Comments on actuator fault accommodation

Thibaut Raharijaona, Rachid Outbib, Mustapha Ouladsine, Ghachem Sahbi

Comments on actuator fault accommodation

T. Raharijaona * **R. Outbib** * **M. Ouladsine** * **S. Ghachem** **

* *Laboratoire des Sciences de l'Information et des systèmes (LSIS) UMR CNRS 6168, Université Paul Cézanne Aix-Marseille III, Domaine universitaire de St Jérôme, Ave. Escadrille Normandie Niemen, 13 397 Marseille Cedex France (Tel: +33 4 91 05 60 23; e-mail: (thibaut.raharijaona@lsis.org).*
** *Université de Tunis El Manar (e-mail: sahbi.ghachem@lsis.org)*

Abstract: The present work concerns the problem of progressive accommodation to actuator failure. An optimal nonlinear controller synthesis approach is formulated on the basis of the closed loop stability objective. The authors show the interest of the proposed method even for a local analysis when a linear approximation is used. This work focuses on a solution to ensure stability while accommodating to actuator failure. The approach is illustrated in an academic example.

1. INTRODUCTION

The demand for dynamical systems to become safer, more reliable and respectful of the environment is increasing. A cost-effective way to improve dependability in automated systems is to design and introduce Fault Tolerant Control (FTC). The primary objective of such a control is to maintain acceptable performances with respect to nominal system operation see Blanke, Kinnaert, Lunze and Staroswiecki [2003] considering some graceful performance degradation see Jiang and Zhang [2006].
The main task to be tackled in achieving fault tolerance is the design of a controller with stability guarantee and satisfying dynamic performances also in presence of faults. A fault tolerant controller can be designed as a fixed or varied structure to ensure its fault-tolerant functions. If the structure and/or the parameters of the controller are computed online with respect to fault effects and thanks to the real-time measurements, the strategy is so called an active fault tolerant control see Chen and Patton [1999].
Generally speaking, in order to be realistic, the processes are represented by nonlinear models. However and for a seek of simplicity, linear approximation is usually investigated.
Indeed, in the literature, a conventional strategy to solve a nonlinear reconfigurable control problem consists in designing a linear approximation of the model around operating points. Recent papers such as multiple model in Kanev and Verhaegen [2003], Theillol, Sauter and Ponsart [2003] have been presented. In order to handle nonlinear systems beyond using a linearized approximation, reconfigurable control methods have been proposed using backstepping in Zhang, Polycarpou and Parisini [2001] and nonlinear regulator in Bajpai, Chang and Kwatny [2002].
Moreover, few papers concern the delays associated with computation times see Staroswiecki, Yang and Jiang [2007], Staroswiecki [2004] and Zhang, Parisini and Polycarpou [2004]. The former introduced the concept of progressive accommodation whose the objective is to minimize the effect of the accommodation delay. To this end, the reconfigurable control design method is based on a linear quadratic approach.
The goal of this work is to study the validity of the linear approximation approach when the fault holds. More precisely, this paper proposes an analysis of the accommodation delay and its effects on the closed loop stability. This work considers a linear system as an approximation of a nonlinear one around an equilibrium point. The limitation of the linear approach is emphasized when the actuator fault occurs near the boundary of the validity domain of the linearization. In this case, an appropriate nonlinear approach which is valid on the whole physical domain can be helpful.
The present paper is organized as follows: In section 2, the class of affine nonlinear systems is introduced and a necessary background is provided on the main idea of the actuator fault accommodation and optimal regulation problem. Section 3 presents the analysis of the closed loop system stabilization during the fault occurrence with the use of the domain of attraction and the linear approximation validity domain. In Section 4, simulation studies have been conducted in an example to illustrate the proposed analysis.

2. PRELIMINARIES AND MOTIVATION

In the present work, input-affine nonlinear continuous-time dynamic systems are considered with a state-space representation :

$$\begin{cases} \dot{x} = f(x) + Bu, \\ y = h(x), \end{cases} \tag{1}$$

where $x \in \mathbb{R}^n$ is the vector of state variables, $u \in \mathbb{R}^m$ is the control vector and $y \in \mathbb{R}^l$ is the output vector. f and h are smooth functions with $f(0) = 0$. B is a constant matrix of dimension $(n \times m)$.

