{\sigma L_s L_r \tau_r}\varepsilon_\phi - \Gamma_d(w) - u_d sign_{eq}\varepsilon_d = 0 \tag{40}
$$

$$
-(\hat{\omega}_s - \omega_s) i_{ds} - \frac{L_m}{\sigma L_s L_r} P\Omega\varepsilon_\phi - \Gamma_q(w) - u_q sign_{eq}\varepsilon_q = 0 \tag{41}
$$

$$
\frac{d\varepsilon_\phi}{dt} = -\frac{1}{\tau_r}\varepsilon_\phi \tag{42}
$$

Equation (42) shows that ε_ϕ converges to zero as $t \to \infty$, then $\hat{\omega}_s \to \omega_s$ and the estimates of the faults are given by:

$$
\begin{aligned}
\hat{\Gamma}_d &= -u_d sign_{eq}(\varepsilon_d) \\
\hat{\Gamma}_q &= -u_q sign_{eq}(\varepsilon_q)
\end{aligned}
\tag{43}
$$

Remark 1: The function $sign_{eq}$ represents the average value of the $sign$ function, it can be obtained by the use of a low passe filter or by a continuous approximation of the sign function. Here the $sign$ function is approximated by an hyperbolic $tanh$ function.

VI. DESIGN OF FAULT TOLERANT CONTROL

The structure of the global controller is given by:

$$
\begin{aligned}
V_{ds} &= V_{dsn} + V_{df} \\
V_{qs} &= V_{qsn} + V_{qf}
\end{aligned}
\tag{44}
$$

with V_{dsn} and V_{qsn} are the backstepping control laws (21) and (22) designed in un-faulty mode $(w(t) = 0)$ to steer the tracking errors to zero and to compensate the load disturbance.

V_{df} and V_{qf} are additional control laws (compensation units) that will be designed in order to compensate the faults.

Proposition 3: Let V_{dsn} and V_{qsn} be the backstepping control laws given by (21) and (22) and let:

$$
V_{df} = -\sigma L_s \hat{\Gamma}_d \tag{45}
$$

$$
V_{qf} = -\sigma L_s \hat{\Gamma}_q \tag{46}
$$

be the additional control laws, where the estimate $\hat{\Gamma}_d, \hat{\Gamma}_q$ are given by (43). Then the faults are compensated.

Proof. The dynamics of the tracking errors are given by:

$$\begin{aligned}
\frac{de_d}{dt} =& - ai_{ds} + \omega_s i_{qs} + \frac{L_m}{\sigma L_s L_r \tau_r}\phi_{dr} + \frac{V_{ds}}{\sigma L_s} \\
& - (\frac{1-\tau_r k_\phi}{L_m})(\frac{L_m}{\tau_r} i_{ds} - \frac{\phi_{dr}}{\tau_r}) - \frac{\tau_r k_\phi}{L_m}\dot{\phi}^*_{dr} \\
& - \frac{\tau_r}{L_m}\ddot{\phi}^*_{dr} + \Gamma_d(w) \\
\frac{de_q}{dt} =& - ai_{qs} - \omega_s i_{ds} - \frac{L_m}{\sigma L_s L_r} P\Omega\phi_{dr} + \frac{V_{qs}}{\sigma L_s} \\
& - \frac{JL_r}{L_m P\phi_{dr}} F_1(e_\Omega)(\frac{PL_m}{L_r J} i_{qs}\phi_{dr} - \frac{f}{J}\Omega) \\
& - \frac{JL_r}{L_m P\phi_{dr}}(\frac{f}{J} - F_1(e_\Omega))\dot{\Omega}^* - \frac{JL_r}{L_m P\phi_{dr}}\ddot{\Omega}^* \\
& + \frac{L_r F_1(e_\Omega)}{PL_m\phi_{dr}} T - F_2(e_\Omega, \Omega, \phi_{dr}) + \Gamma_q(w) \\
\frac{de_\phi}{dt} =& - k_\phi e_\phi + \frac{L_m}{\tau_r} e_d \\
\frac{de_\Omega}{dt} =& \frac{PL_m}{L_r J} e_q \phi_{dr} - k_\Omega e_\Omega - k_1 tanh(\frac{k_1 h}{\varepsilon_1} e_\Omega) - \frac{T}{J}
\end{aligned} \tag{47}$$

