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Abstract

We propose in this paper a model of optimal Carbon Capture and Storage in which the reservoir of sequestered carbon is leaky, and pollution eventually is released into the atmosphere. We formulate the social planner problem as an optimal control program and we describe the optimal consumption paths as a function of the initial conditions, the physical constants and the economical parameters. In particular, we show that the presence of leaks may lead to situations which do not occur otherwise, including that of non-monotonous price paths for the energy.

1 Introduction

The fact that the carbon emissions generated by the use of the fossil fuels could be captured and sequestered is now well documented both empirically and theoretically, and it is now included in the main empirical models of energy uses. Were this option open at a sufficiently low cost for the most potentially polluting primary resource, that is coal, its competitive full cost, including the shadow cost of its pollution power, could be drastically reduced given that coal is abundant at a low extraction cost and can be transformed into energy ready to use for final users at moderately transformation costs. The main problem concerning its future competitiveness is the cost at which its pollution damaging effects can be abated.

Abating the emissions involves two different types of costs. The first one is a monetary cost : capturing, compressing and transporting the captured CO_2 into reservoirs involves money outlays. The second one is a shadow cost because this type of garbage has to be stockpiled somewhere. This problem has been attacked in Lafforgue *et alii* (2008-a, 2008-b). It is not quite clear that sufficient storage capacities would be available for low CO_2 capture and storage (CCS) costs, in which case the reservoir capacities themselves could have to be seen as scarce resources to which some rents should have to be imputed along an optimal or equilibrium path.

As far as equilibrium paths are concerned there is a very difficult problem about property rights. The reservoirs into which the captured CO_2 is assumed to be confined are in underground places, on which property rights are more or less defined, and differently defined all over the world.

Even if sufficiently large reservoirs are available there exists another problem concerning the security of such reservoirs. Most reservoirs are leaking in the long run, a well-known problem in engineering. The fact that captured CO_2 will eventually return into the atmosphere cannot be ignored when assessing the economic relevance of CCS.

A first investigation of this last problem has been given by Ha-Duong and Keith (2003). Their main conclusion is that "leakage rates on order of magnitude below the discount rate are negligible" (p. 188). Hence leakage is a second order problem as far as the rate of discount is sufficiently high, and probably that other characteristics of the empirical model they use are sufficiently well profiled.

A second batch of investigations has recently been conducted by Gerlagh, Smekens and Van der Zwaan.[1] These papers are mainly empirical papers using and comparing DEMETER and MARKAL models to assess the usefulness of CCS policies. Their results are twofold. First using CCS policies with leaky reservoirs does not permit to escape a big switch to renewable non polluting primary resources if a 450ppmv atmospheric pollution ceiling has to be enforced. But CCS with leaky reservoirs is smoothing the optimal path. A second point concerns the relative competitiveness of coal : "The large scale application of CCS needed for a significantly lower contribution of renewable would be consistent, in terms of climate change control, with the growing expectation that fossil fuels, and in particular coal, will continue to be a dominant form of energy supply during the twenty-first century" (Van der Zwaan and Gerlagh, 2009, p. 305). As they point out "The economic implications of potential CO_2 leakage associated with the large scale development of CCS have so far been researched in a few studies" (ibidem, p. 306). To our knowledge theoretical studies are even fewer.

The objective of this paper is to try to elucidate some theoretical features of optimal CCS policies with leaky reservoirs and specifically the dynamics of the shadow cost of both carbon stocks and their relation with the mining rent of the nonrenewable resource, determining the long run relative competitiveness of coal and solar energies. The paper has to be seen as mainly exploratory. To conduct the inquiry we adopt the most simple model permitting to isolate the dynamics of captured CO_2, leakage and atmospheric pollution.

Naturally, the presence of leaks, producing an additional flow of pollutant, makes the pressure on the atmospheric stock larger than when there is none, and should favor capture. On the other hand, for the same reason, it is not necessarily good to sequestrate too much pollution, since this will make economic conditions worse in the future.