The infinite-time horizon nonlinear regulation problem is defined with the following quadratic performance index in u:

$$V(x) = \min_{u(t)} \int_0^{\infty} \left(x^T Q(x) x + u^T R(x) u \right) dt \tag{2}$$

in which $Q(x) \geq 0$ and $R(x) > 0$ for all x. Moreover, it is assumed that Q and R are sufficiently smooth so that the value function $V(x)$ is continuously differentiable.

In this case, the Hamilton-Jacobi equation (HJE) is quadratic in $\frac{\partial V}{\partial x}(x)$ such that:

$$\frac{\partial V}{\partial x}(x) f(x) - \frac{1}{4} \frac{\partial V}{\partial x}(x) B R^{-1} B^T \frac{\partial V^T}{\partial x}(x) + x^T Q x = 0 \tag{3}$$

and the optimal feedback control can be designed from :

$$u_n = -\frac{1}{2} R^{-1}(x) B^T \frac{\partial V}{\partial x}(x) \tag{4}$$

In this paper, we consider as defined in Staroswiecki [2004] in a linear representation, that one (or several) actuator fault(s) occur at time t_f. The system can be described by:

$$\dot{x} = f(x) + B_\theta(u) \tag{5}$$

where :

$$B_\theta(u) = \begin{cases} Bu, & t \in [0, t_f[\\ \beta_f(u, \theta), & t \in [t_f, +\infty[\end{cases} \tag{6}$$

The function $\beta_f(u, \theta)$ and the parameter θ represent the contribution of the faulty actuator.The complex structure of the system (1) introduces difficulties in solving the optimal control problem.

The calculation of an optimal nonlinear state feedback for nonlinear systems requires the development of numerical algorithms Lawton and Beard [1998], Mousavere and Kravaris [2005] because the optimization problem needs a resolution of the Hamilton-Jacobi equation. Otherwise, the control problem makes mandatory an approximation by system with a simpler structure.

Notice that in a local area, the linear system is given by :

$$\dot{x} = Ax + Bu, \tag{7}$$

where $A = \frac{\partial f}{\partial x}|_{x=0}$ is the Jacobian matrix of f at point $x = 0$. Therefore, the optimal regulation problem is characterized by an Algebraic Riccati Equation (ARE).

As mentioned in Staroswiecki [2004], in the FTC problem, one has to consider four time periods in order to analyze the system behavior under actuator fault.

(1) $t \in [0, t_f[$: nominal system and control u_n
(2) $t \in [t_f, t_{fdi}[$: faulty system under the nominal control u_n and FDI algorithm in process for fault detection, isolation and estimation.
(3) $t \in [t_{fdi}, t_{ftc}[$: faulty system under the nominal control u_n and the fault is detected, isolated and estimated.
(4) $t \in [t_{ftc}, +\infty[$: faulty system under the accommodated control u_f.

These four time periods are presented in Fig. 1.

In practical applications, even if the diagnosis is perfect that is not realistic, the system control is inappropriate

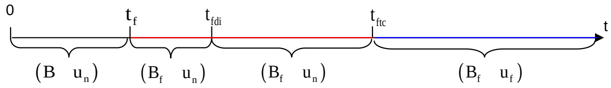

Fig. 1. Description of the fault tolerant control strategy

on the interval $[t_f, t_{ftc}[$ since the faulty system is controlled by u_n. The progressive accommodation presented in Staroswiecki [2003],Staroswiecki [2004], Staroswiecki, Yang and Jiang [2007], aims at minimizing the interval $[t_{fdi}, t_{ftc}[$. Therefore, thanks to an online control computation, the authors propose an improvement of the closed loop behavior of the faulty system in a linear context.

The present paper exposes the limitations of the linear approach and develops an extension of the actuator fault accommodation to a class of affine nonlinear system.

3. ANALYSIS OF THE CLOSED LOOP SYSTEM STABILIZATION DURING THE FAULT OCCURRENCE

In practice, an actuator fault in a controlled system generates changes in inputs/outputs signals and in the parameters of the differential system which describes the dynamics.

