Appendix to the paper entitled: Soft ELECTRE TRI outranking method based on belief functions

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Abstract

This short note provides the detailed results of the steps of the ELECTRE TRI (ET) and the Soft ELECTRE TRI (SET) methods presented in [1] that has been applied in the example on a waste disposal selection in an environmental context scenario. ELECTRE TRI (ET) and the Soft ELECTRE TRI (SET) methods have already been published elsewhere: in this note, we only provide a helpful support to engineers or researchers for verifying and testing the implementation of these methods according to their needs.

Index Terms

ELECTRE TRI, information fusion, belief functions, outranking methods, multicriteria analysis, operational research.

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I. Recall of the example under analysis

The example under analysis in [1] to test SET method had been introduced by Maystre in [2] and concerns the choice of the location of an urban waste resource recovery disposal which aims to re-use the recyclable part of urban waste produced by several communities. Indeed, this disposal must collect at least $20000m^3$ of urban waste per year to be economically viable. It must be a collective unit and the best possible location has to be identified. Each community has to bring its urban waste production to the disposal: the transport costs are valuated in tons by kilometer per year ($t.km/year$). Building such a disposal is generally not easily accepted by population, particularly when the environmental inconveniences are already high. This initial environmental status is measured by a specific criterion. Building an urban waste disposal implies to use a wide area that could be used for other activities such as a sport terrain, touristic equipments, a natural zone, etc. This competition with other activities is measured by a specific criterion (g_5).

A. Alternatives, criteria and profiles definition

In this example, seven possible locations (alternatives/choices) a_i, $i = 1, 2, \ldots, 7$, for urban waste resource recovery disposal are compared according to the following five criteria $g_j(.)$, ($j = 1, 2, \ldots, 5$):

$$g_1 = \text{Terrain price (decreasing preference)};$$
$$g_2 = \text{Transport costs (decreasing preference)};$$
$$g_3 = \text{Environment status (increasing preference)};$$
$$g_4 = \text{Impacted population (increasing preference)};$$
$$g_5 = \text{Competition activities (increasing preference)}.$$

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- Price of terrain (g_1) is expressed in $€/m^2$ with decreasing preferences (the lower is the price, the higher is the preference);
- Transport costs (g_2) are expressed in $t.km/year$ with decreasing preferences (the lower is the cost, the higher is the preference);
- The environment status (g_3) corresponds to the initial environmental inconvenience level expressed by population with an increasing direction of preferences. The higher is the environment status, the lower are the initial environmental inconveniences. It is rated with an integer between 0 and 10 (highest environment status corresponding to the lowest initial environmental inconveniences);
- Impacted population (g_4) is an integrated criterion to measure negative effects based on subjective and qualitative criteria. It corresponds to the status of the environment with an increasing direction of preferences. The higher is the evaluation, the lower are the negative effects. It is rated with a real number between 0 (great number of impacted people) and 10 (very few people impacted);
- Activities competition (g_5) is an integrated criterion, evaluated by a real number, that measures the competition level between activities with an increasing direction of preferences. The higher is the evaluation, the lower is the competition with other activities on the planned location (tourism, sport, natural environment ...).

The evaluations of the seven alternatives are summarized in Table I.

Criteria $g_j \rightarrow$ **Choices** $a_i \downarrow$	g_1 ($€/m^2$)	g_2 ($t \cdot km$/year)	g_3 $\{0,1,\ldots,10\}$	g_4 $[0,10]$	g_5 $\{0,1,\ldots,100\}$
a_1	-120	-284	5	3.5	18
a_2	-150	-269	2	4.5	24
a_3	-100	-413	4	5.5	17
a_4	-60	-596	6	8.0	20
a_5	-30	-1321	8	7.5	16
a_6	-80	-734	5	4.0	21
a_7	-45	-982	7	8.5	13

Table I: Inputs of ET (7 alternatives according to 5 criteria).