By substituting (44) in (47) we get:

$$\begin{aligned}
\frac{de_d}{dt} =& - k_d e_d - \frac{L_m}{\tau_r} e_\phi + \Gamma_d(w) - \hat{\Gamma}_d(w) \\
\frac{de_q}{dt} =& - k_q e_q - k_2 tanh(\frac{k_2 h}{\varepsilon_2} e_q) - \frac{PL_m}{JL_r} e_\Omega \phi_{dr} \\
& + \frac{L_r F_1(e_\Omega)}{PL_m \phi_{dr}} T + \Gamma_q(w) - \hat{\Gamma}_q(w) \\
\frac{de_\phi}{dt} =& - k_\phi e_\phi + \frac{L_m}{\tau_r} e_d \\
\frac{de_\Omega}{dt} =& \frac{PL_m}{L_r J} e_q \phi_{dr} - k_\Omega e_\Omega - k_1 tanh(\frac{k_1 h}{\varepsilon_1} e_\Omega) - \frac{T}{J}
\end{aligned} \tag{48}$$

Section (5) shows that $\hat{\Gamma}_d(w) \to \Gamma_d(w)$ and $\hat{\Gamma}_q(w) \to \Gamma_q(w)$ then the faults are compensated and the resulting closed loop system is stable. Its stability is proved in Section 3 by the Lyapunov function (24).

VII. SIMULATION RESULTS

Numerical simulations have been performed to validate the proposed control scheme. The induction motor parameters are given in the appendix. The controller parameters are chosen as follows: $k_d = 650, k_q = 500, k_\Omega = 0.5, k_\phi = 25, k_1 = 300, k_2 = 300, u_d = 26000, u_q = 26000$. The speed and flux references are fixed at $\Omega_* = 100 rad/s$ and $\phi^*_{dr} = 0.9Wb$, respectively, also a load disturbance $T = 3N.m$ is applied. Figure 1 shows the responses of the induction motor in the absence of faults in case just the backstepping controller is present in the loop. The speed and the flux trajectories converge to their desired references and the load disturbance is rejected. Figure 2 shows the responses of the induction motor controlled by the backstepping controller when stator and rotor faults occur. The IM responses exhibit oscillations and deviation from their desired references. Figure 3 shows the responses of the IM in case the backstepping controller is augmented with the fault compensation units. The effectiveness of the compensation units in compensating the effect of the faults is evident. Figure 4 shows a good estimation of the faults. In order to test the robustness of the proposed control scheme, the variations of the rotor resistance $R_r$and stator resistance R_s are introduced. the simulation results are given by the figure 5. We see that the proposed controller is insensitive to the rotor and stator resistance variations.

VIII. CONCLUSION

In this paper a sliding mode observer based approach to fault compensation for induction motors has been presented. In absence of faults a backstepping controller permits to steer the flux and the speed variables to their desired references and to reject the load disturbance, however the presence of faults degradates the performances of the induction motor. In order, to detect and estimate the faults a sliding mode observer is used. The use of a sliding mode observer provides a good estimation of the faults. Then, additional control laws based on the resulting faults estimates permit to eliminate the effect of the faults.

