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► **To cite this version:**

Lina Ye, Philippe Dague. Diagnosability analysis for self-observed distributed discrete event systems. VALID - 4th International Conference on Advances in System Testing and Validation Lifecycle, Nov 2012, Lisbonne, Portugal. 2012. <hal-00790135>

**HAL Id: hal-00790135**

**<https://hal.inria.fr/hal-00790135>**

Submitted on 19 Feb 2013

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# Diagnosability Analysis for Self-observed Distributed Discrete Event Systems

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**Abstract**—Diagnosability is a crucial property that determines at design stage how accurate any diagnosis algorithm can be on a partially observable system and thus has a significant impact on the performance and reliability of complex systems. Most existing approaches assumed that observable events in the system are globally observed. But sometimes it is not possible to obtain global information. Thus a recent work has proposed a new framework to check diagnosability in a system where each component can only observe its own observable events to keep the internal structure private in terms of observations. However, the authors implicitly assume that local paths in components can be exhaustively enumerated, which is not true in a general case where there are embedded cycles. In this paper, we get some new results about diagnosability in such a system, i.e., what we call joint diagnosability in a self-observed distributed system. First we prove the undecidability of joint diagnosability with unobservable communication events by reducing Post’s Correspondence Problem to an observation problem. Then we propose an algorithm to check a sufficient but not necessary condition of joint diagnosability. Finally we briefly discuss about the decidable case with observable communication events.

## I. INTRODUCTION

Over the latest decades, with more performance requirements imposed on the complex systems, they are subject to more errors. However, it is unrealistic to detect faults manually for such systems. Automated diagnosis mechanisms are thus required for large distributed applications. Generally speaking, diagnosis reasoning aims at detecting possible faults explaining the observations. The efficiency of diagnosis reasoning depends on system diagnosability, which is a crucial property that determines at design stage how accurate any diagnosis algorithm can be on a partially observable system. The systems we discuss here are Discrete Event Systems (DES).

Some existing works analyzed diagnosability in a centralized way ([1], [2], etc.), i.e., a monolithic model of a given system is hypothesized, which is unrealistic due to combinatorial explosion of state space. This is why very recently distributed approaches began to be investigated ([3], [4], etc.), relying on local objects. However, all these approaches assumed that observable events in the system are globally observed. But sometimes it is not possible to obtain global information. Then [5] has proposed a new framework to check diagnosability in a system where each component can only observe its own observable events to keep the internal structure private in terms of observations. However, the authors implicitly assume that local paths can be exhaustively enumerated, which is not true in a general case where there are embedded cycles. In this

paper, we generalize this work to get some new results about the diagnosability of what we call self-observed distributed systems, where observable events can only be observed by their own component.

We make several contributions in this paper. First, we extend diagnosability of globally observed systems to what we call joint diagnosability of self-observed systems and then to prove its undecidability with unobservable communication events. Secondly, we propose an algorithm for testing a sufficient condition, where we obtain pairs of local trajectories in the faulty component, such that for each pair only one trajectory contains the fault but both have the same observations, and then check their global consistency through two phases. We provide the proof that it is a sufficient condition and point out why it is not necessary. Thirdly, the decidable case where communication events are observable is discussed.

## II. PRELIMINARIES

In this section, we model self-observed distributed DES and then recall joint diagnosability features [5].

We consider a self-observed distributed DES composed of a set of components  $\{G_1, G_2, \dots, G_n\}$  that communicate by communication events, where each component can only observe its own observable events. Such a system is modeled by a set of finite state machines (FSM), each one representing the local model of one component. The local model of a component  $G_i$  is a FSM, denoted by  $G_i = (Q_i, \Sigma_i, \delta_i, q_i^0)$ , where  $Q_i$  is the set of states;  $\Sigma_i$  is the set of events;  $\delta_i \subseteq Q_i \times \Sigma_i \times Q_i$  is the set of transitions; and  $q_i^0$  is the initial state. The set of events  $\Sigma_i$  is partitioned into four subsets:  $\Sigma_{i_o}$ , the set of locally observable events that can be observed only by their own component  $G_i$ ;  $\Sigma_{i_n}$ , the set of unobservable normal events;  $\Sigma_{i_f}$ , the set of unobservable fault events; and  $\Sigma_{i_c}$ , the set of communication events shared by at least one other component, which are the only shared events between components. Figure 1 depicts a self-observed distributed system with two components:  $G_1$  (left) and  $G_2$  (right), where the events  $O_i$  denote locally observable events, the event  $F$  denotes an unobservable fault event, the events  $U_i$  denote unobservable normal events and the events  $C_i$  denote communication events.