The alternatives (possible locations for an urban waste resource recovery disposal) are compared to two decision profiles b_1 and b_2 as described in Table II. At the end, the goal is to decide to which category (bounded by the decision profiles), each alternative belongs. The weights, indifference, preference and

Profiles $b_h \rightarrow$ **Criteria** $g_j \downarrow$	b_1	b_2
g_1: $€/m^2$	-100	-50
g_2: $t \cdot km$/year	-1000	-500
g_3: $\{0,1,\ldots,10\}$	4	7
g_4: $[0,10]$	4	7
g_5: $\{0,1,\ldots,100\}$	15	20

Table II: Evaluation profiles.

veto thresholds for criteria g_j are given in Table III.

Thresholds $\rightarrow$ **Criteria** $g_j \downarrow$	w_j (weight)	q_j (indifference)	p_j (preference)	v_j (veto)
g_1: €$/m^2$	0.25	15	40	100
g_2: $t \cdot km/$year	0.45	80	350	850
g_3: $\{0, 1, \ldots, 10\}$	0.10	1	3	5
g_4: $[0, 10]$	0.12	0.5	3.5	4.5
g_5: $\{0, 1, \ldots, 100\}$	0.08	1	5	8

Table III: Thresholds.

APPENDIX 1: RESULTS OF ET STEPS

• Results of ET-Step 1 (Partial indexes):

The partial concordance indexes $c_j(a_i, b_1)$ and $c_j(b_1, a_i)$ for profile b_1 are listed in Table IV and the partial concordance indexes $c_j(a_i, b_2)$ and $c_j(b_2, a_i)$ for profile b_2 are listed in Table V.

$c_j(a_i, b_1)$ – *Alternative versus Profile.*					
Criteria $g_j \rightarrow$ **Choices** $a_i \downarrow$	g_1	g_2	g_3	g_4	g_5
a_1	0.8000	1.0000	1.0000	1.0000	1.0000
a_2	0.0000	1.0000	0.5000	1.0000	1.0000
a_3	1.0000	1.0000	1.0000	1.0000	1.0000
a_4	1.0000	1.0000	1.0000	1.0000	1.0000
a_5	1.0000	0.1074	1.0000	1.0000	1.0000
a_6	1.0000	1.0000	1.0000	1.0000	1.0000
a_7	1.0000	1.0000	1.0000	1.0000	0.7500

$c_j(b_1, a_i)$ – *Profile versus Alternative.*					
Criteria $g_j \rightarrow$ **Choices** $a_i \downarrow$	g_1	g_2	g_3	g_4	g_5
a_1	1.0000	0.0000	1.0000	1.0000	0.5000
a_2	1.0000	0.0000	1.0000	1.0000	0.0000
a_3	1.0000	0.0000	1.0000	0.6667	0.7500
a_4	0.0000	0.0000	0.5000	0.0000	0.0000
a_5	0.0000	1.0000	0.0000	0.0000	1.0000
a_6	0.8000	0.3111	1.0000	1.0000	0.0000
a_7	0.0000	1.0000	0.0000	0.0000	1.0000

Table IV: (ET Step 1) Partial concordance indexes for profile b_1.

$c_j(a_i, b_2)$ – *Alternative versus Profile.*					
Criteria $g_j \rightarrow$ **Choices** $a_i \downarrow$	g_1	g_2	g_3	g_4	g_5
a_1	0.0000	1.0000	0.5000	0.0000	0.7500
a_2	0.0000	1.0000	0.0000	0.3333	1.0000
a_3	0.0000	1.0000	0.0000	0.6667	0.5000
a_4	1.0000	0.9407	1.0000	1.0000	1.0000
a_5	1.0000	0.0000	1.0000	1.0000	0.2500
a_6	0.4000	0.4296	0.5000	0.1667	1.0000
a_7	1.0000	0.0000	1.0000	1.0000	0.0000

$c_j(b_2, a_i)$ – *Profile versus Alternative.*					
Criteria $g_j \rightarrow$ **Choices** $a_i \downarrow$	g_1	g_2	g_3	g_4	g_5
a_1	1.0000	0.4963	1.0000	1.0000	1.0000
a_2	1.0000	0.4407	1.0000	1.0000	0.2500
a_3	1.0000	0.9741	1.0000	1.0000	1.0000
a_4	1.0000	1.0000	1.0000	0.8333	1.0000
a_5	0.8000	1.0000	1.0000	1.0000	1.0000
a_6	1.0000	1.0000	1.0000	1.0000	1.0000
a_7	1.0000	1.0000	1.0000	0.6667	1.0000

Table V: (ET Step 1) Partial concordance indexes for profile b_2.

