

On the complexity of the Arora-Ge Algorithm against LWE

Martin Albrecht, Carlos Cid, Jean-Charles Faugère, Robert Fitzpatrick,
Ludovic Perret

► **To cite this version:**

Martin Albrecht, Carlos Cid, Jean-Charles Faugère, Robert Fitzpatrick, Ludovic Perret. On the complexity of the Arora-Ge Algorithm against LWE. SCC 2012 – Third international conference on Symbolic Computation and Cryptography, Jul 2012, Castro Urdiales, Spain. pp.93-99, 2012, <http://wmc2012.unican.es/SCC_WMC_2012.pdf>. <hal-00776434>

HAL Id: hal-00776434

<https://hal.inria.fr/hal-00776434>

Submitted on 15 Jan 2013

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

On the complexity of the Arora-Ge Algorithm
against LWE
**Martin R. Albrecht, Carlos Cid, Jean-Charles
Faugère, Robert Fitzpatrick, and Ludovic
Perret**

Abstract

Arora & Ge [5] recently showed that solving LWE can be reduced to solve a high-degree non-linear system of equations. They used a linearization to solve the systems. We investigate here the possibility of using Gröbner bases to improve Arora & Ge approach.

Introduction

The Learning With Errors (LWE) Problem was introduced by Regev in [27, 26]. It is a generalisation for large primes of the well-known LPN (Learning Parity with Noise) problem. Since its introduction, LWE has become a source of many innovative cryptosystems, such as the oblivious transfer protocol by Peikert et al. [25], a cryptosystem by Akavia et al. [1] that is secure even if almost the entire secret key is leaked, homomorphic encryption [21, 10, 4], etc. . . Reasons of LWE’s success in cryptography include its simplicity as well as convincing theoretical arguments regarding its hardness, i.e. a reduction from (worst-case) assumed hard lattice problems to (average-case) LWE.

The purpose of this paper is to investigate whether algebraic techniques (e.g. [16, 17, 18, 19, 3, 2, 20]) can be used in the context of LWE. This is motivated by a recent result Arora & Ge [5] who showed that solving LWE can be reduced to solve a high-degree non-linear system of equations.

Learning With Errors

We reproduce below the definition of the LWE problem from [27, 26].

Definition 1 (LWE). *Let $n \geq 1$ be the number of variables, q be an odd prime integer, χ be a probability distribution on \mathbb{Z}_q and \mathbf{s} be a secret vector in \mathbb{Z}_q^n . We denote by $L_{\mathbf{s},\chi}^{(n)}$ the probability distribution on $\mathbb{Z}_q^n \times \mathbb{Z}_q$ obtained by choosing $\mathbf{a} \in \mathbb{Z}_q^n$ at random, choosing $e \in \mathbb{Z}_q$ according to χ , and returning $(\mathbf{a}, c) = (\mathbf{a}, \langle \mathbf{a}, \mathbf{s} \rangle + e) \in \mathbb{Z}_q^n \times \mathbb{Z}_q$. LWE is the problem of finding $\mathbf{s} \in \mathbb{Z}_q^n$ given pairs $\mathbb{Z}_q^n \times \mathbb{Z}_q$ sampled according to $L_{\mathbf{s},\chi}^{(n)}$.*

The modulus q is typically taken to be polynomial in n , and χ is the discrete Gaussian distribution on \mathbb{Z}_q with mean 0 and standard deviation $\sigma = \alpha \cdot q$, for some α . To discretize the Gaussian distribution $\mathbb{N}0, \sigma^2$ modulo q , we sample according to $\mathbb{N}0, \sigma^2$ and round to the nearest integer mod q . In what follows, $\chi_{\alpha,q}$ will then denote this discretized distribution.

A typical setting for the standard deviation (std) is $\sigma = n^\epsilon$, with $\epsilon, 0 \leq \epsilon \leq 1$. For example, [27] suggests $q \approx n^2$ and $\alpha = 1/(\sqrt{n} \cdot \log^2 n)$. Indeed, as soon as $\epsilon \geq 1/2$ (worst-case) GAPSVP $-\tilde{O}(n/\alpha)$ reduces to (average-case) LWE¹. Thus, any algorithm solving LWE (when $\epsilon \geq 1/2$) can be used for GAPSVP $-\tilde{O}(n/\alpha)$. We emphasize that it is widely believed that only exponential algorithm exists for solving GAPSVP $-\tilde{O}(n/\alpha)$.

