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Abstract

In this paper, we enhance the MIP formulation for the Network Power Consumption problem, proposed by Giroire et al. We derive cutting planes, extending the well-known cutset inequalities, and report on preliminary computations.

Keywords: Green Networking, Mixed Integer Programming, Cutting Planes

1 Introduction

Green networking and energy-efficient routing are major aspects of today's backbone networks. Recent studies [8] indicate that the power consumption of the network mostly depends on the number of active links and routers. Based on this idea, energy-efficient routing [9] has been proposed so that traffic flows are aggregated into fewer links while preserving connectivity and QoS. Thus energy consumption can be reduced. Recently, Anand et al. [10] have analyzed data redundancy elimination opportunities between certain routers. Extending on this, Cisco, Juniper, and others have proposed the so called WAN Optimization Controllers (WOC). These can be enabled at each router

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allowing for caching/compressing of traffic flows and thus potentially reducing traffic amounts [11]. Such a compressed flow can not be compressed again and it must be uncompressed somewhere on the network so that the receiver receives uncompressed flow. Based on this idea, Giroire et al. have proposed the GreenRE model [1], in which the necessary amount of active links can be reduced further via compressions, compared to standard energy aware routing (minimal #link usage), and thus, more energy can be saved.

In this paper, we will extend the work in [1] from a methodical point of view. As shown in [1], heuristic algorithms are used commonly to find approximate solutions since it is time-consuming to find optimal solutions. We will focus on enhancements of the ILP formulation to benchmark these heuristic solutions. Therefore, we propose two classes of valid inequalities and rate their potentials with an exact separation algorithm.

2 The Network Power Consumption Problem

Let $G = (V, E)$ be an undirected graph. The nodes V describe routers and the edges E describe potential immediate connections between those routers. The (energy) costs of an active WOC at node v is given by $PN_v \geq 0$ and the costs of using edge $e = \{v, w\}$ are given by $PE_e \geq 0$. We denote by D^{vw} the demand of (uncompressed) traffic to be routed between v and $w \in V$, $v \neq w$. Further, we assume that the capacity of all edges is a constant $c \in \mathbb{R}_{>0}$ and that the compression rate is given by $\gamma \in \mathbb{R}$, $\gamma > 1$. The task is to find a feasible routing between all participants, fulfilling all demands and being minimal in energy consumption.

We formulate the above presented problem as (mixed) integer linear program. Hereby, we use variables $x_{uv} \in \{0, 1\}$, indicating the usage of link $\{u, v\} \in E$ and variables $w_u \in \{0, 1\}$ denoting the activation of a WOC in $u \in V$. Further, we use variables $f^{st}_{uv}, g^{st}_{uv} \geq 0$ for describing the (un-) compressed flows for demand $s \neq t$ on edge uv from u to v of a feasible routing. Hereby f describes the amount of uncompressed flow on edge uv of demand st whereas g describes the amount of compressed flow respectively. Then, denoting by $N(v)$ all neighbors of a node v, the problem can be stated as

$$\min \sum_{(uv)\in E} PE_{uv} x_{uv} + \sum_{u \in V} PN_u w_u$$

$$\text{s.t.} \sum_{v \in N(u)} \left(f^{st}_{vu} + \gamma g^{st}_{vu} - f^{st}_{uv} - \gamma g^{st}_{uv} \right) = \begin{cases} -D^{st} & \text{if } u = s, \\ D^{st} & \text{if } u = t, \\ 0 & \text{else} \end{cases} \quad \begin{array}{l} \forall u \in V, \\ \forall s \neq t \in V \times V \end{array} \tag{1}$$

$$\sum_{s\in V}\sum_{\substack{t\in V\\ t\neq s}}\left(f^{st}_{uv}+f^{st}_{vu}+g^{st}_{uv}+g^{st}_{vu}\right)\leq cx_{uv}\ \forall uv\in E \tag{2}$$

$$\sum_{v\in N(u)}\left(g^{st}_{uv}-g^{st}_{vu}\right)\leq \frac{D^{st}}{\gamma}w_u \qquad \begin{array}{l}\forall u\in V,\\ \forall s\neq t\in V\times V\end{array} \tag{3}$$