The partial discordance indexes $d_j(a_i, b_1)$ and $d_j(b_1, a_i)$ for profile b_1 are listed in Table VI and the partial discordance indexes $d_j(a_i, b_2)$ and $d_j(b_2, a_i)$ for profile b_2 are listed in Table VII.

$d_j(a_i, b_1)$ – *Alternative versus Profile.*					
Criteria $g_j \rightarrow$ **Choices** $a_i \downarrow$	g_1	g_2	g_3	g_4	g_5
a_1	0.0000	0.0000	0.0000	0.0000	0.0000
a_2	0.1667	0.0000	0.0000	0.0000	0.0000
a_3	0.0000	0.0000	0.0000	0.0000	0.0000
a_4	0.0000	0.0000	0.0000	0.0000	0.0000
a_5	0.0000	0.0000	0.0000	0.0000	0.0000
a_6	0.0000	0.0000	0.0000	0.0000	0.0000
a_7	0.0000	0.0000	0.0000	0.0000	0.0000

$d_j(b_1, a_i)$ – *Profile versus Alternative.*					
Criteria $g_j \rightarrow$ **Choices** $a_i \downarrow$	g_1	g_2	g_3	g_4	g_5
a_1	0.0000	0.7320	0.0000	0.0000	0.0000
a_2	0.0000	0.7620	0.0000	0.0000	1.0000
a_3	0.0000	0.4740	0.0000	0.0000	0.0000
a_4	0.0000	0.1080	0.0000	0.5000	0.0000
a_5	0.5000	0.0000	0.5000	0.0000	0.0000
a_6	0.0000	0.0000	0.0000	0.0000	0.3333
a_7	0.2500	0.0000	0.0000	1.0000	0.0000

Table VI: (ET Step 1) Partial discordance indexes for profile b_1.

$d_j(a_i, b_2)$ – *Alternative versus Profile.*					
Criteria $g_j \rightarrow$ **Choices** $a_i \downarrow$	g_1	g_2	g_3	g_4	g_5
a_1	0.5000	0.0000	0.0000	0.0000	0.0000
a_2	1.0000	0.0000	1.0000	0.0000	0.0000
a_3	0.1667	0.0000	0.0000	0.0000	0.0000
a_4	0.0000	0.0000	0.0000	0.0000	0.0000
a_5	0.0000	0.9420	0.0000	0.0000	0.0000
a_6	0.0000	0.0000	0.0000	0.0000	0.0000
a_7	0.0000	0.2640	0.0000	0.0000	0.6667

$d_j(b_2, a_i)$ – *Profile versus Alternative.*					
Criteria $g_j \rightarrow$ **Choices** $a_i \downarrow$	g_1	g_2	g_3	g_4	g_5
a_1	0.0000	0.0000	0.0000	0.0000	0.0000
a_2	0.0000	0.0000	0.0000	0.0000	0.0000
a_3	0.0000	0.0000	0.0000	0.0000	0.0000
a_4	0.0000	0.0000	0.0000	0.0000	0.0000
a_5	0.0000	0.0000	0.0000	0.0000	0.0000
a_6	0.0000	0.0000	0.0000	0.0000	0.0000
a_7	0.0000	0.0000	0.0000	0.0000	0.0000

Table VII: (ET Step 1) Partial discordance indexes for profile b_2.

• Results of ET-Step 2 (Global concordance and credibility indexes):

The values of the concordance indexes $c(a_i, b_h)$ and $c(b_h, a_i)$ for the profiles b_1 and b_2 are listed in Table VIII and the credibility indexes $\rho(a_i, b_h)$ and $\rho(b_h, a_i)$ for profiles b_1 and b_2 are listed in Table IX.