Recently, Arora & Ge [5] introduced a variant of LWE with *structured* errors. In this setting, you have given an oracle such that given LWE samples returns polynomials which vanish on the errors.

¹The reduction is quantum if q is polynomial but can be made [24] classical if q is super polynomial.

They showed that the (discretized) Gaussian intrinsically induced a structure on the errors. This feature can be used to reduce LWE to the problem of solving a non-linear system of multivariate equations.

The total complexity (time and space) of their approach is $2^{\tilde{O}(n^{2\varepsilon})}$. It is then subexponential when $\varepsilon < 1/2$, but remains exponential when $\varepsilon \geq 1/2$. It is interesting that Arora&Ge reach with a completely different approach the $\varepsilon = 1/2$ hardness limit advised by Regev [27, 26].

Note that an improvement on Arora&Ge could allow to challenge the ‘subexponentiality’ of GAPSVP – $\tilde{O}(n/\alpha)$. Remark that [5] uses linearization to solve the non-linear system. It is then natural to investigate whether more advanced tools, such as Gröbner bases [11, 12, 13], could improve the algorithm of Arora&Ge.

In this note, we will show that Gröbner bases can bring a practical improvement on the complexity of [5]. We also briefly discuss whether Gröbner bases can (or can not) allow to change the complexity class of Arora&Ge. Before that, we need to recall some basic complexity results about Gröbner bases.

Gröbner bases – Complexity Results

Gröbner basis is probably the main tool allowing to solve non-linear system of finite fields. From an algorithmic point of view, Lazard [22] showed that computing the Gröbner basis for a system of polynomials is equivalent to perform a Gaussian elimination on the *Macaulay matrices* [23] $\mathcal{M}_{d,m}^{\text{acaulay}}$ for $d, 1 \leq d \leq D$ for some integer D . Moreover, the most efficient known algorithms such as F_5 [15] reduce Gröbner basis computations to a series of Gaussian eliminations on matrices of increasing sizes.

Definition 2 (Macaulay Matrix [23]). *Let $f_1, \dots, f_m \in \mathbb{Z}_q[x_1, \dots, x_n]$. The Macaulay matrix $\mathcal{M}_{d,m}^{\text{acaulay}}(f_1, \dots, f_m)$ of degree d is defined as follows: list “horizontally” all the degree d monomials from smallest to largest sorted by some fixed admissible monomial ordering. The smallest monomial comes last. Multiply each f_i by all monomials $t_{i,j}$ of degree $d - d_i$ where $d_i = \deg(f_i)$. Finally, construct the coefficient matrix for the resulting system:*

$$\mathcal{M}_{d,m}^{\text{acaulay}}(f_1, \dots, f_m) := \begin{matrix} & & & & \text{monomials of degree } \leq d \text{ sorted for } < \\ & (t_{1,1}, f_1) & & & & \\ & (t_{1,2}, f_1) & & & & \\ & \vdots & & & & \\ & (t_{m,1}, f_m) & & & & \\ & (t_{m,2}, f_m) & & & & \\ & \vdots & & & & \end{matrix} \left(\begin{matrix} \\ \\ \\ \\ \\ \\ \end{matrix} \right)$$

Theorem 3 ([22]). *Let $\mathbf{f} = (f_1, \dots, f_m) \in (\mathbb{Z}_q[x_1, \dots, x_n])^m$ and $<$ be a monomial ordering. There exists a positive integer D for which Gaussian elimination on all $\mathcal{M}_{d,m}^{\text{acaulay}} = (f_1, \dots, f_m)$ matrices for $d, 1 \leq d \leq D$ computes a Gröbner basis of $\langle f_1, \dots, f_m \rangle$ w.r.t. to $<$. The degree D will be called degree of regularity of f_1, \dots, f_m .*

Consequently, the complexity of computing a Gröbner basis is bounded by the complexity of performing Gaussian elimination on the Macaulay matrix in some degree D . Roughly, the complexity of computing a Gröbner basis with an algorithm based on the degree of regularity (such as – but not limited too – Buchberger’s algorithm, F_4, F_5 [15, 11, 12, 14]) is:

$$o \left(\binom{n+D}{D}^\omega \right), \quad (1)$$

where $2 \leq \omega < 3$ is the linear algebra constant, and D is the degree of semi-regularity of the system.

In general, computing the degree of regularity of a system is a difficult problem. However, it is known for a specific family of polynomial systems [6, 8, 7, 9].