$$\sum_{v\in N(u)}\left(g^{st}_{vu}-g^{st}_{uv}\right)\leq \frac{D^{st}}{\gamma}w_u \qquad \begin{array}{l}\forall u\in V,\\ \forall s\neq t\in V\times V\end{array} \tag{4}$$

$$x_{uv}\in\{0,1\}, w_u\in\{0,1\}, f^{st}_{uv}\geq 0, g^{st}_{uv}\geq 0$$

The first class of constraints (1) consists of flow conservation constraints (either compressed or not). The second class (2) describes link capacity constraints, and the remaining class (3),(4) constitutes the possibility to (de-) compress flow at a node if a WOC is enabled. In the following, we will refer to this MIP as the NPC (Network Power Consumption) formulation.

3 Valid Inequalities

We present the following inequalities for strengthening the NPC formulation. Hereby we align ourselves closely to the notation of Raack [4]. Given a set $S\subset V$, the total demand, which needs to be routed between S and $V\setminus S =: \overline{S}$ is denoted by

$$D^S := \sum_{v\in S}\sum_{w\in\overline{S}} D^{vw} + \sum_{v\in\overline{S}}\sum_{w\in S} D^{vw}.$$

Further, let $\delta(S,\overline{S})$ be the corresponding cut between both sets. Now, the well known cutset inequality for network design [2], [3] can be adapted.

Theorem 3.1 *Let $S,\overline{S}\subset V$ be a partition of V. Then the **cutset** inequality*

$$\sum_{uv\in\delta(S,\overline{S})} x_{uv}\geq\left\lceil\frac{D^S}{c\gamma}\right\rceil. \tag{5}$$

holds for NPC.

This inequality assumes that at least one WOC is available in each of the two subsets. Assuming the contrary, we could increase the right-hand side. The following result takes the actual number of WOCs into account.

Theorem 3.2 *Let $S, \overline{S} \subset V$ be a partition of V. Then the* ***extended cutset*** *inequality*

$$\left(\left\lceil \frac{D^S}{c} \right\rceil - \left\lceil \frac{D^S}{c\gamma} \right\rceil\right) \sum_{u \in S} w_u + \sum_{uv \in \delta(S,\overline{S})} x_{uv} \geq \left\lceil \frac{D^S}{c} \right\rceil \tag{6}$$

holds for NPC.

Proof. Let S be given. Since w_u is integer, we distinguish two cases:

Case 1: Let $w_u = 0$ for all $u \in S$. Then (6) becomes

$$\sum_{uv \in \delta(S,\overline{S})} x_{uv} \geq \left\lceil \frac{D^S}{c} \right\rceil,$$

which is equivalent to the cutset inequality in absence of WOCs.

Case 2: Let $\sum_{u \in S} w_u \geq 1$. We obtain

$$\begin{aligned} &\left(\left\lceil \frac{D^S}{c} \right\rceil - \left\lceil \frac{D^S}{c\gamma} \right\rceil\right) \sum_{u \in S} w_u + \sum_{uv \in \delta(S,\overline{S})} x_{uv} \\ \geq\ &\left(\left\lceil \frac{D^S}{c} \right\rceil - \left\lceil \frac{D^S}{c\gamma} \right\rceil\right) + \sum_{uv \in \delta(S,\overline{S})} x_{uv} \geq \left\lceil \frac{D^S}{c} \right\rceil \end{aligned}$$

which holds, because of the cutset inequality (5). □

Comparing inequality (5) with (6), we conclude: both inequalities are equal if exactly one WOC is deployed (in S). If no WOC is deployed, the latter one strictly dominates the first and vice versa if more than one WOC is available. In fractional solutions, no dominance relation can be given: the latter inequality has a weaker left-hand side while the first inequality has a weaker right-hand side. If S contains WOCs but $\overline{S}$ does not, an exchange of S and $\overline{S}$ yields the stronger inequality.