	$c(a_i, b_1)$	$c(b_1, a_i)$	$c(a_i, b_2)$	$c(b_2, a_i)$
a_1	0.9500	0.5100	0.5600	0.7733
a_2	0.7000	0.4700	0.5700	0.6883
a_3	1.0000	0.4900	0.5700	0.9883
a_4	1.0000	0.0500	0.9733	0.9800
a_5	0.5983	0.5300	0.4900	0.9500
a_6	1.0000	0.5600	0.4433	1.0000
a_7	0.9800	0.5300	0.4700	0.9600

Table VIII: (ET Step 2) Global concordance indexes.

	$\rho(a_i, b_1)$	$\rho(b_1, a_i)$	$\rho(a_i, b_2)$	$\rho(b_2, a_i)$
a_1	0.9500	0.2789	0.5600	0.7733
a_2	0.7000	0.0000	0.0000	0.6883
a_3	1.0000	0.4900	0.5700	0.9883
a_4	1.0000	0.0247	0.9733	0.9800
a_5	0.5983	0.5300	0.0557	0.9500
a_6	1.0000	0.5600	0.4433	1.0000
a_7	0.9800	0.0000	0.2956	0.9600

Table IX: (ET Step 2) credibility indexes.

• Results of ET-Step 3 (Fuzzy and crisp outranking):

Using a $\lambda = 0.75$ for the λ-cut strategy, one gets the outranking relations listed in Table X.

	b_0	b_1	b_2	b_3
a_1	$>$	$>$	$<$	$<$
a_2	$>$	R	R	$<$
a_3	$>$	$>$	$<$	$<$
a_4	$>$	$>$	I	$<$
a_5	$>$	R	$<$	$<$
a_6	$>$	$>$	$<$	$<$
a_7	$>$	$>$	$<$	$<$

Table X: (ET Step 3) Outranking relations obtained with ET ($\lambda = 0.75$).

• Results of ET-Step 4 (hard assignment):

The final hard (binary) assignments obtained by ET method using the pessimistic and optimistic decisional attitudes are listed in Table XI.

	C_1	C_2	C_3
a_1	0	1	0
a_2	1	0	0
a_3	0	1	0
a_4	0	1	0
a_5	1	0	0
a_6	0	1	0
a_7	0	1	0

(a) Pessimistic decisions.

	C_1	C_2	C_3
a_1	0	1	0
a_2	0	0	1
a_3	0	1	0
a_4	0	0	1
a_5	0	1	0
a_6	0	1	0
a_7	0	1	0

(b) Optimistic decisions.

Table XI: (ET Step 4) Hard assignments obtained with ET ($\lambda = 0.75$).

APPENDIX 2: RESULTS OF SET STEPS

The results obtained in SET Steps for both SET-B (Bayesian SET version) based on the weighted averaging fusion rule, and in SET-NB (non-Bayesian SET version) based on Proportional Conflict Redistribution fusion rule #5 (PCR5) proposed in [3] are detailed in this second Appendix.

• Results of SET-Step 1 (Partial indexes):

The partial concordances obtained for the sigmoidal model detailed in [1] and proposed in Step 1 of the SET method are listed in Tables XII–XIII.

$c_j(a_i, b_1)$ – *Alternative versus Profile.*					
Criteria $g_j \rightarrow$ **Choices** $a_i \downarrow$	g_1	g_2	g_3	g_4	g_5
a_1	0.7674	1.0000	0.9975	0.8808	0.9975
a_2	0.0217	1.0000	0.4997	0.9656	1.0000
a_3	0.9878	1.0000	0.9820	0.9907	0.9933
a_4	1.0000	0.9999	0.9997	0.9997	0.9997
a_5	1.0000	0.1662	1.0000	0.9993	0.9820
a_6	0.9995	0.9992	0.9975	0.9350	0.9999
a_7	1.0000	0.9692	1.0000	0.9998	0.7311

$c_j(b_1, a_i)$ – *Profile versus Alternative.*					
Criteria $g_j \rightarrow$ **Choices** $a_i \downarrow$	g_1	g_2	g_3	g_4	g_5
a_1	0.9995	0.0002	0.8808	0.9656	0.4999
a_2	1.0000	0.0001	0.9997	0.8808	0.0001
a_3	0.9878	0.0021	0.9820	0.6608	0.7311
a_4	0.1121	0.0499	0.4997	0.0362	0.1121
a_5	0.0006	0.9996	0.0093	0.1121	0.8808
a_6	0.7674	0.3161	0.8808	0.9350	0.0333
a_7	0.0090	0.9487	0.1121	0.0052	0.9933

Table XII: (SET Step 1) Partial concordance indexes for profile b_1.