Definition 4 (Semi-regular Sequence [8]). *Let $m > n$, and $f_1, \dots, f_m \in \mathbb{Z}_q[x_1, \dots, x_n]$ be homogeneous polynomials of degrees d_1, \dots, d_m respectively and I the ideal generated by these polynomials. The system is said to be a semi-regular sequence if the Hilbert series [13] of I w.r.t. the grevlex order is:*

$$H_I(z) = \left[\frac{\prod_{i=1}^m (1 - z^{d_i})}{(1 - z)^n} \right]_+, \quad (2)$$

where $[S]_+$ denotes the series obtained by truncating S before the index of its first non-positive coefficient. Thus, the degree of regularity D involved in Theorem 3 for a semi-regular sequence is:

$$1 + \deg(H_I).$$

Improving Arora-Ge Approach

We briefly detail below the linearization approach of Arora-Ge. We then discuss whether Gröbner bases can be used in this context.

Basic Arora-Ge Algorithm – A Linearization Approach

The idea of [5] is to generate a non-linear noise-free system of equations from LWE samples. This is due to the following well-known feature of a Gaussian noise:

Lemma 5. *Let $C > 0$ be a constant. It holds that:*

$$\Pr[e \stackrel{\$}{\leftarrow} \chi_{\alpha, q} : |e| > C \cdot \sigma] \leq e^{o(-C^2)}.$$

As a consequence, elements sampled from a Gaussian distribution only takes values on a (small) subset $[-C \cdot \sigma, \dots, C \cdot \sigma]$ of \mathbb{Z}_q with high probability. From now on, we set $t = C \cdot \sigma$. We can re-interpret Lemma 5 algebraically by considering the polynomial:

$$P(X) = X \prod_{i=1}^t (X + i)(X - i).$$

Clearly P is of degree $2t + 1 \in o(\sigma)$. Thus, if $e \stackrel{\$}{\leftarrow} \chi_{\alpha, q}$, then $P(e) = 0$ with probability at least $1 - e^{o(-C^2)}$.

For $i \geq 1$, let $(\mathbf{a}_i, \langle \mathbf{a}_i, s \rangle + e_i) = (\mathbf{a}_i, b_i) \in \mathbb{Z}_q^n \times \mathbb{Z}_q$. If $e_i \stackrel{\$}{\leftarrow} \chi_{\alpha, q}$, then

$$P(\mathbf{a}_i, \langle \mathbf{a}_i, s \rangle - b_i) = 0, \quad (3)$$

with probability at least $1 - e^{o(-C^2)}$. As a consequence, each sample $(\mathbf{a}_i, \langle \mathbf{a}_i, s \rangle + e_i) = (\mathbf{a}_i, b_i) \in \mathbb{Z}_q^n \times \mathbb{Z}_q$ allows to generate a non-linear equation of degree $2t + 1$ in the n components of the secret \mathbf{s} .

The idea of Arora & Ge is then to generate sufficiently many equations as in (3) to perform a linearization. However, one has to choose the constant – denoted by C_{AG} – occurring in Lemma 5 sufficiently big so that all errors generated lies with high probability in $[-C_{AG} \cdot \sigma, \dots, C_{AG} \cdot \sigma] \subseteq \mathbb{Z}_q$, i.e. the secret \mathbf{s} is indeed a common solution of the M_{AG} equations constructed as in (3). To this end, we set:

$$p_f = \frac{M_{AG}}{e^{o(C_{AG}^2)}}.$$

This is the probably that the secret $s \in \mathbb{Z}_q^n$ is not solution of the system \mathcal{S}_{AG} generated from M_{AG} equations as in (3), i.e. the probability of failure of Arora-Ge approach. Let also $D_{AG} = 2C_{AG} \cdot \sigma + 1$ be the degree of the equations occurring in \mathcal{S}_{AG} . According to [5], taking $C_{AG} \in \tilde{o}(\sigma)$ allows to make the probability of failure negligible.

To summarize, Arora-Ge approach reduces to linearize at system of M_{AG} equations of degree $D_{AG} = 2C_{AG} \cdot \sigma + 1 \in \tilde{o}(\sigma^2)$. Moreover, correctness of this approach can be proven:

Theorem 6. [5] Let $D_{AG} < q$. The system obtained by linearizing $M_{AG} = O\left(q \cdot \log(q) \binom{n+D_{AG}}{D_{AG}} \sigma\right) = n^{O(D_{AG})} = 2^{\tilde{O}(D_{AG})}$ equations as in (3) has at most one solution with high probability.