4 Recognizing violated Inequalities

Employing all inequalities for all possible cuts in the NPC formulation is not a realistic option. In the following, we will present a straightforward approach for separating these inequalities via an integer linear program generalizing the separation of cutset inequalities in [4]. While the objective function 'rebuilds' the extended cut inequality (6) for the current LP solution(w^*, x^*), we need the following variables:

For each $v \neq w \in V$, let $z_{vw} \in \{0,1\}$ denote, whether v and w are in separate sides of the cut. Further, let $d, d_\gamma \in \mathbb{Z}_{\geq 0}$, which represent the values $\lceil \frac{D^S}{c} \rceil$ and $\lceil \frac{D^S}{c\gamma} \rceil$, respectively. For every node $v \in V$, $\alpha_v \in \{0,1\}$ denotes whether v is in S or in $\overline{S}$. Finally, given a sufficiently large constant $M \in \mathbb{N}$, for $k = 1, \ldots, M$, and $v \in V$, let $\alpha_v^k \in \{0,1\}$ denote if k equals $\lceil \frac{D^S}{c} \rceil - \lceil \frac{D^S}{c\gamma} \rceil$ and v is in S, i.e., α_v^k is an enumeration of all possible coefficients of the WOC-variable coefficients in the extended cut. Consequentially, we have that

$$\sum_{k=1}^{M} k\alpha_v^k w_v^* = \left(\left\lceil \frac{D^S}{c} \right\rceil - \left\lceil \frac{D^S}{c\gamma} \right\rceil \right) \sum_{v \in V} \alpha_v w_v^* = \left(\left\lceil \frac{D^S}{c} \right\rceil - \left\lceil \frac{D^S}{c\gamma} \right\rceil \right) \sum_{v \in S} w_v^*.$$

Then the problem of finding a violated extended cut can be written as an extension of the triangle-formulation of the cut-polytope [12] as

$$\begin{aligned}
\min \ & \sum_{k=1}^{M} k w_v^* \alpha_v^k + \sum_{vw \in E} x_{vw}^* z_{vw} - d & & \\
\text{s.t.} \ & -1 + \epsilon \leq \frac{1}{c} \sum_{v \in V} \sum_{\substack{w \in V \\ w \neq v}} D^{vw} z_{vw} - d \leq 0 & & (7) \\
& -1 + \epsilon \leq \frac{1}{c\gamma} \sum_{v \in V} \sum_{\substack{w \in V \\ w \neq v}} D^{vw} z_{vw} - d_\gamma \leq 0 & & (8) \\
& z_{vw} + z_{uw} + z_{uv} \leq 2 & \forall \ \{u, v, w\} \subset V & \quad (9) \\
& z_{vw} + z_{uw} \geq z_{uv} & \forall \ \{u, v, w\} \subset V & \quad (10) \\
& \alpha_v + \alpha_w \leq 2 - z_{vw} & \forall \ v, w \in V, v \neq w & \quad (11) \\
& \alpha_v + \alpha_w \geq z_{vw} & \forall \ v, w \in V, v \neq w & \quad (12) \\
& \sum_{k=1}^{M} \alpha_v^k = \alpha_v & \forall \ v \in V & \quad (13) \\
& M(1 - \alpha_v) + \sum_{k=1}^{M} k\alpha_v^k \geq d - d_\gamma & \forall \ v \in V & \quad (14) \\
& z_{vw}, \alpha_v, \alpha_v^k \in \{0,1\}, d, d_\gamma \in \mathbb{Z}_{\geq 0} & &
\end{aligned}$$

In this model, the inequalities (9), (10) establish a feasible cut and the inequalities (7) and (8) recognize the rounded traffic across this cut. (11) and (12) determine which nodes are within the one side (S) of the cut, and the inequalities (13) - (14) choose the correct α_v^k values for each α_v.

If the resulting objective value is strictly smaller than zero, the optimal solution of the corresponding MIP describes a partition of V, violating an extended cutset inequality (the z_{vw} correspond to the edge variables and the α_v to the WOC variables in S). If the contrary holds, none of these extended cuts is violated.

This integer program can be adapted to separate cutset inequalities (5) easily by omitting the unnecessary parts (d, α_v, α_v^k) and restricting to the constraints (9), (10) and (8).