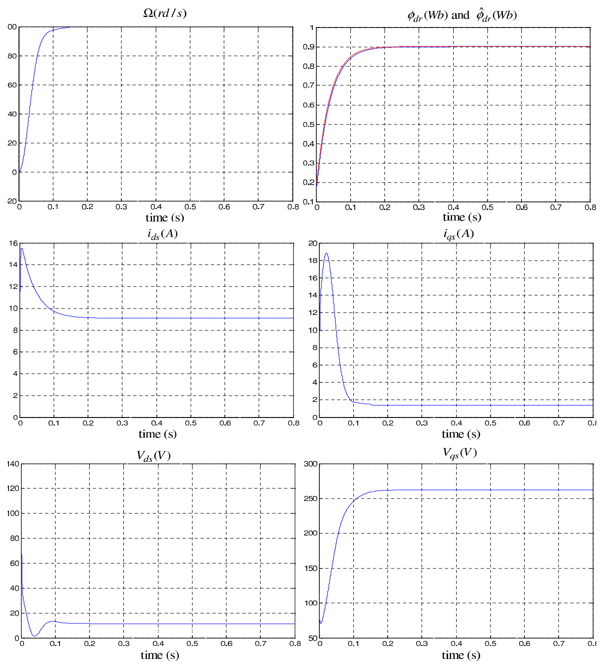

Fig. 1. Responses of the IM controlled by the backstepping controller in un-faulty mode

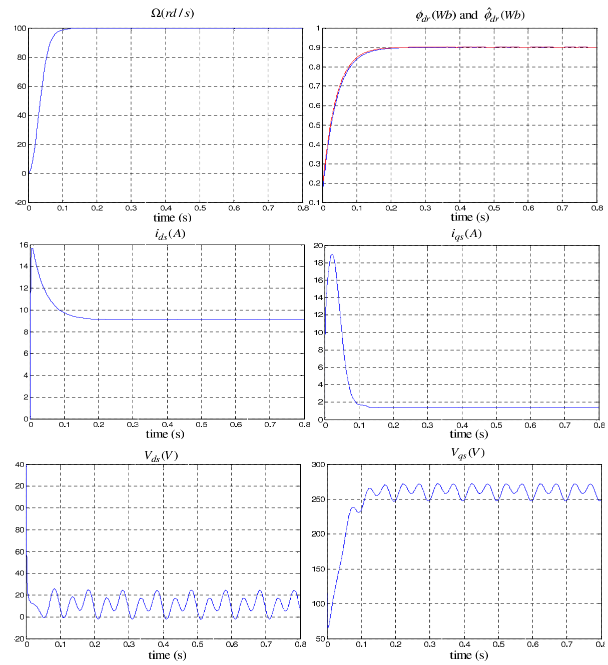

Fig. 3. Responses of the IM in case the backstepping controller is augmented with the fault compensation units

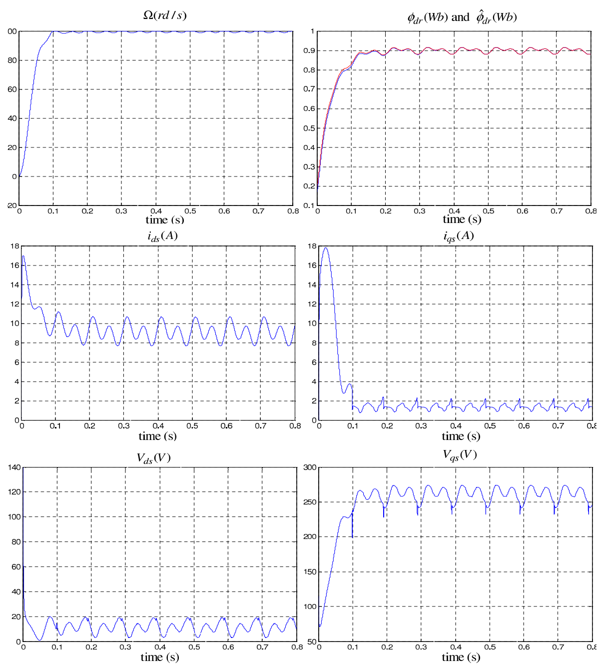

Fig. 2. Responses of the IM controlled by the backstepping controller when faults occur