The time complexity of the basic Arora-Ge approach is then

$$C_{AG}^{\text{plx}} = n^{O(D_{AG})} = 2^{\tilde{O}(\sigma^2)} = 2^{\tilde{O}(n^{2\varepsilon})}.$$

Note also this algorithm also requires $2^{\tilde{O}(n^{2\varepsilon})}$ LWE samples for performing the linearization.

From Linearization to Gröbner Bases

The question we try to address here is whether the complexity C_{AG}^{plx} can be improved by using Gröbner bases instead of linearization. The rationale is that you can decrease the constant C_{AG} (and so the degree of the equations) to a value smaller than $\tilde{O}(n^{2\varepsilon})$ by considering less equations (whilst keeping the probability p_f of failure similar in both approaches). However, the cost of the solving step increases since one has to compute a Gröbner basis. The question is then to find – if any – a tradeoff allowing to improve upon linearization.

To do so, we will consider a number of equations of the form $\sqrt[\theta]{M_{AG}}$, with $\theta > 1$ ($\theta = 1$ is the basic Arora-Ge). We want to keep the probability of failure similar for the linearization and Gröbner basis approaches. As a consequence, we need to take a constant C_θ such that:

$$p_f = \frac{\sqrt[\theta]{M_{AG}}}{e^{O(C_\theta^\theta)}}.$$

An easy calculation leads to $C_\theta \in \tilde{O}\left(\frac{C_{AG}}{\sqrt{\theta}}\right)$. Thus, decreasing the number of equations from M_{AG} to $\sqrt[\theta]{M_{AG}}$ allows to divide the constant C_{AG} by a factor $\sqrt{\theta}$. The degree of the equations we are doing to consider is then equal to $2\sigma \cdot C_\theta + 1 \in \tilde{O}\left(\frac{\sigma^2}{\sqrt{\theta}}\right)$.

The question is now to find a good candidate for θ . Typically, if θ is too big then you will greatly decrease the number of equations, but the cost of the solving step will become prohibitive and the total complexity will be worse than for a linearization.

We have considered a θ of the form: $\theta = n^{2\beta}$, for some $\beta \geq 0$ (note that we get the basic Arora-Ge by taking $\beta = 0$). In this new setting, we get a constant $C_\beta = n^{\varepsilon-\beta}$. We have then to solve a system having $M_\beta = \sqrt[n^{2\beta}]{M_{AG}} \in 2^{\tilde{O}(n^{2(\varepsilon-\beta)})}$ equations of degree $D_\beta = \tilde{O}(n^{2\varepsilon-\beta})$. We denote such a system by $S_{GB}(\beta)$.

The question is to determine the complexity $C_{GB-AG}^{\text{plx}}(\beta)$ of solving $S_{AG}(\beta)$. This reduces to studying its degree of regularity D_{reg}^β . Given current algorithms, the specific structure of the system does not allow to solve it faster than random systems. As a consequence, we assume that D_{reg}^β is not bigger than the degree of regularity of a semi-regular system of the same size², namely:

$$D_{reg}^\beta \leq 1 + \deg(H_\beta),$$

where:

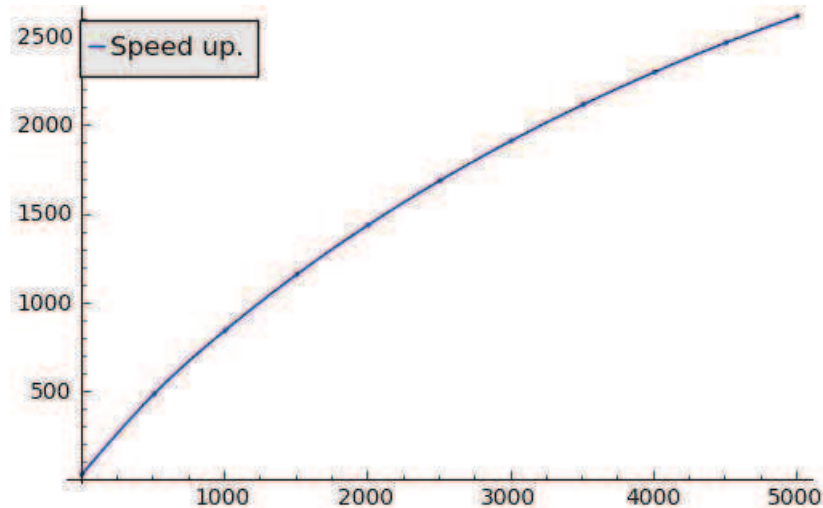
$$H_\beta(z) = \left[\frac{(1-z^{D_\beta})^{M_\beta}}{(1-z)^n} \right]_+,$$

where $[\cdot]_+$ denotes the series obtained by truncating before the index of its first non-positive coefficient.