[1] c.f. Van der Zwaan (2005), Van der Zwaan and Gerlagh (2009) and Van der Zwaan and Smekens (2009)

The results presented in this paper show how the optimal consumption paths are modified with respect to the benchmark situation where there are no leaks. In particular, it turn out that over some optimal path, the price of energy is not necessarily monotonous. Non-monotonous price paths in the exploitation of nonrenewable resources have been described before: for a first paper in this direction, see for instance Livernois and Martin (2001). In the present situation, the lack of monotonicity results from a combination of a constraint on the present atmospheric stock of pollution, and a lag effect for the sequestered stock of pollution; such an effect has not been reported in the literature, to the best of our knowledge.

Our analysis reveals other interesting features. First of all, not every possible configuration of atmospheric and sequestered stock is acceptable, thus causing a possible *viability* problem. Other results quantitatively confirm that the presence of leakage does reduce the economic incentive to sequestrate pollution.

The paper is organized as follows. We develop the model, its assumptions and notations in Section 2. In Section 3, we state the mathematical optimization program representing the social planner problem, and derive the necessary optimality conditions. In Section 4, we describe the behavior of optimal consumption paths in the different parametric situations, and draw our conclusions. Technical details are provided in Section 5.

2 Model and Preliminaries

We consider a global economy in which the energy consumption can be supplied by two primary resources: a nonrenewable polluting source like coal and a clean renewable one as solar plants.

2.1 Energy consumption and gross surplus

Let us denote by q the instantaneous energy consumption rate of the final users and by $u(q)$ the instantaneous gross surplus thus generated. The gross surplus function is assumed to satisfy the following standard assumptions:

Assumption 1. The function $u : \mathbb{R}_+ \to \mathbb{R}_+$ is a function of class C^2, strictly increasing and strictly concave, and which satisfies the first Inada condition $\lim_{q \to 0} u'(q) = +\infty$.

The function $u'(q)$, the inverse demand function, is also denoted by $p(q)$ and its inverse, the direct demand function, is denoted by $q^d(p)$. Under Assumption 1, the function q^d is strictly positive and strictly decreasing.

2.2 The non renewable polluting resource

Let $X(t)$ be the stock of coal available at time t, $X^0 = X(0)$ be its initial endowment, and $x(t)$ be the instantaneous extraction rate: $\dot{X}(t) = -x(t)$. The current average transformation cost of coal into useful energy is assumed to be constant and is denoted by c_x. We denote by $\widetilde{x}$ the non renewable energy consumption when its marked price is equal to c_x and coal is the only energy supplier: $u'(\widetilde{x}) = c_x$.

Burning coal for producing useful energy implies a flow of pollution emissions proportional to the coal thus burned. Let ζ be the unitary pollution contents of coal so that the gross emission flow amounts to $\zeta x(t)$. This gross emission flow can be either freely relaxed into the atmosphere or captured to be stockpiled into underground reservoirs however at some cost.

Let c_s be the average capturing and sequestrating cost of the potential pollution generated by the exploitation of coal. Let us denote by $s(t)$ this part of the potential flow $\zeta x(t)$ which is captured and sequestered. Then the sequestration cost amounts to $c_s s(t)$. The remaining flow of carbon $\zeta x(t) - s(t) \geq 0$ goes directly into the atmosphere.

2.3 Pollution stocks and leakage effects

We take two pollution stocks explicitly into account, the atmospheric stock denoted by $Z(t)$ and the sequestered stock denoted by $S(t)$.

The atmospheric stock Z is first fed by the non-captured pollution emissions, resulting from the use of coal, that is $\zeta x(t) - s(t)$. This atmospheric stock is self-regenerating at some constant proportional rate α.[2] However, Z is also fed by the leaks of the sequestered pollution stock S. We assume that leaks are proportional to the stock and denote by β the leakage rate. In total:

$$\dot{S}(t) \; = \; s(t) - \beta S(t) \; .$$

We assume that the sequestering capacities are sufficiently large to be never saturated and that no cost has to be incurred for maintaining the captured stock S into reservoirs. The only costs are the above capture costs $c_s s(t)$.