We denote the synchronized FSM of components  $G_1, \dots, G_n$  by  $\|(G_1, \dots, G_n)$ , where the synchronized events are the shared events between components and any one of them always occurs simultaneously in all components that define it. The global model of the entire system is implicitly defined as

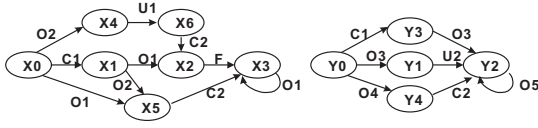


Fig. 1. A system with two components  $G_1$  (left) and  $G_2$  (right).

the synchronized FSM of all components based on their shared events, i.e., communication events. However, the global model will not be calculated since in a self-observed distributed system, the global occurrence order of observable events is not accessible. In the following, we call subsystem of  $G$  the synchronization of a subset of components of  $G$ , i.e.,  $\|(G_{s_1}, \dots, G_{s_m})$ , where  $\{s_1, \dots, s_m\} \subseteq \{1, \dots, n\}$ . One component or the entire system can be considered as a subsystem.

Given a system model  $G = (Q, \Sigma, \delta, q^0)$ , the set of words produced by the FSM  $G$  is a prefix-closed language  $L(G)$  that describes the normal and faulty behaviors of the system. Formally,  $L(G) = \{s \in \Sigma^* \mid \exists q \in Q, (q^0, s, q) \in \delta\}$ , where the transition  $\delta$  has been extended from events to words. In the following, we call a word of  $L(G)$  also a **trajectory** in the system  $G$  and a sequence  $q_0\sigma_0q_1\sigma_1\dots$  a **path** in  $G$ , where  $\sigma_0\sigma_1\dots$  is a trajectory and for all  $i$ , we have  $(q_i, \sigma_i, q_{i+1}) \in \delta$ . Given  $s \in L(G)$ , we denote the post-language of  $L(G)$  after  $s$  by  $L(G)/s$  and denote the projection of  $s$  to observable events of  $G$  (resp.  $G_i$ ) by  $P(s)$  (resp.  $P_i(s)$ ). We adopt the assumption in [3], i.e., the projection of the global language on each local model is observable live, i.e., there is no unobservable cycle in any component. For the sake of simplicity, our approach is shown for only one fault, which can be extended to the case with multiple faults by running one time for each fault. Next we rephrase reconstructibility introduced in [7].

**Definition 1:** (Reconstructibility). Given a system  $G$  that is composed of several subsystems, i.e.,  $G = \|(G_{s_1}, \dots, G_{s_m})$ , a set of trajectories in these subsystems is said to be reconstructible with respect to  $G$  if it is obtained by projection on this set of subsystems of a trajectory in  $G$ .

If there is no common communication event between two subsystems, then any trajectory in one subsystem and any one in the other subsystem are reconstructible.

For the sake of consistency, now we rename what is called cooperative diagnosability in [5] as joint diagnosability. We denote a trajectory ending with the fault  $f$  by  $s^f$ .

**Definition 2:** (Joint diagnosability). A fault  $f$  is jointly diagnosable in a self-observed distributed system  $G$  with components  $\{G_1, \dots, G_n\}$ , iff

$$\begin{aligned} \exists k \in \mathbb{N}, \forall s^f \in L(G), \forall t \in L(G)/s^f, (\forall i \in \\ \{1, \dots, n\}, |P_i(t)| \geq k) \Rightarrow (\forall p \in L(G) \\ (\forall i \in \{1, \dots, n\}, P_i(p) = P_i(s^f.t)) \Rightarrow f \in p). \end{aligned}$$

Joint diagnosability of  $f$  means that for each faulty trajectory  $s^f$  in  $G$ , for each extension  $t$  with enough locally observable events in all components, every trajectory  $p$  in  $G$  that is equivalent to  $s^f.t$  for local observations in each component should contain in it  $f$ . In other words, the fault can be detected after finite non bounded trajectory prolongation in at least one component. In a self-observed system, we call a pair of trajectories  $p$  and  $p'$  satisfying the three conditions a (global)

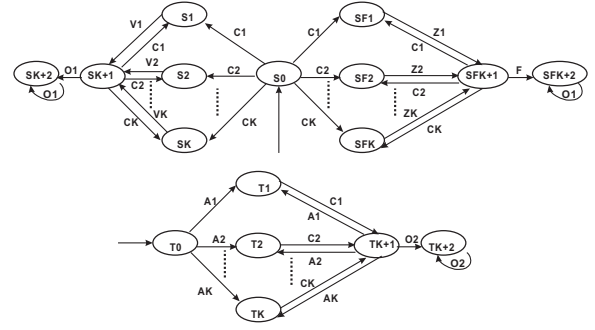


Fig. 2. A system with two components  $G_1$  (top) and  $G_2$  (bottom) for proving undecidability of joint diagnosability.