5 Preliminary Computational Experiments

In this section, we want to show benefits of incorporating the inequalities (5) and (6) within the (standard) MIP-solution process. In this preliminary study, for every LP solution, the inequalities (5) are separated. Only if no violated cut is found, the inequalities (6) are separated. We used modified instances of the SNDlib [7]. Taking care of the great variety within that library, all demands have been scaled such that a routing (without compression) is feasible at a capacity of $10,000$ per link and infeasible at a capacity of $9,000$. In this study, we used the instances ABILENE (scaling factor: 102), ATLANTA (2.6), DFN-BWIN (4.5), FRANCE (1.1) and POLSKA (0.17).

All computations have been done with CPLEX 12.4, with CPU-/thread-usage limited to one. For obtaining clear results, CPLEX internal cutting-planes have been disabled. We report on progress after the root node and compare ourselves to the usage of plain CPLEX (with the same settings). All scenarios have been tested with capacities $5,000$ (halved), $10,000$ (normal) and $20,000$ (doubled) and a compression factor of $\gamma = 2$. The price for an edge has been determined as 200 [9] and the price of an WOC as 30 [1]. Success of the separation routine is measured by the relatively closed gap at the root, i.e. let DB denote the LP relaxation, DB_s the best dual bound obtained by our separation approach and PB the best primal bound available. Then we define the gap closed as $GC := \frac{DB_s - DB}{PB - DB}$. Clearly, an improvement is given, as soon as this value is positive and a higher value corresponds to a bigger improvement.

The results presented in Figure 1 are throughout positive. The gain from separating one or both of the two classes of inequalities is shown. The total improvement amounts to an average gap closed of 46.1%. This improvement was achieved by an average amount of 45 cuts per instance, 33 cutset inequalities (5) and 12 extended cuts (6). While the relation between the amounts of both cutting plane families is clear by the lazy separation approach of the extended

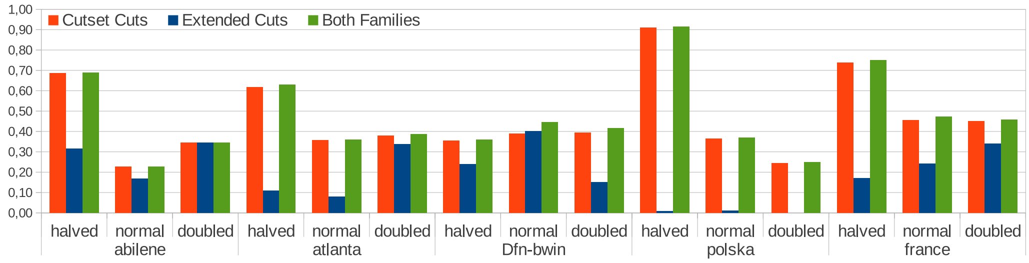

Fig. 1. Relative Gap Closing

cuts, it appears that the extended cuts (6) can still improve on the cutset inequalities (5). However, the success and the relation between both classes is highly dependant on the underlying network topology (and the edge/WOC prices). For example the POLSKA (doubled) instance seems to be very unfortunate for extended cuts, while the DFN-BWIN (normal) seems to favor exactly these. On the ABILENE (normal) instance, both behave equal.

The drawback of this separation approach is it's time consumption. The time needed for finding violated cuts and re-optimizing the linear relaxation is significant. Depending on the amount of cuts found, this procedure can amount to an increase of time consumption of more than 100%.

6 Conclusions

Concluding our paper, we rate our findings and point out potential further research. We derived two different classes of valid inequalities for the NPC-problem. These inequalities have been used in a straightforward cutting plane approach. Exemplary calculations showed that these inequalities are helpful in strengthening the formulation of the root node. However, the increase in time-consumption is a major drawback.

We believe that further research should be focused on a more sophisticated (but probably heuristic) separating algorithm. Clearly, this has to be joint with a more detailed analysis of success and effect of these inequalities. Finally, we still believe that there is potential for further improvement in the form of additional classes of valid inequalities and we hope to extend our finding to the robust counterpart of our model in the future.

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