$c_j(a_i, b_2)$ – *Alternative versus Profile.*					
Criteria $g_j \rightarrow$ **Choices** $a_i \downarrow$	g_1	g_2	g_3	g_4	g_5
a_1	0.0006	0.9983	0.4997	0.1121	0.7311
a_2	0.0000	0.9986	0.0003	0.3392	0.9991
a_3	0.0217	0.9887	0.1121	0.6608	0.4999
a_4	0.9424	0.8533	0.8808	0.9820	0.9526
a_5	0.9995	0.0000	0.9975	0.9656	0.2679
a_6	0.3977	0.4278	0.4997	0.2083	0.9820
a_7	0.9945	0.0139	0.9820	0.9907	0.0064

$c_j(b_2, a_i)$ – *Profile versus Alternative.*					
Criteria $g_j \rightarrow$ **Choices** $a_i \downarrow$	g_1	g_2	g_3	g_4	g_5
a_1	1.0000	0.4945	0.9997	0.9993	0.9933
a_2	1.0000	0.4388	1.0000	0.9975	0.2679
a_3	1.0000	0.8692	1.0000	0.9907	0.9975
a_4	0.9975	0.9901	0.9975	0.7914	0.9526
a_5	0.7674	1.0000	0.8808	0.8808	0.9991
a_6	0.9999	0.9987	0.9997	0.9987	0.8808
a_7	0.9732	1.0000	0.9820	0.6608	1.0000

Table XIII: (SET Step 1) Partial concordance indexes for profile b_2.

The partial discordances indexes obtained from the sigmoidal model [1] in Step 1 of SET method are listed in Tables XIV–XV.

$d_j(a_i, b_1)$ – *Alternative versus Profile.*					
Criteria $g_j \rightarrow$ **Choices** $a_i \downarrow$	g_1	g_2	g_3	g_4	g_5
a_1	0.0091	0.0000	0.0000	0.0000	0.0000
a_2	0.2080	0.0000	0.0093	0.0000	0.0000
a_3	0.0002	0.0000	0.0000	0.0000	0.0000
a_4	0.0000	0.0000	0.0000	0.0000	0.0000
a_5	0.0000	0.0862	0.0000	0.0000	0.0000
a_6	0.0000	0.0000	0.0000	0.0000	0.0000
a_7	0.0000	0.0003	0.0000	0.0000	0.0007

$d_j(b_1, a_i)$ – *Profile versus Alternative.*					
Criteria $g_j \rightarrow$ **Choices** $a_i \downarrow$	g_1	g_2	g_3	g_4	g_5
a_1	0.0000	0.7167	0.0003	0.0000	0.0047
a_2	0.0000	0.7404	0.0000	0.0000	0.9655
a_3	0.0002	0.4740	0.0000	0.0000	0.0007
a_4	0.1121	0.1700	0.0093	0.4963	0.1121
a_5	0.5000	0.0000	0.4997	0.1121	0.0001
a_6	0.0091	0.0475	0.0003	0.0000	0.3373
a_7	0.2688	0.0006	0.1121	0.8797	0.0000

Table XIV: (SET Step 1) Partial discordance indexes for profile b_1.