Taking into account both this leakage effect and the above self-regeneration effect, we get the dynamics of the atmospheric stock:

$$\dot{Z}(t) \; = \; \zeta x(t) - s(t) + \beta S(t) - \alpha Z(t) \; .$$

[2] This self-regeneration effect may be seen as some kind of leakage of the atmosphere reservoir towards some other natural reservoirs not explicitly modeled in the present setting. For models taking explicitly into account such questions, see for example Lontzek and Rickels (2008) or Rickels and Lontzek (2008).

2.4 Atmospheric pollution damages

There are two main ways for modeling the atmospheric pollution damages. A most favored way by some economists is to postulate some damage function, the higher is the atmospheric pollution stock $Z(t)$, the larger are the current damages at the same time t. Generally, this function is assumed to be convex. The other way is to assume that, as far as the atmospheric pollution stock is kept under some critical level $\overline{Z}$, the damages are not so large. However, around the critical level $\overline{Z}$, the damages are strikingly increasing, so that, whatever what could have been gained by following a path generating an overrun at $\overline{Z}$, the damages would counterbalance the gains.[3] We assume that the loss generated by Z are negligible provided that Z be maintained under some level $\overline{Z} \geq Z^0 \geq 0$, $Z^0 \equiv Z(0)$, but is infinitely costly once $Z(t)$ overruns $\overline{Z}$.

We denote by $\overline{x}$ the maximum coal consumption when the atmospheric pollution stock is at its ceiling $\overline{Z}$, no part of the gross pollution flow ζx is captured ($s = 0$) and the stock of sequestered pollution is nil:

$$\dot{Z} \;=\; 0 \;= \zeta\overline{x} - \alpha\overline{Z} \qquad \Longrightarrow \qquad \overline{x} \;=\; \frac{\alpha}{\zeta}\overline{Z}\;.$$

We denote by $\overline{p}$ the corresponding energy price assuming that coal is the only energy supplier: $\overline{p} \equiv q^d(\overline{x})$.

Clearly there exists an effective constraint on coal consumption if and only if $\overline{p} > c_x$ or equivalently $\overline{x} < \widetilde{x}$ and simultaneously the coal initial endowment X^0 is sufficiently large.

2.5 The renewable clean energy

The other primary resource is a renewable clean energy. Let $y(t)$ be its instantaneous consumption rate. We assume that its average cost, denoted by c_y, is constant. We denote by $\widetilde{y}$ the renewable energy consumption when the renewable one is the only energy supplier: $u'(\widetilde{y}) = c_y$.

Both c_x and c_y include all that has to be supported to supply ready to use energy to the final users. Hence, once these costs are supported the two types of energy are perfect substitutes for the final user, and we may define $q(t)$ as the sum of $x(t)$ and $y(t)$.

We assume not only that the cost of the renewable energy is higher than the cost of the nonrenewable one, but furthermore that c_y is higher than $\overline{p}$. In summary:

Assumption 2. It is assumed that $c_s > 0$, and

$$c_x \;<\; \overline{p} \;<\; c_y\;. \tag{1}$$

[3]Some authors use simultaneously both approaches.

Equivalently, under Assumption 1, $\widetilde{y} < \overline{x} < \widetilde{x}$.

More specific assumptions will be introduced when necessary.

The flows and stocks of energy and pollution are illustrated in Figure 1.