The design of a passive fault tolerant controller is sufficient to ensure degraded dynamic performances when the changes in the parameters and signals are small. When the effects of the fault are significant, the global stability of the system may not be ensured, therefore the stabilization of the dynamic system with a fixed controller may be impossible.

In this paper, the authors consider an actuator fault occurrence under the constraint that the faulty actuator can't be switched-off and replaced. This last strategy is usually called system reconfiguration. In this section, the authors focus their attention on the fault accommodation in a nonlinear context. They first refer to a fault tolerant control designed beforehand when failure is identified and secondly to an online accommodation scheme. They are led to evaluate the limitations of the linear approach and they propose a solution in order to tackle the issue for a class of nonlinear systems.

3.1 Analysis - Closed loop stabilization and accommodation to the actuator fault

Number of methods for determining the stability region of nonlinear systems have been proposed in the literature for example Zubov's method see Genesio, Tartaglia and Vicino [1985]. It computes the entire stability region via a Lyapunov function. Regardless an eventual actuator fault occurrence, the solution of the Zubov's partial differential equation is used to estimate the closed-loop stability region.

Let the evolution of the nonlinear system be described by the equation (1). At any given point in time t, assume that it is always possible to integrate the dynamic equation (1) for all admissible input control $u(t)$. An optimal control design is computed thanks to the optimization of the performance index (2). The problem of local output regulation involves the design of a feedback controller which ensures that the closed loop system is locally asymptotically stable

at the origin, and the regulated output $y(t)$ asymptotically decays to 0 as $t \to +\infty$.
In order to accomplish the above task, the problem of nonlinear control may be solved in a local area using a linear approximation of the system.
In this paper, the authors introduce the notion of the validity domain $\mathcal{V}$ of the linear approximation which allows to synthesize an optimal controller by an Algebraic Riccati Equation. This study stands for an extension to the class of nonlinear system (1) of the linear approach proposed in Staroswiecki, Yang and Jiang [2007] and Staroswiecki [2004]. As shown on Figure 2, starting from the initial condition $x_0 \in \mathcal{V}$, the stability of the nonlinear system in closed loop is ensured in the domain of attraction $\mathcal{B}(B, u_n)$ using the linear optimal controller. The system converges to the equilibrium point x_{eq}. D_φ stands for the physical operating domain of the system.

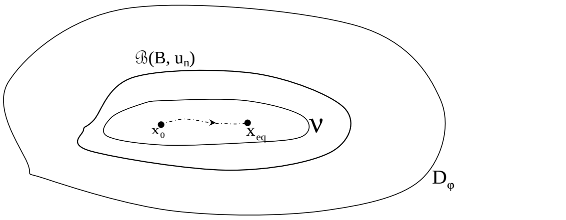

Fig. 2. Description of the validity domain of the linear approximation

When an actuator fault occurs, the nominal model is changed at time t_f and the quadratic performance index (2) is modified. During the time interval $[t_f, t_{ftc}[$, the domain of attraction of the closed loop becomes $\mathcal{B}(B_f, u_n)$.
If the system is tolerant to the fault Staroswiecki et al. in Staroswiecki, Yang and Jiang [2007] proposed in a linear approach, an optimal way to progressively accommodate the fault such that the closed loop system is stable. The algorithm is based on the Newton-Raphson algorithm developped in a linear context in Kleinman [1968]. It is considered here that the diagnostic algorithm is computed with no delay, no error that is not realistic. Consequently, as shown on Figure 1, the diagnostic strategy is characterized by the time delay $t_{ftc} - t_{fdi}$.
Therefore, depending on the nonlinearity of the system, the linear approach to the progressive accommodation may not be able to stabilize the closed loop. The system may leave the validity domain $\mathcal{V}$ of the linear approximation and the domain of attraction $\mathcal{B}(B_f, u_n)$ as shown on Figure 3.

