

# Role of electric synapses in spike train statistics of integrate and fire neural networks

Rodrigo Cofré, Bruno Cessac

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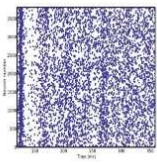
# Dynamics and spike trains statistics in conductance-based Integrate-and-Fire with chemical and electric synapses

Rodrigo Cofre, Bruno Cessac  
NeuroMathComp project team (INRIA, UNSA, ENS, CNRS)



We consider conductance-based Integrate-and-Fire models coupled with gap junctions and chemical synapses, where conductances depend upon the spike-history of the network. We compute explicitly the time evolution operator and show that given the spike-history of the network and the membrane potentials at a given time, the further dynamical evolution can be written in a closed form. Moreover, spike train statistics is described by a Gibbs distribution whose potential can be approximated with an explicit formula, when the noise is weak.

## Spike Trains



Spike state	$\omega_k(n) \in \{0, 1\}$
Spike pattern	$\omega(n) = (\omega_k(n))_{k=1}^N$
Spike block	$\omega_m^n = \{\omega(m), \omega(m+1), \dots, \omega(n)\}$
Raster plot	$\omega \stackrel{\text{def}}{=} \omega_0^T$

## The equation

We consider a network of  $N$  neurons, where dynamics depend in continuous and discrete dynamical variables. The sub-threshold variation of the membrane potential of neuron  $k$  at time  $t$  is given by:

$$C_k \frac{dV_k}{dt} = -g_{L,k}(V_k - E_L) - \sum_j g_{kj}(t, \omega)(V_k - E_j) + \sum_j \bar{g}_{kj}(V_j - V_k) + I_k(t)$$

Where :

$C_k$  is the capacitance of each neuron  
 $g_{L,k}$  is the leak conductance.

$\bar{g}_{kj}$  is the electrical conductance which is symmetric.

$I_k(t) = i_k^{(ext)}(t) + \sigma_B \xi_k(t)$  is the current term which is composed by:

$i_k^{(ext)}(t)$  the external current, and white noise term  $\xi_k(t)$  whose magnitude is controlled by  $\sigma_B > 0$

## Conductance adaptation

$$g_{kj}(t) = g_{kj}(t_j^{(r)}(\omega)) + G_{kj} \alpha_{kj}(t - t_j^{(r)}(\omega)), \quad t > t_j^{(r)}(\omega)$$

$G_{kj} \geq 0$  is the maximal conductance

$\alpha_{kj}(t) = h(t) e^{-\frac{t}{\tau_{kj}}} H(t)$  is called "alpha function"

## Matrix-Vector representation

$$C \frac{dV}{dt} + [G(t, \omega) - \bar{G}] V = I(t, \omega)$$

$$G(t, \omega) = (g_{L,k} + \sum_{j=1}^N g_{kj}(t, \omega)) \delta_{kk}$$

$\bar{G}$  is the symmetric matrix of electric conductances, with entries  $\bar{g}_{kj}$

$$I(t, \omega) = I^{cs}(t, \omega) + I^{ext}(t) + I^B(t)$$

## The linear SDE

$$\frac{dV}{dt} = \underbrace{C^{-1}(\bar{G} - G(t, \omega)) V}_{\Phi(t, \omega)} + \underbrace{C^{-1} I^{cs}(t, \omega)}_{f(t, \omega)} + C^{-1} I^{ext}(t) + C^{-1} I^B(t)$$

## The flow and the solution

If we define recursively the following matrices:

$$M_0(t_0, t, \omega) = \mathcal{I}_N$$

$$M_k(t_0, t, \omega) = \mathcal{I}_N + \int_{t_0}^t \Phi(s, \omega) M_{k-1}(s, t) ds$$

Under very general conditions, this limits exist and is called "flow"

$$\Gamma(t_0, t, \omega) \stackrel{\text{def}}{=} \lim_{k \rightarrow \infty} M_k(t_0, t, \omega)$$

Considering both chemical and electric synapses, there exist a unique strong solution to the stochastic differential equation given by

$$V(t_0, t, \omega) = \Gamma(t_0, t, \omega) v + \int_{t_0}^t \Gamma(s, t, \omega) f(s, \omega) ds + \frac{\sigma_B}{c} \int_{t_0}^t \Gamma(s, t, \omega) dW(s)$$

This are examples when the flow takes an exponential form

(i)  $\bar{G}$  is diagonal;

(ii)  $\bar{G} = 0$ ;

(iii)  $G(t, \omega) = G(t) = \kappa(t) \mathcal{I}_N$  where  $\kappa(t)$  is a real function.

## Spike Trains Statistics

The membrane potential can be decomposed in their deterministic and stochastic part:

$$V(t, \omega) = V^{(d)}(t, \omega) + V^{(noise)}(t, \omega)$$

The main result establishes that spike trains are distributed according to a Gibbs distribution whose potential can be obtained under Gaussian approximation of  $V^{(noise)}(t, \omega)$

$$\hat{\theta}_k(t, \omega) = \theta - V_k^{(d)}(t, \omega) \quad \mathcal{J}_{k(n, \omega)} = \begin{cases} 1 - \infty, \hat{\theta}_k(n-1, \omega), & \text{if } \omega_k(n) = 0; \\ \hat{\theta}_k(n-1, \omega), +\infty, & \text{if } \omega_k(n) = 1; \end{cases}$$

$$\mathcal{J}(n, \omega) = \prod_{k=1}^N \mathcal{J}_k(n, \omega) \quad dv = \prod_{i=1}^N dv_i$$

$$\mathbb{P}[\omega(n) | \omega_{-\infty}^{n-1}] = \int_{\mathcal{J}(n, \omega)} \frac{e^{-V^T \mathcal{Q}^{-1} (n-1, \omega) V}}{(2\pi)^{\frac{N}{2}} |\mathcal{Q}(n-1, \omega)|^{\frac{1}{2}}} dV$$

In the case without electric synapses:

$$\mathbb{P}[\omega(n) | \omega_{-\infty}^{n-1}] = \prod_{k=1}^N \mathbb{P}[\omega_k(n) | \omega_{-\infty}^{n-1}]$$

## Gibbs distribution

The transition probabilities define a stochastic process in the set of rasters, where the probability of having a spike pattern depends on an infinite past. Such process are called "Chains with complete connections". Under suitable conditions define a unique probability distribution called Gibbs distribution.

$$\phi(n, \omega) = \log \mathbb{P}[\omega(n) | \omega_{-\infty}^{n-1}] \quad \text{is called Gibbs potential}$$

$$\mathbb{P}[\omega_m^n | \omega_{-\infty}^{m-1}] = e^{\sum_{i=m}^n \phi(i, \omega)} \quad \text{is the conditional probability in terms of the Gibbs potential}$$

## Conclusions:

Including electric synapses spike statistics becomes **indecomposable**.

$$\mathbb{P}[\omega(n) | \omega_{-\infty}^{n-1}] \neq \prod_{k=1}^N \mathbb{P}[\omega_k(n) | \omega_{-\infty}^{n-1}]$$

Correlations are due to stimulus and **dynamics**.

Electric synapses have an effect in the **memory** of the system.

The **Gibbs potential** is largely **more complex** than the Ising Model used in retina spike train analysis.

## Bibliography :

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