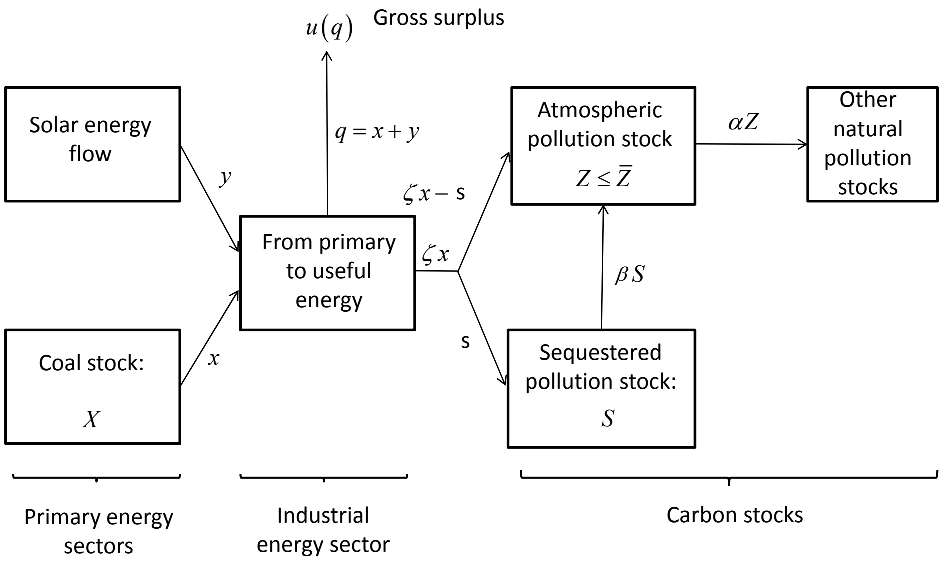

Figure 1: Energy and pollution flow and stocks

2.6 The admissible domain of S and Z

Once coal extraction is closed for some time, the atmospheric pollution stock is fed only by the leaks of the sequestered pollution stock. Then the dynamics of S and Z are given by:

$$\dot{S}(t) \;=\; -\beta S(t) \quad \text{and} \quad \dot{Z}(t) \;=\; \beta S(t) - \alpha Z(t) \;.$$

Let t_0 be the time at which such a phase begins and let us denote by S_0 and Z_0 the stocks of S and Z at this time: $S_0 \equiv S(t_0)$ and $Z_0 \equiv Z(t_0)$. Integrating the above system, we obtain for all t:[4]

$$S(t) \;=\; S^0 e^{-\beta(t-t^0)} \tag{2}$$

$$Z(t) \;=\; Z^0 e^{-\alpha(t-t^0)} - S^0 \frac{\beta}{\alpha - \beta}\left(e^{-\alpha(t-t^0)} - e^{-\beta(t-t^0)}\right) \;. \tag{3}$$

[4]We write these formulas for the general case $\alpha \neq \beta$. Special forms for $\alpha = \beta$ are easily established.

Eliminating t, we get the family of trajectories in the (S, Z) space:

$$Z(S; S_0, Z_0) \;=\; \left(\frac{S}{S_0}\right)^{\alpha/\beta} \left(Z_0 - \frac{\beta}{\alpha-\beta} S_0\right) \;+\; \frac{\beta}{\alpha-\beta} S \; .$$

These curves depend upon α and β and, structurally, only upon α/β. As a function of S, Z is first increasing and next decreasing whatever $\alpha > 0$ and $\beta > 0$ may be. The maximum is attained when $Z = (\alpha/\beta)S$. The family of these curves is illustrated in Figure 2. The movement is going from the right to the left though time. Under the line $Z = (\alpha/\beta)S$, the leaks flow βS is higher than the self-regeneration flow αZ so that the atmospheric stock of pollutant increases, whereas above the line the reverse holds and the atmospheric stock decreases.