$d_j(a_i, b_2)$ – *Alternative versus Profile.*					
Criteria $g_j \rightarrow$ **Choices** $a_i \downarrow$	g_1	g_2	g_3	g_4	g_5
a_1	0.5000	0.0000	0.0093	0.1121	0.0007
a_2	0.8808	0.0000	0.8808	0.0016	0.0000
a_3	0.2080	0.0001	0.1121	0.0000	0.0047
a_4	0.0013	0.0028	0.0003	0.0000	0.0000
a_5	0.0000	0.8542	0.0000	0.0000	0.0262
a_6	0.0425	0.0313	0.0093	0.0145	0.0000
a_7	0.0001	0.2798	0.0000	0.0000	0.6604

$d_j(b_2, a_i)$ – *Profile versus Alternative.*					
Criteria $g_j \rightarrow$ **Choices** $a_i \downarrow$	g_1	g_2	g_3	g_4	g_5
a_1	0.0000	0.0241	0.0000	0.0000	0.0000
a_2	0.0000	0.0300	0.0000	0.0000	0.0262
a_3	0.0000	0.0024	0.0000	0.0000	0.0000
a_4	0.0000	0.0001	0.0000	0.0000	0.0000
a_5	0.0091	0.0000	0.0003	0.0000	0.0000
a_6	0.0000	0.0000	0.0000	0.0000	0.0001
a_7	0.0005	0.0000	0.0000	0.0000	0.0000

Table XV: (SET Step 1) Partial discordance indexes for profile b_2.

The partial uncertainties indexes obtained from the sigmoidal model [1] in Step 1 of SET method are listed in Tables XVI–XVII.

$u_j(a_i, b_1)$ – *Alternative versus Profile.*					
Criteria $g_j \rightarrow$ **Choices** $a_i \downarrow$	g_1	g_2	g_3	g_4	g_5
a_1	0.2235	0.0000	0.0025	0.1192	0.0025
a_2	0.7703	0.0000	0.4910	0.0344	0.0000
a_3	0.0120	0.0000	0.0180	0.0093	0.0067
a_4	0.0000	0.0001	0.0003	0.0003	0.0003
a_5	0.0000	0.7476	0.0000	0.0007	0.0180
a_6	0.0005	0.0008	0.0025	0.0650	0.0001
a_7	0.0000	0.0305	0.0000	0.0002	0.2683

$u_j(b_1, a_i)$ – *Profile versus Alternative.*					
Criteria $g_j \rightarrow$ **Choices** $a_i \downarrow$	g_1	g_2	g_3	g_4	g_5
a_1	0.0005	0.2832	0.1189	0.0344	0.4953
a_2	0.0000	0.2595	0.0003	0.1192	0.0344
a_3	0.0120	0.5239	0.0180	0.3392	0.2683
a_4	0.7758	0.7801	0.4910	0.4675	0.7758
a_5	0.4994	0.0004	0.4910	0.7758	0.1191
a_6	0.2235	0.6364	0.1189	0.0650	0.6294
a_7	0.7222	0.0508	0.7758	0.1151	0.0067

Table XVI: (SET Step 1) Partial uncertainty indexes for profile b_1.

$u_j(a_i, b_2)$ – *Alternative versus Profile.*					
Criteria $g_j \rightarrow$ **Choices** $a_i \downarrow$	g_1	g_2	g_3	g_4	g_5
a_1	0.4994	0.0017	0.4910	0.7758	0.2683
a_2	0.1192	0.0013	0.1189	0.6591	0.0009
a_3	0.7703	0.0112	0.7758	0.3392	0.4953
a_4	0.0563	0.1439	0.1189	0.0180	0.0474
a_5	0.0005	0.1458	0.0025	0.0344	0.7059
a_6	0.5598	0.5410	0.4910	0.7772	0.0180
a_7	0.0054	0.7064	0.0180	0.0093	0.3331

$u_j(b_2, a_i)$ – *Profile versus Alternative.*					
Criteria $g_j \rightarrow$ **Choices** $a_i \downarrow$	g_1	g_2	g_3	g_4	g_5
a_1	0.0000	0.4814	0.0003	0.0007	0.0067
a_2	0.0000	0.5312	0.0000	0.0025	0.7059
a_3	0.0000	0.1284	0.0000	0.0093	0.0025
a_4	0.0025	0.0098	0.0025	0.2086	0.0474
a_5	0.2235	0.0000	0.1189	0.1192	0.0009
a_6	0.0001	0.0013	0.0003	0.0013	0.1191
a_7	0.0263	0.0000	0.0180	0.3392	0.0000

Table XVII: (SET Step 1) Partial uncertainty indexes for profile b_2.