**indeterminate pair:** 1)  $p$  contains  $f$  and  $p'$  does not; 2)  $p$  has arbitrarily long local observations in all components after the occurrence of  $f$ ; 3)  $\forall i \in \{1, \dots, n\}, P_i(p) = P_i(p')$ . Here arbitrarily long local observations can be considered as infinite local observations. Now we have the following theorem [5].

**Theorem 1:** Given a self-observed distributed system  $G$ , a fault  $f$  is jointly diagnosable in  $G$  iff there is no (global) indeterminate pair in  $G$ .

### III. UNDECIDABLE CASE

To discuss about joint diagnosability, we consider two cases: communication events being unobservable and observable. We first consider the general case, i.e., communication events being unobservable.

Theorem 1 implies that checking joint diagnosability boils down to check the existence of indeterminate pairs that witnesses non joint diagnosability. Inspired from [6], where undecidability of joint observability is proved by reducing the Post's Correspondence Problem (PCP) to an observation problem, we discuss first about whether joint diagnosability is decidable or not.

For the sake of simplicity, we give now a simplified proof for undecidability of joint diagnosability <sup>1</sup>.

**Theorem 2:** Given a self-observed distributed system where communication events are unobservable, checking joint diagnosability of a given fault is undecidable.

*Proof:* :

1)PCP: given a finite alphabet  $\Sigma$ , two sets of words  $v_1, v_2, \dots, v_k$  and  $z_1, z_2, \dots, z_k$  over  $\Sigma$ , then a solution to PCP is a sequence of indices  $(i_m)_{1 \leq m \leq n}$  with  $n \geq 1$  and  $1 \leq i_m \leq k$  for all  $m$  such that  $v_{i_1}v_{i_2}\dots v_{i_n} = z_{i_1}z_{i_2}\dots z_{i_n}$ . 2)Now consider the example depicted in figure 2, where the system is composed of two components  $G_1$  and  $G_2$ . In  $G_1$ , each one of  $V_i, i \in \{1, \dots, k\}$ , and each one of  $Z_i, i \in \{1, \dots, k\}$ , denotes a sequence of observable events all different from  $O_1$ ,  $C_1, \dots, C_k$  are unobservable communication events,  $F$  denotes a fault event and  $O_1$  is an observable event. In  $G_2$ , each one of  $A_i, i \in \{1, \dots, k\}$ , denotes an observable event different from  $O_2$ ,  $C_1, \dots, C_k$  are unobservable communication events and  $O_2$  is an observable event. Then the observations in  $G_1$  can be described as  $V_{i_1}V_{i_2}\dots V_{i_n}O_1^*$  without fault or

<sup>1</sup>Thanks to Gaul Gastin for proof discussion

$Zi_1Zi_2\dots Zi_nO1^*$  with fault, where  $\forall i_j, j \in \{1, \dots, n\}, i_j \in \{1, \dots, k\}$ . In  $G_2$ , the observations are  $Ai_1Ai_2\dots Ai_nO2^*$ . In this system, the occurrence of the fault can be confirmed by the observation of  $O1$ .

3) Without the observation of  $O1$ , the local observations are  $wO1^+$  for  $G_1$  and  $Ai_1Ai_2\dots Ai_nO2^*$  for  $G_2$ , where  $w = Vi_1Vi_2\dots Vi_n$  when there is no fault or  $w = Zi_1Zi_2\dots Zi_n$  when there is a fault. Clearly, if PCP has a solution, i.e.,  $\exists (i_m)_{1 \leq m \leq n}$  such that  $Vi_1Vi_2\dots Vi_n = Zi_1Zi_2\dots Zi_n$ , we have two trajectories  $p$  and  $p'$  such that the observations of  $p$  in  $G_1$  are  $Vi_1Vi_2\dots Vi_nO1^+$ , which is a trajectory without fault, while the observations of  $p'$  in  $G_1$  are  $Zi_1Zi_2\dots Zi_nO1^+$ , which is a trajectory with a fault. And both  $p$  and  $p'$  have the same observations for  $G_2$ , i.e.,  $Ai_1Ai_2\dots Ai_nO2^*$ . Thus we get that  $p$  and  $p'$  have the same observations for both  $G_1$  and  $G_2$ , i.e.,  $Vi_1Vi_2\dots Vi_nO1^+ = Zi_1Zi_2\dots Zi_nO1^+$  for  $G_1$  and  $Ai_1Ai_2\dots Ai_nO2^*$  for  $G_2$ , then the fault is not jointly diagnosable.