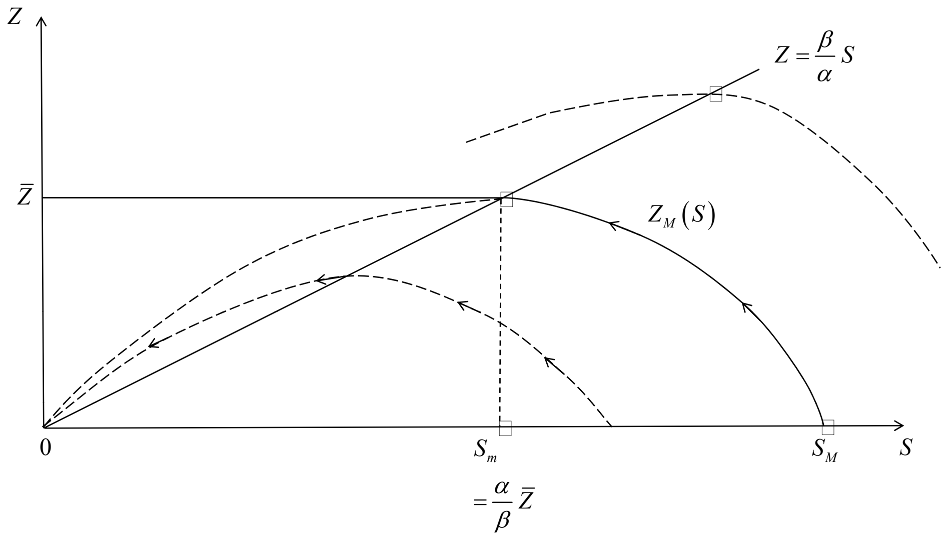

Figure 2: Admissible levels of pollution stocks

Among these trajectories, let $Z_M(S)$ be the one, the maximum of which is equal to $\overline{Z}$, S_m the value of S for which this maximum is attained, and S_M the positive value of S for which $Z_M(S) = 0$.[5] Clearly, $S_M > S_m$. Given that the maxima of $Z(\cdot)$ are located along the line $Z = (\alpha/\beta)S$, we get for $Z = \overline{Z}$: $S_m = (\alpha/\beta)\overline{Z}$. Then

$$Z_M(S) \;=\; Z(S; S_m, \overline{Z}) \;=\; \frac{\beta}{\alpha-\beta} \left(S - \overline{Z}\left(\frac{S}{S_m}\right)^{\alpha/\beta}\right) \; .$$

It follows that $S_M = \overline{Z}(\alpha/\beta)^{\alpha/(\alpha-\beta)}$, and it can be verified that $S_M > S_m$ for all values of α and β.

[5]The other value is $S = 0$.

For any $S \in (S_m, S_M]$, the control vector $(s, \zeta x - s)$ points *outwards*, and it is easy to see that for any initial position located above the curve $Z = Z_M(S)$, and for any control, the trajectory will necessarily exit the domain $\{Z \leq \overline{Z}\}$. Such a trajectory is not viable. Likewise, if a non-zero control is applied at any point of the curve $(S, Z_M(S))$, then the trajectory will necessarily exit the domain $\{Z \leq \overline{Z}\}$, whatever control is applied later on.

Therefore, the set of *viable* initial states (S^0, Z^0) is delimited by the constraints $Z \leq \widetilde{Z}(S)$, where the function $\widetilde{Z}$ is defined on $[0, S_M]$ as:

$$\widetilde{Z}(S) \;=\; \begin{cases} \overline{Z}, & 0 \leq S \leq S_m \\ Z_M(s), & S_m \leq S \leq S_M. \end{cases} \tag{4}$$

2.7 Other modeling issues

With respect to previous theoretical models, we introduce the possibility of leakage. On the other hand, we do not consider limits on the flow of renewable resource y nor capacity constraints on the reservoir S. Those features should be of course added to a complete model. As stated above, we purposedly keep the model simple in this first analysis, in order to better isolate the influence of the self-regeneration rate α, the leakage rate β and the capture cost c_s on the shape of optimal extraction paths. However, most of what is reported in this paper would remain true if the sequestrated stock would be assumed to have a maximal capacity $\bar{S}$, as long as $\bar{S} > S_m = (\alpha/\beta)\overline{Z}$.

3 The Social Planner Problem

The social planner problem is to maximize the social welfare. The social welfare W is the sum of the discounted net current surplus, taking into account the gross surplus $u(q)$ and the production or capture costs. We assume that the social rate of discount ρ, $\rho > 0$, is constant throughout time.