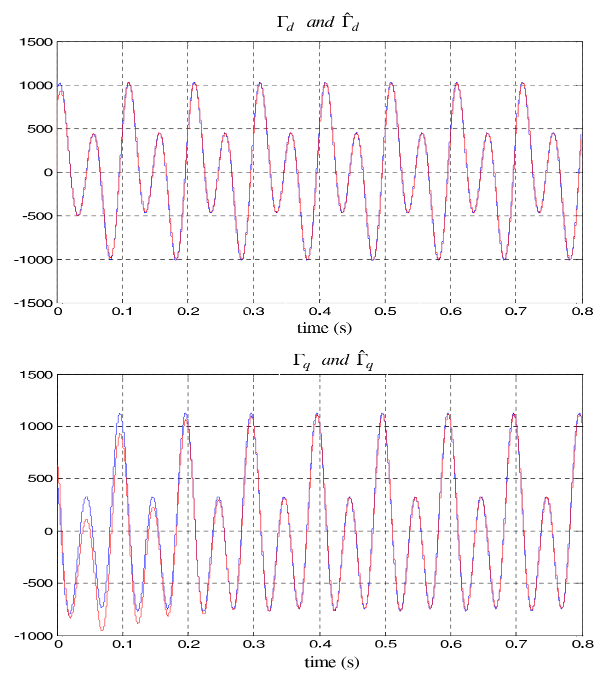

Fig. 4. The faults and their estimates

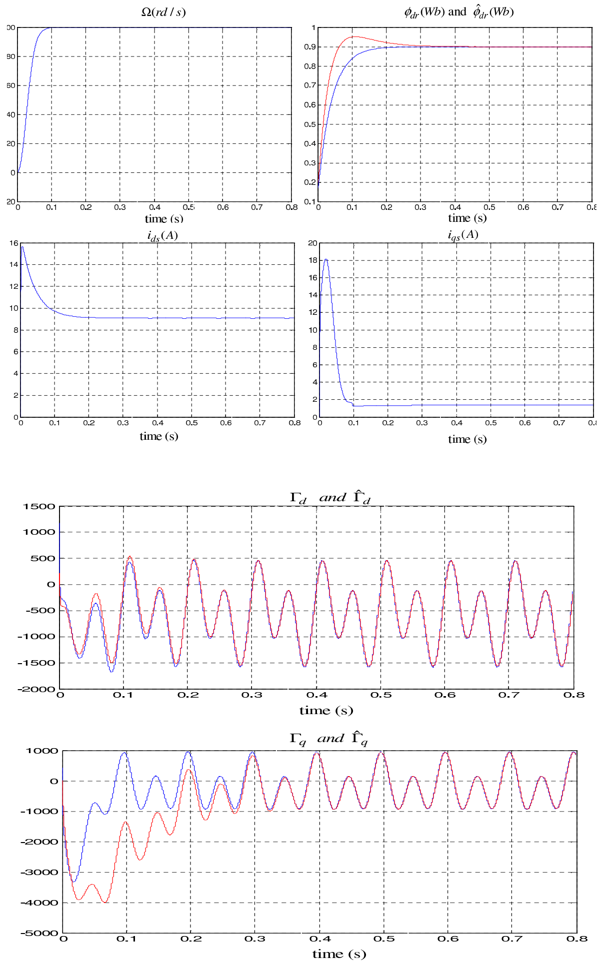

Fig. 5. Responses of the IM with an increase of $+50\% R_r$ and $+50\% R_s$

IX. Appendix

The induction motor used in this work is a $1.5KW, U = 220V, 50Hz, I_n = 7.5A$. The parameters are: $R_s = 1.633\Omega, R_r = 0.93\Omega, L_r = 0.076H, L_s = 0.142H, L_m = 0.099H, J = 0.0111Kg.m^2, f = 0.0018N.m/rad/s$ and $P = 2$.

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