4) On the other hand, if the fault is not jointly diagnosable, then we obtain at least one indeterminate pair, denoted by  $p$  and  $p'$  such that the projection of  $p$  on  $G_1$  is  $Ci_1Vi_1Ci_2Vi_2\dots Ci_nVi_nO1^*$ , on  $G_2$  is  $Ai_1Ci_1Ai_2Ci_2\dots Ai_nCi_nO2^*$  and that of  $p'$  on  $G_1$  is  $Cj_1Zj_1Cj_2Zj_2\dots Cj_mZj_mFO1^*$  and on  $G_2$  is  $Aj_1Cj_1Aj_2Cj_2\dots Aj_mCj_mO2^*$ . From the fact that  $p$  and  $p'$  have the same observations for  $G_2$ , we get  $Ai_1Ai_2\dots Ai_nO2^* = Aj_1Aj_2\dots Aj_mO2^*$  and thus we have  $m = n$  and  $i_1 = j_1, \dots, i_n = j_n$ . And then from the same observations of  $p$  and  $p'$  on  $G_1$ , we get  $Vi_1Vi_2\dots Vi_nO1^* = Zi_1Zi_2\dots Zi_nO1^*$ , i.e.,  $Vi_1Vi_2\dots Vi_n = Zi_1Zi_2\dots Zi_n$ , which means that there is a solution for PCP.

5) The above proves that the existence of a solution for PCP is equivalent to that of a fault being not jointly diagnosable. Since PCP is an undecidable problem, then checking joint diagnosability is undecidable. ■

There are two major differences between joint diagnosability in our framework and joint observability in [6]. One is that the former assumes that local observers are attached to local components that are synchronized by common communication events to get a global model while the latter separates arbitrarily the observable events in the global model into several sets. The other one is that joint diagnosability consists in separating infinite trajectories while joint observability consists in separating finite ones. Thus, if any communication event is assumed to be unobservable, joint diagnosability checking boils down to infinite PCP. But this one has also been proved to be undecidable [8], which gives the result.

#### IV. SUFFICIENT ALGORITHM

We have proved that joint diagnosability with unobservable communication events is undecidable. We can nevertheless propose an algorithm to test a sufficient condition, which is still quite useful in some circumstances. We first construct the local diagnoser from a given local model to show fault information for any local trajectory. Then we show how to build a structure called local twin plant to obtain original information

about indeterminate pairs (also called local indeterminate pairs in the following), based on the local diagnoser. The next step is to check the global consistency, i.e., to check whether the local indeterminate pairs can be extended into (global) indeterminate pairs, whose existence testifies non joint diagnosability. Actually our algorithm remains trivially applicable when the assumption of unobservability of communication events is partially relaxed, i.e., in the most general case where some communication events are observable and others unobservable.

##### A. Original diagnosability information

To check the existence of indeterminate pairs, in the distributed framework, we use the structure called local twin plant defined in [2]. In particular, the considered fault is assumed to only occur in one component, denoted by  $G_f$ . Then the local twin plant for  $G_f$  contains original information of indeterminate pairs: actually this twin plant is a FSM that compares every pair of local trajectories to search for the pairs with the same arbitrarily long local observations, but exactly one of the two containing a fault, which are local indeterminate pairs. First, we define delay closure operation with respect to a subset  $\Sigma_d$  of  $\Sigma$  to preserve only the information about the events in  $\Sigma_d$ .

*Definition 3: (Delay Closure).* Given a FSM  $G = (Q, \Sigma, \delta, q^0)$ , its delay closure with respect to  $\Sigma_d \subseteq \Sigma$  is  $\mathcal{C}_{\Sigma_d}(G) = (Q, \Sigma_d, \delta_d, q^0)$  where  $(q, \sigma, q') \in \delta_d$  iff  $\exists s \in (\Sigma \setminus \Sigma_d)^*, (q, s\sigma, q') \in \delta$ .

We now describe how to construct the local diagnoser of a given component, based on which we build the local twin plant. Given a local model, we get a modified one by attaching fault label, denoted by  $l \in \{N, F\}$ , where  $N$  for normal and  $F$  for fault, to each state. In other words, before the occurrence of the fault, each state is labeled with label  $N$  and, after its occurrence, with label  $F$ .

*Definition 4: (Local diagnoser).* Given a local model  $G_i$ , its local diagnoser  $D_i$  is obtained by operating the delay closure with respect to the set of communication events and observable events on the modified model:  $D_i = \mathcal{C}_{\Sigma_{i_o} \cup \Sigma_{i_c}}(G_i^m)$ , where  $G_i^m$  is the modified version of  $G_i$ .