Accordingly, the social planner faces the following optimization problem:

$$\max_{s(\cdot),x(\cdot),y(\cdot)} \int_0^\infty \left[u(x(t)+y(t)) - c_s s(t) - c_x x(t) - c_y y(t)\right] e^{-\rho t} \mathrm{d}t \tag{5}$$

given the controlled dynamics

$$\begin{cases} \dot{X} &= -x \\ \dot{Z} &= -\alpha Z + \beta S + \zeta x - s \\ \dot{S} &= -\beta S + s \;, \end{cases} \tag{6}$$

the initial conditions $(X(0), Z(0), S(0)) = (X^0, Z^0, S^0)$, and the constraints on state variables and controls: $Z(t) \leq \overline{Z}$, $X(t) \geq 0$, $y(t) \geq 0$, $x(t) \geq 0$, $s(t) \geq 0$, $s(t) \leq \zeta x(t)$, for all t. Other physically relevant constraints ($S \geq 0$, $Z \geq 0$) are automatically satisfied by the dynamics and are not explicitly taken into account.

Let us denote by L the current-value Lagrangian of the problem:

$$\begin{aligned} L(y,x,s,X,Z,S) \;=\;& u(x+y) - c_s s - c_x x - c_y y \\ & +\lambda_X[-x] + \lambda_Z[-\alpha Z + \beta S + \zeta x - s] + \lambda_S[-\beta S + s] \\ & +\nu_Z[\overline{Z} - Z] + \nu_X X \\ & +\gamma_s s + \gamma_x x + \gamma_{sx}(\zeta x - s) + \gamma_y y \; . \end{aligned} \tag{7}$$

The first order conditions are then the following.

First, optimality of the control yields:

$$\frac{\partial L}{\partial s} = 0 \quad \Longleftrightarrow \quad 0 = -c_s - \lambda_Z + \lambda_S + \gamma_s - \gamma_{sx} \tag{8}$$

$$\frac{\partial L}{\partial x} = 0 \quad \Longleftrightarrow \quad 0 = u'(x+y) - c_x - \lambda_X + \zeta\lambda_Z + \gamma_x + \zeta\gamma_{sx} \tag{9}$$

$$\frac{\partial L}{\partial y} = 0 \quad \Longleftrightarrow \quad 0 = u'(x+y) - c_y + \gamma_y \; , \tag{10}$$

together with the complementary slackness constraints:

$$\gamma_{sx} \geq 0, \qquad \zeta x - s \geq 0 \quad \text{and} \quad \gamma_{sx}[\zeta x - s] = 0 \tag{11}$$

$$\gamma_s \geq 0, \qquad s \geq 0 \quad \text{and} \quad \gamma_s s = 0 \tag{12}$$

$$\gamma_x \geq 0, \qquad x \geq 0 \quad \text{and} \quad \gamma_x x = 0 \tag{13}$$

$$\gamma_y \geq 0, \qquad y \geq 0 \quad \text{and} \quad \gamma_y y = 0 \; . \tag{14}$$

Next, the dynamics of the costate variables are

$$\dot{\lambda}_X = \rho\lambda_X - \frac{\partial L}{\partial X} \quad \Longleftrightarrow \quad \dot{\lambda}_X \;=\; \rho\lambda_X - \nu_X \tag{15}$$

$$\dot{\lambda}_Z = \rho\lambda_Z - \frac{\partial L}{\partial Z} \quad \Longleftrightarrow \quad \dot{\lambda}_Z \;=\; (\rho+\alpha)\lambda_Z + \nu_Z \tag{16}$$

$$\dot{\lambda}_S = \rho\lambda_S - \frac{\partial L}{\partial S} \quad \Longleftrightarrow \quad \dot{\lambda}_S \;=\; (\rho+\beta)\lambda_S - \beta\lambda_Z \; , \tag{17}$$

with the constraints:

$$\nu_X \geq 0, \qquad X \geq 0 \quad \text{and} \quad \nu_X X = 0 \tag{18}$$

$$\nu_Z \geq 0, \qquad \overline{Z} - Z \geq 0 \quad \text{and} \quad \nu_Z[\overline{Z} - Z] = 0 \; . \tag{19}$$

Finally, we have the transversality conditions:

$$\lim_{t\to\infty} e^{-\rho t}\lambda_X X \;=\; 0 \tag{20}$$

$$\lim_{t\to\infty} e^{-\rho t}\lambda_Z Z \;=\; 0 \tag{21}$$

$$\lim_{t\to\infty} e^{-\rho t}\lambda_S S \;=\; 0 \; . \tag{22}$$

4 Optimal Consumption Paths

We devote this section to the general description of optimal consumption and capture paths. Some technical details that validate the scenarios will be provided in Section 5.