We present below some experiments performed for $\beta = 1/5$. We have computed explicitly the complexities for both approaches: linearization and Gröbner bases. As suggested in [27],

²We have performed few experiments for small parameters. The experiments seem to confirm this hypothesis.

we considered $q \approx n^2$ and $\alpha = 1/(\sqrt{n} \cdot \log^2 n)$. We plotted below the speed-up we obtained, i.e. $\log_2 \left(\frac{C_{GB-AG}^{plx}(\beta)}{C_{AG}^{plx}} \right)$ (y-axis) for $n, 0 \leq n \leq 5000$. We can see that Gröbner bases allow to improve the complexity of the basic Arora-Ge when $n \leq 5000$ (x-axis). Note that further experiments are required to confirm this behavior when n tends to infinity³



However, the form of the speed-up also tends to suggest that we only improve from a constant C_{AG}^{plx} . change the asymptotical behavior of the Arora&Ge approach. we mention that we are currently considering several forms for the β . In particular, β which is not a constant but a function of n . As a conclusion, we also emphasize that Arora-Ge needs exponential (or subexponential) number of LWE samples. For most cryptosystems based on LWE, you have access to much less samples, typically polynomially-many. In this situation, you have then not enough samples to perform the linearization and the only option to mount the Arora&Ge approach is to solve the system by using Gröbner bases.

References

- [1] A. Akavia, S. Goldwasser, and V. Vaikuntanathan. Simultaneous hardcore bits and cryptography against memory attacks. In O. Reingold, editor, *TCC*, volume 5444 of *Lecture Notes in Computer Science*, pages 474–495. Springer Verlag, 2009.
- [2] M. R. Albrecht and C. Cid. Cold boot key recovery by solving polynomial systems with noise. In J. Lopez and G. Tsudik, editors, *ACNS*, volume 6715 of *Lecture Notes in Computer Science*, pages 57–72, 2011.
- [3] M. R. Albrecht and K. G. Paterson. Breaking an identity-based encryption scheme based on dhies. In L. Chen, editor, *IMA Int. Conf.*, volume 7089 of *Lecture Notes in Computer Science*, pages 344–355. Springer Verlag, 2011.
- [4] M. R. Albrecht, P. Farshim, J.-C. Faugère, and L. Perret. Polly Cracker, revisited. In *Advances in Cryptology – ASIACRYPT 2011*, volume 7073 of *Lecture Notes in Computer Science*, pages 179–196, Berlin, Heidelberg, New York, 2011. Springer Verlag. full version available as Cryptology ePrint Archive, Report 2011/289, 2011 <http://eprint.iacr.org/>.

³Note that the degree of the equations involved being huge, it becomes rather costly to just expand the Hilbert series for the systems considered.