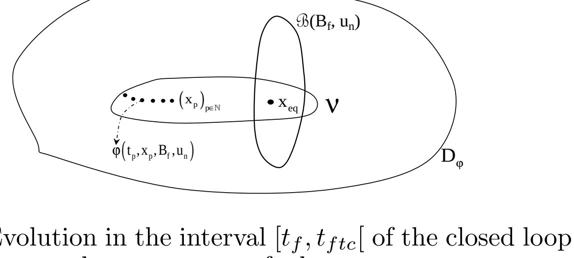

Fig. 3. Evolution in the interval $[t_f, t_{ftc}[$ of the closed loop system under an actuator fault

From Figure 3, x_p stands for the initial condition, t_p is the time delay necessary to cross the boundary of $\mathcal{V}$ and $\varphi(t_p, x_p, B_f, u_n)$ is the solution of the system (1) while an actuator fault occurs.
From now, if the state corresponding to the solution $\varphi(t_p, x_p, B_f, u_n)$ belongs to the domain of attraction $\mathcal{B}(B_f, u_f)$, the control is fault tolerant and the solution converges to the equilibrium point as shown on Figure 4.
Nevertheless, if the state corresponding to the solution $\varphi(t_p, x_p, B_f, u_n)$ doesn't belong to the domain of attraction $\mathcal{B}(B_f, u_f)$, the closed loop system is unstable as presented on Figure 5 and the actuator fault is not accommodated.

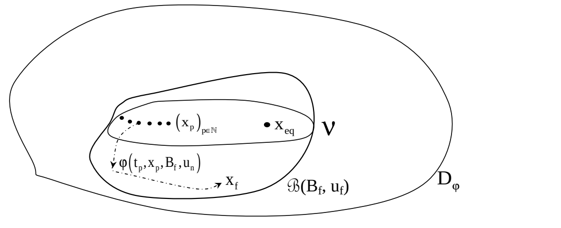

Fig. 4. Evolution in the interval $[t_{ftc}, t[$ of the closed loop system with a fault tolerant control

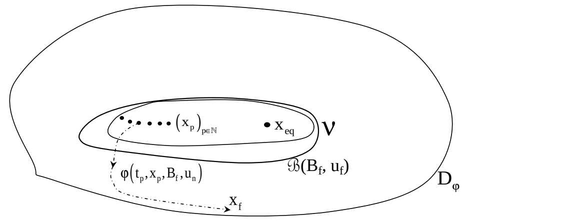

Fig. 5. Evolution in the interval $[t_{ftc}, t[$ of the closed loop system for a non accommodated fault

3.2 Conclusion - Limitations of an online progressive accommodation considering the model validation domain $\mathcal{V}$

As briefly presented in 3.1, for a controlled naturally unstable system, assuming that the fault diagnosis time delay is not equal to zero and depending on the nonlinearity, an actuator fault occurrence that can be accepted with respect the quadratic cost (2), may yield to a critical situation which leads to unstability despite the linear progressive accommodation strategy.
In the following, the authors argue the presented problem, propose an illustration and a solution for the actuator fault accommodation.
Consider the class of nonlinear system defined in (1).
Let $\mathcal{B}(B, u)$ be the domain of attraction of the closed loop system defined from (7) with (B, u). The validity domain $\mathcal{V}$ is included in the domain of attraction $\mathcal{B}(B, u_n)$ where (B, u_n) describes the nominal operating conditions and control. In other words, $\mathcal{V} \subset \mathcal{B}(B, u_n)$. This means that the domain of attraction is at least equal to $\mathcal{V}$ since the validity domain of the linear model is ensured on the whole $\mathcal{V}$.
However, the estimation of the domain of attraction $\mathcal{B}(B_n, u_n)$, when it is possible must be done using the nonlinear model (1) as proposed in Genesio, Tartaglia and Vicino [1985].
In the present paper, the authors consider the case of unstable free dynamics.

In the interval $[t_f, t_{ftc}[$ for the closed loop system defined from (1) with (B_f, u_n), we have $\nu \not\subset \mathcal{B}(B_f, u_n)$ as presented on Figure 3. Otherwise, this means that the actuator fault doesn't affect the performances of the closed loop.
From now, let define the following notations. $\partial\nu$ designates the boundary of ν . $d(x_p, \partial\nu)$ denotes the distance from x_p to $\partial\nu$. t_p is the time delay necessary to cross the boundary $\partial\nu$. $\varphi(t_p, x_p, B, u)$ stands for the solution of the closed loop system (1) defined by the pair (B, u) for t_p with the initial condition x_p.
Consequently, there exists two sequences $(x_p)_{p\in\mathbb{N}} \in \nu$ and $(t_p)_{p\in\mathbb{N}} \in \mathbb{R}^+$ with $t_p < \frac{1}{p}$ such that:

$$d(x_p, \partial\nu) < \tfrac{1}{p} \text{ and } \varphi(t_p, x_p, B_f, u_n) \notin \nu \ .$$