• Results of SET-Step 2 (Global indexes):

The global concordance indexes of SET method (Bayesian and Non-Bayesian versions) are listed in Table XVIII.

	$c(a_i, b_1)$	$c(b_1, a_i)$	$c(a_i, b_2)$	$c(b_2, a_i)$
a_1	0.9271	0.2615	0.5713	0.7719
a_2	0.7013	0.0000	0.0000	0.6886
a_3	0.9935	0.4839	0.5808	0.9398
a_4	0.9998	0.0642	0.9017	0.9658
a_5	0.6233	0.5348	0.0550	0.9155
a_6	0.9915	0.5371	0.4454	0.9897
a_7	0.9646	0.0000	0.2985	0.9508

(a) SET-B Step 2.

	$c(a_i, b_1)$	$c(b_1, a_i)$	$c(a_i, b_2)$	$c(b_2, a_i)$
a_1	0.9729	0.1771	0.7610	0.6845
a_2	0.7716	0.0000	0.0000	0.6269
a_3	0.9999	0.3750	0.7542	0.9730
a_4	1.0000	0.0189	0.9628	0.9941
a_5	0.4376	0.7582	0.0325	0.9719
a_6	0.9994	0.4693	0.4031	0.9991
a_7	0.9942	0.0000	0.2325	0.9854

(b) SET-NB Step 2..

Table XVIII: (SET Step 2) Global concordance indexes $c(a_i, b_h)$ and $c(b_h, a_i)$.

The global discordance indexes of SET method (Bayesian and Non-Bayesian versions) are listed in Table XIX.

	$d(a_i,b_1)$	$d(b_1,a_i)$	$d(a_i,b_2)$	$d(b_2,a_i)$
a_1	0.0000	0.0000	0.0000	0.0000
a_2	0.0000	0.0000	0.0000	0.0000
a_3	0.0000	0.0000	0.0000	0.0000
a_4	0.0000	0.1053	0.0000	0.0000
a_5	0.0000	0.0000	0.0000	0.0000
a_6	0.0000	0.0000	0.0000	0.0000
a_7	0.0000	0.0000	0.0000	0.0000

(a) SET-B Step 2.

	$d(a_i,b_1)$	$d(b_1,a_i)$	$d(a_i,b_2)$	$d(b_2,a_i)$
a_1	0.0000	0.0000	0.0000	0.0000
a_2	0.0000	0.0000	0.0000	0.0000
a_3	0.0000	0.0000	0.0000	0.0000
a_4	0.0000	0.0496	0.0000	0.0000
a_5	0.0000	0.0000	0.0000	0.0000
a_6	0.0000	0.0000	0.0000	0.0000
a_7	0.0000	0.0000	0.0000	0.0000

(b) SET-NB Step 2..

Table XIX: (SET Step 2) Global discordance indexes $d(a_i,b_h)$ and $d(b_h,a_i)$.

The global uncertainty indexes of SET method (Bayesian and Non-Bayesian versions) are listed in Table XX.

	$u(a_i,b_1)$	$u(b_1,a_i)$	$u(a_i,b_2)$	$u(b_2,a_i)$
a_1	0.0729	0.7385	0.4287	0.2281
a_2	0.2987	1.0000	1.0000	0.3114
a_3	0.0065	0.5161	0.4192	0.0602
a_4	0.0002	0.8305	0.0983	0.0342
a_5	0.3767	0.4652	0.9450	0.0845
a_6	0.0085	0.4629	0.5546	0.0103
a_7	0.0354	1.0000	0.7015	0.0492

(a) SET-B Step 2.

	$u(a_i,b_1)$	$u(b_1,a_i)$	$u(a_i,b_2)$	$u(b_2,a_i)$
a_1	0.0271	0.8229	0.2390	0.3155
a_2	0.2284	1.0000	1.0000	0.3731
a_3	0.0001	0.6250	0.2458	0.0270
a_4	0.0000	0.9316	0.0372	0.0059
a_5	0.5624	0.2418	0.9675	0.0281
a_6	0.0006	0.5307	0.5969	0.0009
a_7	0.0058	1.0000	0.7675	0.0146

(b) SET-NB Step 2..