Each optimal path can be decomposed into a sequence of "phases", characterized by the fact that some constraints on the control or on the state are active or not. The precise way phases are chained depends on the initial conditions (X^0, Z^0, S^0) as well as on the parameters of the model. It turns out with respect to the model without leakage analyzed in Lafforgue *et alii* (2008-b), new phases appear, and there are more possible combination of phases. Also, the fact that some combinations are possible or not depends on thresholds involving the leakage rate β.

We shall focus our attention on optimal paths which are such that the threshold $\overline{Z}$ is reached at some point in time.[6] Those trajectories have the common feature that the atmospheric pollution stock Z initially increases until it reaches $\overline{Z}$, then stays at the ceiling, then ultimately decrease when the stock of coal X is close to exhaustion. While the two end parts of such a path are quite easy to characterize, what happens during the period where $Z(t) = \overline{Z}$ is more complex.

The discussion is organized as follows. We start with general facts on optimal trajectories (Section 4.1), then we describe three qualitatively different behaviors, depending on the value of the unitary capture cost c_s. The analysis reveals the existence of following critical values:

$$\hat{c}_s = \frac{\rho}{\rho+\beta}\frac{\overline{p}-c_x}{\zeta} \tag{23}$$

$$\bar{c}_s = \frac{\rho}{\rho+\beta}\frac{c_y-c_x}{\zeta} , \tag{24}$$

which are such that $\hat{c}_s < \bar{c}_s$ under Assumption 2. Then, either $c_s < \hat{c}_s$ (situation of a "small" c_s), or $\hat{c}_s < c_s < \bar{c}_s$ (situation of an "intermediate" c_s), or $c_s > \bar{c}_s$ ("large" c_s).

4.1 General features

The first general fact, which is also common to all similar models, is that whenever there is coal left in stock ($X(t) > 0$), the adjoint variable λ_X evolves according to the differential equation (15) with $\nu_X = 0$, that is: $\dot{\lambda}_X = \rho\lambda_X$. As a consequence, if one defines as T the time instant at which extraction ceases definitely: for all $t \leq T$:

$$\lambda_X(t) = (c_y - c_x)\, e^{\rho(t-T)} . \tag{25}$$

[6]Naturally, the other situation can be studied with a model which does not involve restrictions on the atmospheric stock of carbon.

Next, when none of the constraints on the state of the system (including the implicit constraint $Z \leq \widetilde{Z}(S)$ identified in Section 2.6) are binding, some configurations of the control variables x, y and s are excluded. On the one hand, it is not possible that both the renewable and the nonrenewable resource be both extracted at the same time. When the renewable resource is extracted, then necessarily the stock of coal is depleted. On the other hand, the capture control $s(t)$ obeys a sort of "bang-bang" principle: in the absence of constraints on the state variable, it is optimal, either to capture nothing at all ($s = 0$) or to capture everything ($s = \zeta x$). We shall see however that intermediate situations do occur when the stock of atmospheric carbon $Z(t)$ has reached its maximal level. The statements above are justified in Sections 5.1 and 5.2.

When the stock of coal is exhausted, then necessarily $x = s = 0$, and the surplus is produced by the consumption of the renewable resource at the constant level $y(t) = \widetilde{y}$. The evolution of the stock variables $S(t)$ and $Z(t)$ follow the "natural" dynamics described in Section 2.6, under which the trajectory asymptotically approaches the point $Z = S = 0$. We refer to this phase of every optimal path as Phase "T" (see also Section 5.2).

Before an optimal path reaches this terminal phase, it may visit a variety of other phases, to which we also refer with different letters "A", "P", "Q", "R", "S". They will be described in more details in Section 5.