Finally, for any t_{ftc}, there exists an initial condition $x^{ftc} \in \nu$ such that:

$$\varphi(t_{ftc}, x^{ftc}, B_f, u_n) \notin \nu \tag{8}$$

and it is not ensured that:

$$\lim_{n\to+\infty} \varphi(t, \varphi(t_{ftc}, x^{ftc}, B_f, u_n), B_f, \bar{u}) = 0 \tag{9}$$

for any used $\bar{u}$ which is valid in the domain ν in the interval $[t_{ftc}, +\infty[$.

3.3 Proposal - Online progressive accommodation for nonlinear dynamic systems

In the nonlinear case, the infinite-time horizon nonlinear optimal control problem (1), (2), is characterized in terms of Hamilton-Jacobi Equation (3). The complexity of the HJE prevents any solution excepted in some very simple systems. In order to make real-time implementation possible, one has to avoid solving any partial differential equation. With application to online progressive accommodation and in order to design a suboptimal control design, an alternative is to investigate the State Dependent Riccati Equation (SDRE) presented in Huang and Lu [1996].
Based on the LQR formulation, the state-feedback controller is similarly obtained such that:

$$u(x) = -R^{-1}(x)B^T P(x)x \tag{10}$$

where $P(x)$ is the unique, symmetric, positive-definite solution of the algebraic SDRE:

$$P(x)A(x) + A^T(x)P(x) - P(x)BR^{-1}B^T P(x) + Q = 0 \tag{11}$$

4. ILLUSTRATIVE EXAMPLE

Consider an affine nonlinear continuous-time dynamic system modeled by :

$$\begin{cases} \dot{x} = x + x^2 + 2u, \\ y = x, \end{cases} \tag{12}$$

The following problem is first to define an optimal control u_n with respect to a quadratic performance index (2), in nominal conditions given an initial value of the state x_0. Secondly, a fault tolerant control u_f must be synthesized given an acceptable actuator fault.

4.1 Optimal control in nominal conditions

If the problem is local, a linear approximation of the system around the operating point $x = 0$ is computed. The optimal control u_n is synthesized thanks to the LQ problem:
Find the optimal control u_n, such that the cost:

$$J(u, x_0) = \int_0^{+\infty} \left(x^T Q x + u^T R u\right) dt \tag{13}$$

is minimal.
The nominal solution is known to be :
$u_n = -R^{-1}B^T P x = -F_n x$
where $B = 2$, Q and R are symmetric positive definite matrices and P is the solution to the algrebaic Riccati equation (ARE).

$$PA + A^T P + Q - PBR^{-1}B^T P = 0, \tag{14}$$

With the choice of $Q = R = 1$, one finds $u_n = -1.618x$.

4.2 Progressive accommodation to actuator fault

As defined in (6), an actuator fault occurs at $t_f > 0$. The state-space representation of the nonlinear faulty system becomes:

$$\begin{cases} \dot{x} = x + x^2 + B_f u, \\ y = x, \end{cases} \tag{15}$$

where $B_f = 0.8$. Staroswiecki et al. in Staroswiecki, Yang and Jiang [2007] proposed a linear approach to the progressive fault accommodation. Given the local linearization of the faulty system (15) around the nominal operating point $x = 0$, if the loss of efficiency due the fault occurrence can be admitted, the linear accommodation problem has an admissible solution. Consequently, the linear feedback control law $u_i = -F_i x$ (starting at the time t_{fdi}) is applied on the interval $[t_i, t_{i+1}[$. The description of the linear progressive accommodation strategy is given on Figure 6.