Based on the local diagnoser, the corresponding local twin plant is obtained by synchronizing the local diagnoser with itself based on the locally observable events, allowing one to obtain all pairs of local trajectories with the same observations to search for local indeterminate pairs. To simplify this synchronization, the two identical local diagnosers, denoted by  $D_i^l$  (left instance) and  $D_i^r$  (right instance), can be reduced as follows:  $D_i^l$  is obtained by retaining only paths with at least one fault cycle and  $D_i^r$  is obtained by retaining only paths with at least one non-fault cycle. This reduction keeps all original diagnosability information since what we are interested in are only local indeterminate pairs. However, this reduction is only applicable for the local diagnoser of the faulty component  $G_f$ ; for other components, the local twin plant is obtained by synchronizing the non reduced left instance and the non reduced right instance since there is no fault information. Since this synchronization is based on observable events  $\Sigma_{i_o}$ , the

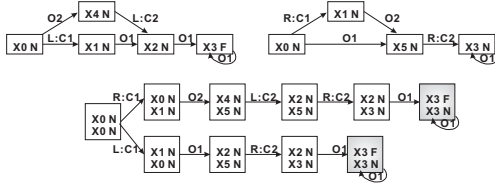


Fig. 3. Two reduced instances of the diagnoser for  $G_1$  (top) and part of the corresponding local twin plant (bottom).

non-synchronized events are distinguished by the prefix  $L$  or  $R$ : in  $D_i^l$  ( $D_i^r$ ), each communication event  $c \in \Sigma_{ic}$  from  $D_i$  is renamed by  $L : c$  ( $R : c$ ). The names of all locally observable events are left unchanged.

**Definition 5:** (Local twin plant). Given a local diagnoser  $D_i$  for the component  $G_i$ , the corresponding local twin plant is a FSM, denoted by  $T_i = D_i^l || D_i^r$ , where the synchronized events are locally observable events in  $G_i$ .

Each state of a local twin plant is a pair of local diagnoser states providing two possible diagnoses with the same local observations. Given a twin plant state  $((q^l, l^l)(q^r, l^r))$ , if the considered fault  $f \in l^l \cup l^r$  but  $f \notin l^l \cap l^r$ , which means that the occurrence of  $f$  is not certain up to this state, then this state is called an ambiguous state with respect to the fault  $f$ . An ambiguous state cycle is a cycle containing only ambiguous states. In a local twin plant, if a path contains an ambiguous state cycle with at least one locally observable event, then it is called a **local indeterminate path**, which corresponds to a local indeterminate pair. Note that local indeterminate paths contain original diagnosability information and can be obtained only in the local twin plant of the component  $G_f$ . If a local indeterminate pair can be extended into a global indeterminate pair, then we say that its corresponding local indeterminate path is globally consistent. Figure 3 shows the left and right instances of the local diagnoser for the faulty component  $G_1$  of Figure 1 (top) as well as a part of the corresponding local twin plant (bottom). Clearly, in the local twin plant, we have local indeterminate paths since they have ambiguous state cycles with observable events.

### B. Global consistency checking

Joint diagnosability verification consists in checking the existence of globally consistent local indeterminate paths, whose existence proves non joint diagnosability. To do this, we have to check the global consistency of the corresponding left trajectories of the local indeterminate paths in the local twin plants as well as that of their corresponding right trajectories, shortly called left consistency checking and right consistency checking.

**Definition 6:** (Left (Right) consistent plant). Given a subsystem  $G_S$  composed of components  $G_{i_1}, \dots, G_{i_m}$  and their corresponding local twin plants  $T_{i_1}, \dots, T_{i_m}$ , to obtain a left (right) consistent plant with respect to the subsystem  $G_S$ , denoted by  $T_f^l$  ( $T_f^r$ ), we perform the following two steps:

1) Distinguish right (left) communication events between local twin plants by renaming them with the prefix of component ID. For example,  $R:C2$  ( $L:C2$ ) in the local twin plant of  $G_2$  is renamed as  $G_2:R:C2$  ( $G_2:L:C2$ ).

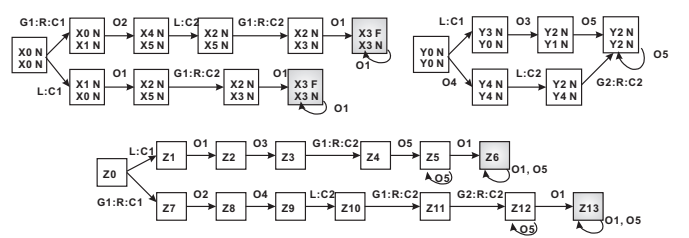


Fig. 4. Part of the renamed local twin plants for  $G_1$  and  $G_2$  (top) and part of the left consistent plant  $T_f^l$  (bottom).