- [5] S. Arora and R. Ge. New algorithms for learning in presence of errors. In L. Aceto, M. Henzinger, and J. Sgall, editors, *ICALP*, volume 6755 of *Lecture Notes in Computer Science*, pages 403–415. Springer Verlag, 2011.
- [6] M. Bardet. *Étude des systèmes algébriques surdéterminés. Applications aux codes correcteurs et à la cryptographie*. PhD thesis, Université de Paris VI, 2004.
- [7] M. Bardet, J.-C. Faugère, and B. Salvy. Complexity study of Gröbner basis computation. Technical report, INRIA, 2002. <http://www.inria.fr/rrrt/rr-5049.html>.
- [8] M. Bardet, J.-C. Faugère, and B. Salvy. On the complexity of Gröbner basis computation of semi-regular overdetermined algebraic equations. In *Proc. International Conference on Polynomial System Solving (ICPSS)*, pages 71–75, 2004.
- [9] M. Bardet, J.-C. Faugère, B. Salvy, and B.-Y. Yang. Asymptotic behaviour of the degree of regularity of semi-regular polynomial systems. In *Proc. of MEGA 2005, Eighth International Symposium on Effective Methods in Algebraic Geometry*, 2005.
- [10] Z. Brakerski and V. Vaikuntanathan. Efficient fully homomorphic encryption from (standard) LWE. In R. Ostrovsky, editor, *IEEE 52nd Annual Symposium on Foundations of Computer Science, FOCS 2011*, pages 97–106. IEEE, 2011.
- [11] B. Buchberger. *Ein Algorithmus zum Auffinden der Basiselemente des Restklassenringes nach einem nulldimensionalen Polynomideal*. PhD thesis, University of Innsbruck, 1965.
- [12] B. Buchberger, G. E. Collins, R. G. K. Loos, and R. Albrecht. Computer algebra symbolic and algebraic computation. *SIGSAM Bull.*, 16(4):5–5, 1982.
- [13] D. A. Cox, J. B. Little, and D. O’Shea. *Ideals, Varieties and Algorithms*. Springer Verlag, 2005.
- [14] J.-C. Faugère. A new efficient algorithm for computing Gröbner bases (F4). *Journal of Pure and Applied Algebra*, 139:61–88, June 1999.
- [15] J.-C. Faugère. A new efficient algorithm for computing Gröbner bases without reduction to zero (F5). In T. Mora, editor, *Proceedings of the 2002 International Symposium on Symbolic and Algebraic Computation ISSAC*, pages 75–83. ACM Press, July 2002. isbn: 1-58113-484-3.
- [16] J.-C. Faugère and A. Joux. Algebraic cryptanalysis of hidden field equation (hfe) cryptosystems using gröbner bases. In D. Boneh, editor, *CRYPTO*, volume 2729 of *Lecture Notes in Computer Science*, pages 44–60. Springer Verlag, 2003.
- [17] J.-C. Faugère and L. Perret. Polynomial equivalence problems: Algorithmic and theoretical aspects. In S. Vaudenay, editor, *Advances in Cryptology – EUROCRYPT 2006*, volume 4004 of *Lecture Notes in Computer Science*, pages 30–47. Springer Verlag, 2006.
- [18] J.-C. Faugère, F. L. dit Vehel, and L. Perret. Cryptanalysis of minrank. In Wagner [28], pages 280–296.
- [19] J.-C. Faugère, A. Otmani, L. Perret, and J.-P. Tillich. In *Advances in Cryptology – EUROCRYPT 2010*, volume 6110 of *Lecture Notes in Computer Science*, pages 279–298. Springer Verlag, 2010.
- [20] J.-C. Faugère, L. Perret, C. Petit, and G. Renault. Improving the complexity of index calculus algorithms in elliptic curves over binary fields. In D. Pointcheval and T. Johansson, editors, *Advances in Cryptology - EUROCRYPT 2012*, volume 7237 of *Lecture Notes in Computer Science*, pages 27–44. Springer Verlag, 2012.

- [21] C. Gentry. *A fully homomorphic encryption scheme*. PhD thesis, Stanford University, 2009. Available at <http://crypto.stanford.edu/craig>.
- [22] D. Lazard. Gröbner-bases, Gaussian elimination and resolution of systems of algebraic equations. In *Proceedings of the European Computer Algebra Conference on Computer Algebra*, volume 162 of *Lecture Notes in Computer Science*, Berlin, Heidelberg, New York, 1983. Springer Verlag.
- [23] F. S. Macaulay. On some formula in elimination. *London Mathematical Society*, 1(33):3–27, 1902.
- [24] C. Peikert. Public-key cryptosystems from the worst-case shortest vector problem: extended abstract. In M. Mitzenmacher, editor, *Proceedings of the 41st Annual ACM Symposium on Theory of Computing, STOC 2009*, pages 333–342. ACM, 2009.
- [25] C. Peikert, V. Vaikuntanathan, and B. Waters. A framework for efficient and composable oblivious transfer. In Wagner [28], pages 554–571.
- [26] O. Regev. On lattices, learning with errors, random linear codes, and cryptography. In H. N. Gabow and R. Fagin, editors, *STOC*, pages 84–93. ACM, 2005.
- [27] O. Regev. On lattices, learning with errors, random linear codes, and cryptography. *J. ACM*, 56(6), 2009.
- [28] D. Wagner, editor. *Advances in Cryptology - CRYPTO 2008, 28th Annual International Cryptology Conference, Santa Barbara, CA, USA, August 17-21, 2008. Proceedings*, volume 5157 of *Lecture Notes in Computer Science*, 2008. Springer Verlag.

M. R. Albrecht INRIA, LIP6, CNRS
malb@lip6.fr

C. Cid Royal Holloway, University of London
carlos.cid@rhul.ac.uk

J.-C. Faugère INRIA, LIP6, CNRS
jean-charles.faugere@inria.fr

R. Fitzpatrick Royal Holloway, University of London
robert.fitzpatrick.2010@live.rhul.ac.uk

L. Perret INRIA, LIP6, CNRS
ludovic.perret@lip6.fr