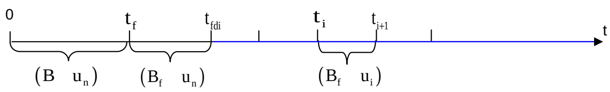

Fig. 6. Description of the progressive accommodation strategy

Based on the linear approximation of the faulty system (A_f, B_f), the feedback control u_i is computed thanks to the Newton-Raphson algorithm presented in Kleinman [1968]. P_i is the unique solution of the Lyapunov equation:

$$\begin{aligned} &P_i(A_f - B_f F_{i-1}) + (A_f - B_f F_{i-1})^T P_i \\ &= -Q - F_{i-1}^T R F_{i-1}, \end{aligned} \tag{16}$$

The initial F_0 is given and for all $i = 1, ..., n$, $F_i = R^{-1}B_f^T P_i$. Moreover, the optimal linear fault tolerant controller is defined by P_f. P_f is the unique positive definite solution of the algebraic Riccati equation :

$$P_f A_f + A_f^T P_f + Q - P_f B_f R^{-1} B_f^T P_f = 0, \tag{17}$$

and $\lim_{i\to+\infty} P_i = P_f$, where P_i is the solution of (16). The optimal control of the faulty system gives $u_f = -2.8508x$.

As an illustration, one can choose the initial condition $x(0) = 0.4$. An actuator fault occurs at the time $t_f = 0.2$ second. According to the definition (6), the system is described by:

$$\dot{x} = \begin{cases} x + x^2 + 2u, & t \in [0, t_f[\\ x + x^2 + 0.8u, & t \in [t_f, +\infty[\end{cases} \quad (18)$$

Let consider the sample computation time $t_e = 0.1$ second and one supposes that the time delay for the fault diagnosis $t_{fdi} - t_f$ is equal to t_e second. Each iteration takes t_e second.
The time delay for the FTC computation $t_{ftc} - t_{fdi}$ is equal to t_e second.

Figure 7 presents an illustration of the linear progressive accommodation to the actuator failure. In the interval $[t_f, t_f + t_e[$, the nonlinear faulty system is driven by the linear optimal nominal feedback control u_n. At the time $t_f + t_e$, the closed-loop is stabilized using the iterative control u_i in dotted line. The first fault tolerant control u_f is applied at the instant $t_{ftc} = t_{fdi} + te$. The corresponding state $x(t)$ is plotted in solid line.

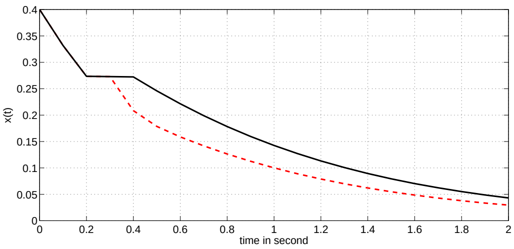

Fig. 7. Illustration of the linear progressive accommodation on the example (12)

As expected, the state $x(t)$ decreases to zero a little bit faster with the progressive accommodation than with the FTC. This result illustrates the schema proposed in Figure 4 to the extent that the closed-loop system is included in the domain of attraction.
Moreover, Table 1 shows the evolution of F_i when linear fault accommodation is applied. The convergence of the Newton-Raphson algorithm on the linear optimal fault tolerant control takes five iterations.

iteration i	0	1	2	3	4	5
F_i	1.6180	4.9154	3.4322	2.9282	2.8526	2.8508

Table 1. Evolution of the iterative state feedback gain F_i such as $u_i = -F_i x$ for $i = 1, ..., n$

By now, for the same actuator fault, there exists an initial condition $x(0)$ such that the nominal closed-loop system stays inside the domain of attraction $\mathcal{B}(B, u_n)$ and the state $x(t)$ doesn't belong to the validity domain $\mathcal{V}$ of the linear approximation. Therefore, from the instant of the fault occurrence, the closed loop system leaves the domain of attraction $\mathcal{B}(B_f, u_n)$ and diverges despite the linear progressive accommodation. Figure 8 illustrates the divergence of the state $x(t)$ with the linear progressive approach for the given initial condition $x(0) = 1.9$ and stands for a representation of the Figure 5. Moreover, one can note that the closed loop system doesn't belong to the domain of attraction $\mathcal{B}(B_f, u_f)$.