2) Synchronize the renamed local twin plants with the synchronized events being the common left (right) communication events, which works because observable events do not intersect between components and non-synchronized right (left) communication events are distinguished by the prefix of component ID.

From definition 1, we know that in the left (right) consistent plant with respect to a subsystem  $G_S$ , each path  $p$  corresponds to a set of paths  $p_{i_1}, \dots, p_{i_m}$  in the local twin plants of all components in  $G_S$  such that the set of left (right) trajectories of  $p_{i_1}, \dots, p_{i_m}$  are reconstructible with respect to  $G_S$ . For our example, the bottom part of figure 4 shows a part of the left consistent plant  $T_f^l$ , which is obtained by synchronizing the renamed local twin plant of  $G_1$  and that of  $G_2$  (top part of figure 4) based on the common left communication events.

### C. Algorithm

Algorithm 1 presents the procedure to verify a sufficient condition of joint diagnosability. As shown in the pseudo-code, algorithm 1 performs as follows. Given the input as the set of component models, the fault  $F$  that may occur in the component  $G_f$ , we initialize the parameters as empty, i.e.,  $G_S^l$  ( $G_S^r$ ), the subsystem for the left (right) consistency checking. The procedure of the algorithm can be separated into two parts: left consistency checking (line 3-12) and right consistency checking (line 13-24).

Left consistency checking begins with the local twin plant construction of  $G_f$ , the subsystem  $G_S^l$  being now  $G_f$  (line 3-4). When both the left consistent plant  $T_f^l$  with respect to the current left subsystem  $G_S^l$  and  $DirectCC(G, G_S^l)$  are not empty (line 5), where  $DirectCC(G, G_S^l)$  is the set of directly connected components to  $G_S^l$  (a directly connected component being one sharing at least one common communication event with the subsystem), the algorithm repeatedly performs the following steps to further check left consistency.

1) Select one directly connected component  $G_i$  to the subsystem  $G_S^l$  and construct its local twin plant  $T_i$  (line 6-7).  
 2) Synchronize  $T_f^l$  with  $T_i$  to obtain left consistent plant for this extended subsystem based on common left communication events (line 8). To do this, non-synchronized right communication events are distinguished by the prefix of component ID.  
 3) Update the subsystem  $G_S^l$  by adding  $G_i$  and reduce the newly obtained  $T_f^l$  by retaining only paths with ambiguous state cycles containing observable events for all components in  $G_S^l$  (line 9-10).

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**Algorithm 1** Sufficient algorithm

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```
1: INPUT:
   the system model  $G = (G_1, \dots, G_n)$ ;
   the fault  $F$  and the faulty component  $G_f$ 
2: Initializations:
    $G_S^l \leftarrow \emptyset$  (subsystem for left consistency checking);
    $G_S^r \leftarrow \emptyset$  (subsystem for right consistency checking)
3:  $T_f^l \leftarrow \text{ConstructLTP}(G_f)$ 
4:  $G_S^l \leftarrow G_f$ 
5: while  $T_f^l \neq \emptyset$  and  $\text{DirectCC}(G, G_S^l) \neq \emptyset$  do
6:    $G_i \leftarrow \text{SelectDirectCC}(G, G_S^l)$ 
7:    $T_i \leftarrow \text{ConstructLTP}(G_i)$ 
8:    $T_f^l \leftarrow T_f^l \parallel T_i$ 
9:    $G_S^l \leftarrow \text{Add}(G_S^l, G_i)$ 
10:   $T_f^l \leftarrow \text{RetainConsisPaths}(T_f^l)$ 
11: if  $T_f^l = \emptyset$  then
12:   return " $F$  is jointly diagnosable in  $G$ "
13: else
14:   $T_f^r \leftarrow \text{AbstractRight}(G_f, T_f^l)$ 
15:   $G_S^r \leftarrow G_f$ 
16:  while  $T_f^r \neq \emptyset$  and  $G_S^l \neq G_S^r$  do
17:     $G_i \leftarrow \text{SelectDirectCC}(G_S^l, G_S^r)$ 
18:     $T_f^r \leftarrow T_f^r \parallel \text{AbstractRight}(G_i, T_f^l)$ 
19:     $G_S^r \leftarrow \text{Add}(G_S^r, G_i)$ 
20:     $T_f^r \leftarrow \text{RetainConsisPaths}(T_f^r)$ 
21:  if  $T_f^r = \emptyset$  then
22:   return " $F$  is jointly diagnosable in  $G$ "
23:  else
24:   return "Joint diagnosability cannot be determined"
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If the left consistent plant  $T_f^l$  is empty, then there is no local indeterminate path that corresponds to a set of paths in the local twin plants of all components in the subsystem such that their left trajectories are reconstructible (definition 1), which implies the non existence of a globally consistent local indeterminate path. In this case joint diagnosability information is returned (line 11-12). Otherwise, if  $T_f^l$  is not empty (line 13), then we proceed to check right consistency of the corresponding paths in  $T_f^l$  that have been already verified to be left consistent in the whole system.