Table XX: (SET Step 2) Global uncertainty indexes $u(a_i,b_h)$ and $u(b_h,a_i)$.

• Results of SET-Step 3 (Probabilized outranking):

Profiles $b_h \rightarrow$ **Outranking probas** $\downarrow$	b_0	b_1	b_2	b_3
P_{1h}	1	0.9506	0.2661	0
P_{2h}	1	0.8506	0.1557	0
P_{3h}	1	0.9937	0.0718	0
P_{4h}	1	1.0000	0.1737	0
P_{5h}	1	0.5950	0.0447	0
P_{6h}	1	0.9908	0.0093	0
P_{7h}	1	0.9823	0.0351	0

(a) SET-B Step 3.

Profiles $b_h \rightarrow$ **Outranking probas** $\downarrow$	b_0	b_1	b_2	b_3
P_{1h}	1	0.9835	0.6212	0
P_{2h}	1	0.8858	0.1866	0
P_{3h}	1	0.9999	0.0550	0
P_{4h}	1	1.0000	0.0795	0
P_{5h}	1	0.2149	0.0145	0
P_{6h}	1	0.9996	0.0007	0
P_{7h}	1	0.9971	0.0095	0

(b) SET-NB Step 3.

Table XXI: (SET Step 3) Probabilities of soft outranking relations given by SET.

Remark 1: Due to space limitation constraint for the conference paper typesettings, the Table XXI-(a) was not included in [1]. Table XXI-(b) corresponds to Table X in [1] with slightly numerical changes because in [1] the values had been computed by a numerical sampling technique which is impossible to reproduce for the readers since it depends on choice of the number of samples used, the random seed used in the numerical sampling and the software. In this note, we give the exact values (rounded at their fourth digits) based on a simple geometrical analysis.

• Results of SET-Step 4 (Soft assignment):

The final soft (probabilistic) assignments obtained by SET method are listed in Table XXII.

	C_1	C_2	C_3	$\emptyset$
a_1	0.0362	$\mathbf{0.6977}$	0.2529	$\delta_1 = 0.0131$
a_2	0.1261	$\mathbf{0.7182}$	0.1324	$\delta_2 = 0.0233$
a_3	0.0059	$\mathbf{0.9224}$	0.0713	$\delta_3 = 0.0005$
a_4	0.0000	$\mathbf{0.8263}$	0.1737	$\delta_4 = 0.0000$
a_5	0.3869	$\mathbf{0.5685}$	0.0266	$\delta_5 = 0.0181$
a_6	0.0091	$\mathbf{0.9815}$	0.0092	$\delta_6 = 0.0001$
a_7	0.0171	$\mathbf{0.9478}$	0.0345	$\delta_7 = 0.0006$

(a) SET-B Step 4.

	C_1	C_2	C_3	$\emptyset$
a_1	0.0062	0.3725	$\mathbf{0.6110}$	$\delta_1 = 0.0102$
a_2	0.0929	$\mathbf{0.7205}$	0.1653	$\delta_2 = 0.0213$
a_3	0.0001	$\mathbf{0.9449}$	0.0550	$\delta_3 = 0.0000$
a_4	0.0000	$\mathbf{0.9205}$	0.0795	$\delta_4 = 0.0000$
a_5	$\mathbf{0.7736}$	0.2118	0.0031	$\delta_5 = 0.0114$
a_6	0.0005	$\mathbf{0.9987}$	0.0007	$\delta_6 = 0.0000$
a_7	0.0029	$\mathbf{0.9876}$	0.0095	$\delta_7 = 0.0000$

(b) SET-NB Step 4.

Table XXII: (SET Step 4) Soft Assignment matrix $[P(a_i \rightarrow C_h)]$ given by SET-B and SET-NB.

Remark 2: For the same reasons as the ones explained in Remark 1, Table XXII-(a) was not included in [1], and Table XXII-(b) corresponds to Table XI in [1] with slightly numerical changes.

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