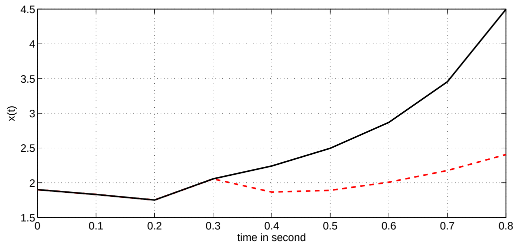

Fig. 8. Illustration of the divergence of the linear progressive accommodation given an initial condition $x(0)$

4.3 Proposed nonlinear approach to the actuator fault accommodation

The point of departure in the present study is an improvement of the linear approach to the progressive accommodation for the class of nonlinear systems defined in (1). As exposed in the paragraph 4.2, the time delay needed to begin the accommodation to the actuator fault is equal to $t_{ftc} - t_f$ for a classical fault tolerant control strategy as resumed in Figure 1 and $t_0 - t_f$ for the linear progressive one whose the description is given in Figure 6 . t_0 stands for the first instant of the correction with respect to the actuator fault. One can remark that $t_0 = t_{ftc}$ for the classical FTC.
Let define t_c, the first instant of correction for the active or passive fault tolerant control. Whatever t_c, there exists an initial condition $x(0)$ such that the nonlinear faulty system in closed-loop leaves the domain of attraction and becomes unstable.
An alternative to the linear approach of the progressive accommodation issue consists in computing a nonlinear optimal control which is abble to accommodate the actuator fault through the minimization of the quadratic performance index (13). Consequently, the optimal control problem in the presence of actuator fault is to find a state feedback control u_{fnl} which minimizes the cost (13) for all possible initial conditions $x(0)$.
To this end, the Hamilton-Jacobi (3) equation must be solved. An analytic solution of such a problem is not accessible in general that's why a numerical approximation is computed in order to produce a suboptimal control. In the paper, because the SDRE approach is much more appealing than solving the HJE in real-time as proposed in Lawton and Beard [1998], Mousavere and Kravaris [2005], a progressive accommodation technique in a nonlinear context is implemented. The SDRE algorithm consists in finding the symmetric positive-definite solution $P(x)$ to the equation (11) and applying at that x, the control (10).
In the example (18), given the initial condition $x(0) = 1.9$, on can find the nonlinear optimal control which accommodate the actuator fault. The optimal state feedback control u_{fnl} is given by a resolution of the following equation which is quadratic in $\frac{\partial V}{\partial x}$:

$$\frac{\partial V}{\partial x}(x)(x + x^2) - \frac{1}{4}\frac{\partial V}{\partial x}(x) B_f B_f^T \frac{\partial V^T}{\partial x}(x) + x^T x = 0 \quad (19)$$

The expression of u_{fnl} is given by :

$$u_{fnl} = -\frac{1}{2} B_f^T \frac{\partial V}{\partial x}(x) \quad (20)$$

Figure 9 (a) presents the optimal control result for the nonlinear actuator fault accommodation in dashed dotted line.

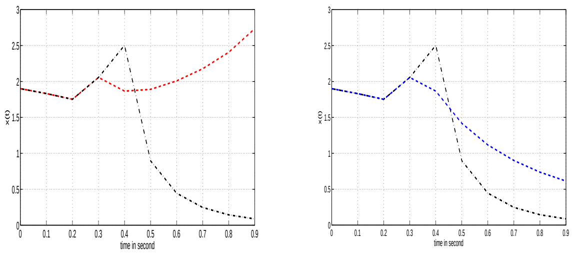

Fig. 9. Illustration of (a) the nonlinear accommodation, (b) the nonlinear progressive accommodation on the example (12)

The plot shows the improvement of the nonlinear approach for the accommodation. In a sense of stability, the used nonlinear control at the instant of correction t_c ensures the decrease of the state $x(t)$ to zero of the damaged system. The closed loop system belongs to the domain of attraction $\mathcal{B}(B_f, u_{fnl})$. Figure 9 (b) exhibits in dashed line the result obtained using the SDRE technique for the online nonlinear progressive accommodation.

Discussion. One can remark from a computational point of view that it is mandatory to find some alternative techniques to get an approximation of the HJE solution. The motivation is the nonlinear optimal controller design which accommodates the actuator fault along the state trajectory $x(t)$. Therefore, based on the linear approach developed in the progressive accommodation, the authors introduce a nonlinear strategy to an online accommodation based on SDRE technique. The state trajectory $x(t)$ of the faulty system is suboptimal compared to the one provided by the analytical solution of the HJE, nevertheless the stability is ensured.