Right consistency checking begins with the function  $\text{AbstractRight}(G_f, T_f^l)$  (line 14), which performs delay closure with respect to right communication events and observable events of  $G_f$ . Then the subsystem  $G_S^r$  is assigned as  $G_f$  (line 15). When the right consistent plant  $T_f^r$  for the current right subsystem  $G_S^r$  is not empty and  $G_S^l \neq G_S^r$  (line 16), we repeatedly perform the following steps to check right consistency in an extended subsystem (since left consistency checking does explore all connected components, for right consistency checking we only consider the subsystem  $G_S^l$  instead of the whole system).

- 1) Select a directly connected component  $G_i$  to  $G_S^r$  from  $G_S^l$  (line 17).
- 2) Perform the function  $\text{AbstractRight}(G_i, T_f^l)$ , which has

been described as above, and then synchronize with  $T_f^r$  based on the common right communication events (line 18). To do this, we rename the right communication events by removing the prefix of component ID, e.g.,  $G_i:R:C2$  renamed as  $R:C2$ . 3) Update the subsystem  $G_S^r$  by adding  $G_i$  and reduce the newly obtained  $T_f^r$  by retaining only paths with ambiguous state cycles containing observable events for all components in  $G_S^r$  (line 19-20).

If the right consistent plant  $T_f^r$  is empty, then there is no local indeterminate path that corresponds to a set of paths in the local twin plants such that their left trajectories and right trajectories are reconstructible respectively, i.e., there is no globally consistent local indeterminate path. In this case, the algorithm returns joint diagnosability information (line 21-22). Otherwise, if  $T_f^r$  is not empty, we cannot determine whether the fault is jointly diagnosable or not. Then the algorithm returns indetermination information (line 23-24). In other words, empty left consistent plant  $T_f^l$  or empty right consistent plant  $T_f^r$  is a sufficient condition but not a necessary condition of joint diagnosability.

*Theorem 3:* In algorithm 1, if the left consistent plant  $T_f^l$  or the right consistent plant  $T_f^r$  is empty, then the fault is jointly diagnosable, but the reverse is not true.

*Proof:* :

( $\Rightarrow$ ) Suppose that  $T_f^l$  or  $T_f^r$  is empty and that the fault is not jointly diagnosable. From non joint diagnosability, it follows that there exists at least one globally consistent local indeterminate path. Since global consistency of a local indeterminate path implies both left consistency and right consistency, from algorithm 1 we know that, after left and right consistency checking, this local indeterminate path must correspond to a path both in  $T_f^l$  and in  $T_f^r$ . Thus neither  $T_f^l$  nor  $T_f^r$  is empty, which contradicts the assumption.

( $\Leftarrow$ ) Now we explain why non emptiness of both  $T_f^l$  and  $T_f^r$  does not necessarily imply that the fault is not jointly diagnosable. Suppose that  $T_f^l$  is not empty and that it contains two paths, denoted by  $\rho_1$  and  $\rho_2$ , corresponding to two local indeterminate paths.  $\rho_1$  corresponds to a set of paths  $\rho_i^1, 1 \leq i \leq n$  in the local twin plants of all components and  $\rho_2$  corresponds to a set of paths  $\rho_i^2, 1 \leq i \leq n$  in all local twin plants. Now suppose that the right trajectories of the set of paths  $\rho_i^1, 1 \leq i \leq n$  are not reconstructible and the same for that of the set of paths  $\rho_i^2, 1 \leq i \leq n$ . It follows that the two local indeterminate paths cannot be extended into global indeterminate pairs and thus are not globally consistent. Then we further suppose that the right trajectories of the set of paths  $\rho_1^1, \dots, \rho_{n-1}^1, \rho_n^2$  are reconstructible or the same for the set of paths  $\rho_1^2, \dots, \rho_{n-1}^2, \rho_n^1$ . In this case, from algorithm 1, it follows that finally the right consistent plant  $T_f^r$  is not empty. Now both  $T_f^l$  and  $T_f^r$  are not empty but there is no globally consistent local indeterminate paths, i.e., the fault is jointly diagnosable. ■

Now illustrate on our example the fact that the condition is not necessary. The top part of figure 5 shows the results of performing delay closure with respect to right

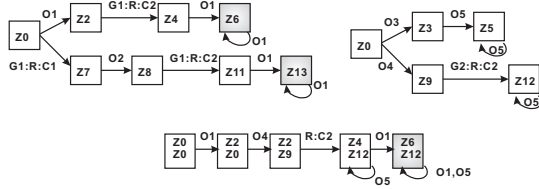


Fig. 5. FSM after delay closure on the left consistent plant (figure 4) for  $G_1$  and  $G_2$  (top) and part of the right consistent plant (bottom).

communication events and observable events both for  $G_1$  and  $G_2$  on the left consistent plant depicted in figure 4. Then, to check right consistency, we rename again the right communication events by removing the component ID such that they can be synchronized. The bottom part of figure 5 shows a part of the right consistent plant, which is not empty. Now both left and right consistent plants are not empty, but this does not imply the existence of global indeterminate pairs that witnesses non joint diagnosability. Actually the part of the left consistent plant depicted here corresponds to two local indeterminate pairs in  $G_1$  with their corresponding left consistent pairs in  $G_2$ , i.e., one local indeterminate pair is  $((C1.O1.F.O1^*), (O1.C2.O1^*))$  in  $G_1$  with its left consistent pair  $((C1.O3.O5^*), (O3.U2.O5^*))$  in  $G_2$  and the other local indeterminate pair is  $((O2.U1.C2.F.O1^*), (C1.O2.C2.O1^*))$  in  $G_1$  with its left consistent pair  $((O4.C2.O5^*), (O4.C2.O5^*))$  in  $G_2$ . While the right consistent plant shown here corresponds to one local indeterminate pair in  $G_1$ , which is  $((C1.O1.F.O1^*), (O1.C2.O1^*))$ , with its right consistent pair in  $G_2$ , i.e.,  $((O4.C2.O5^*), (O4.C2.O5^*))$ . Thus we can see that the same local indeterminate pair does not correspond to the same consistent pair in  $G_2$  in the left consistent plant and in the right consistent plant, which means that this local indeterminate pair cannot be extended into a global indeterminate pair. Our algorithm gives indeterminate information for joint diagnosable systems that satisfy the following condition: for any set of paths including one path in the local twin plant of each non faulty component and one local indeterminate path in that for faulty component, if they are left consistent and right consistent respectively, then their corresponding local trajectories in the components cannot constitute an indeterminate pair through synchronization. Our illustrated example is quite tricky to show the possibility of indeterminate decision given by our algorithm for a joint diagnosable system. However, in reality, a system satisfying the above condition is quite rare and thus this algorithm is suitable to be applicable for a large number of complex systems.

## V. DECIDABLE CASE

We have proved the undecidability of joint diagnosability with unobservable communication events. If we assume their observability, then this problem becomes decidable. Because when any communication event is observable, in the local twin plant, we obtain all pairs of local trajectories with the same observations, including the same observable communication events. Thus each path in the local twin plant corresponds to

a pair of local trajectories with the same sequence of communication events. It follows that during global consistency checking, the separate checkings for left and right consistency becomes now only one checking. While in algorithm 1, the checking into two separate phases is the reason why it gives only a sufficient but not necessary condition. Actually, the observability of communication events makes joint diagnosability equivalent to classical diagnosability since only one checking for global consistency implies the same global occurrence order of observations for global indeterminate pairs.

## VI. CONCLUSION

In this paper, we consider self-observed distributed systems where observable events can only be observed by their own component and thus the distributed and private (w.r.t. observation) nature of real systems is taken into account. Then we prove the undecidability of joint diagnosability checking when communication events are unobservable, before proposing an algorithm to test a sufficient condition. We start from local indeterminate paths and then we check both in sequence left consistency and right consistency. Due to the observation-privacy, the global occurrence order of observable events between different components is not known, which is taken into account through constructing left and right consistent plants separately. At the opposite, in the approaches for DES with globally observable events, twin plant is constructed by incrementally synchronizing local twin plants via both left and right communication events at the same time, which means that in their case, knowledge of the global occurrence order is required. Then we briefly discuss the decidable case where communication events are observable. There is a gap between these two cases as the unobservable case is undecidable and the observable case is decidable. Next interesting work is to investigate where is the frontier between the two cases, i.e., to study the decidability of joint diagnosability for partial observability of communication events.

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