5. CONCLUSION AND PERSPECTIVES

This paper underlines the importance of the analysis of the closed loop system stabilization with the use of the domain of attraction and the linear approximation validity domain in the context of actuator fault accommodation.
This work particularly considers the limitation of the linear approach when the fault occurs next to the boundary of the validity domain of the linearized model. An example aims at illustrating the argued idea which is developed in the article.
Finally, the optimal nonlinear approach for the actuator fault accommodation is proved to be efficient. Moreover, in order to stabilizing the post-fault system, one could be improve the progressive accommodation thanks to an SDRE algorithm taking to account the nonlinearity in the active fault tolerant control synthesis.

REFERENCES

M. Blanke, M. Kinnaert, J. Lunze and M. Staroswiecki. *Diagnosis and fault-tolerant control.* Little, Heildeberg: Springer, Berlin, 2003.

J. Chen, R J. Patton. *Robust model-based fault diagnosis for dynamic systems.* London: Kluwer, 1999.

J. Jiang, Y. Zhang. Accepting performance degradation in fault-tolerant control system design *IEEE Transactions on Control Systems Technology*, Vol 14, Issue 2, pages 284–292,2006.

R. Genesio, M. Tartaglia and A. Vicino. On the estimation of asymptotic stability regions: State of the art and new proposals *IEEE Transactions on Automatic Control*, Vol 30, Issue 8, pages 747–755,1985.

S. Kanev and M. Verhaegen. Controller reconfiguration in the presence of uncertainty in FDI *In Proceedings of the 5th IFAC symposium on fault detection, supervision and safety for technical processes*, pages 145–150, 2003.

D. Theillol, D. Sauter and J.C. Ponsart. a multiple model based approach for fault tolerant control nonlinear systems *In Proceedings of the 5th IFAC symposium on fault detection, supervision and safety for technical processes*, pages 151–156, 2003.

G. Bajpai, B.C. Chang and H.G. Kwatny. Design of fault-tolerant systems for actuator failures in nonlinear systems *In Proceedings of the 2002 American Control Conference*, pages 3618–3623, 2002.

M. Staroswiecki, H. Yang and B. Jiang. Progressive accommodation of parametric faults in linear quadratic control *Automatica*, Vol 43, Issue 12, pages 2070–2076,2007.

D.L Kleinman. On an Iterative Technique for Riccati Equation Computation *IEEE Transactions on Automatic Control*, Vol 13, Issue 1, pages 114–115,1968.

X. Zhang, T. Parisini and M.M. Polycarpou. Adaptative fault-tolerant control of nonlinear uncertain systems: An information-based diagnostic approach *IEEE Transactions on Automatic Control*, Vol 49, Issue 8, pages 1259–1274,2004.

M. Staroswiecki. Progressive accommodation of actuator faults in the linear quadratic control problem *In Proceedings of the 44th IEEE Conference on Decision and Control*, pages 5234–5241,Paradise Island, The Bahamas, 2004.

X. Zhang, M.M. Polycarpou and T. Parisini. Integrated design of fault diagnosis and accommodation schemes for a class of nonlinear systems *In Proceedings of the 40th IEEE Conference on Decision and Control*, pages 1448–1453, 2001.

Y. Huang and W.M Lu. Nonlinear optimal control: alternatives to Hamilton-Jacobi equation *In Proceedings of the 35th IEEE Conference on Decision and Control*, pages 3942–3947, Kobe, Japan, 1996.

M. Staroswiecki. Actuator faults and the linear quadratic control problem *In Proceedings of the 42th IEEE Conference on Decision and Control*, pages 959–965,Maui, Hawaii, 2003.

D. Mousavere and C. Kravaris. Nonlinear Controller Design Via Approximate Solution of Hamilton-Jacobi Equations *In Proceedings of the IEEE International on Mediterrean Conference on Control and Automation*, pages 1137–1142, Limassol, Cyprus, 2005.

J. Lawton and R.W Beard. Numerically efficient approximations to the Hamilton-Jacobi-Bellman equation *In Proceedings of the IEEE International Amercan Control Conference*, pages 195–199, 1998.