4.2 Optimal paths for a small capture cost

In the case $c_s < \hat{c}_s$, the general situation is summarized in Figure 3. The figure is a phase diagram restricted to the space (S, X) with $S \leq S_m$. The value of Z is assumed to be $\overline{Z}$.[7] The phase space is partitioned in several regions corresponding to the phases in which the optimal trajectory is. We will proceed with the description of their characteristics, which are summarized in the figure under the label of each zone/phase.

Several profiles of consumption are possible, depending on the initial conditions. We illustrate them in Figures 4, 5 and 6. Before doing so, observe that sequestration does occur, when the trajectory is in phase "Q" or "S". The sequestration behavior is not "bang-bang" in phase "Q".

Large initial stock of carbon. Figure 4 illustrates the case of some X^0 "large enough". If the initial value Z^0 is less than $\overline{Z}$, the consumption is initially large and $Z(t)$ grows until it reaches $Z = \overline{Z}$ (phase "A", see Section 5.1). The growth of the price of energy $p(t) = u'(x(t)) = c_x + \lambda_X(t) - \zeta\lambda_Z(t)$ is dominated by $\lambda_Z = \lambda_Z^0 e^{(\rho+\alpha)t}$ (see Equation (34)).

Next, the trajectory is in phase "Q" (Section 5.3.2) with a consumption and a capture

[7] With the exception of the zone labeled "A".

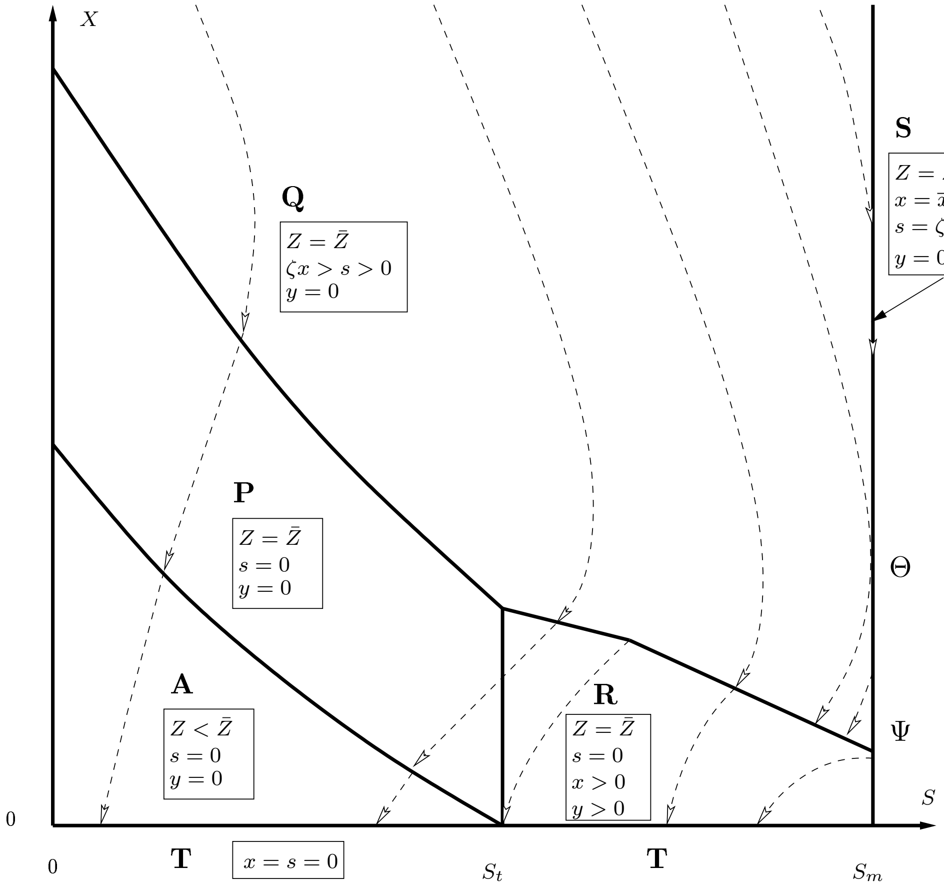

Figure 3: Phases (case $c_s < \hat{c}_s$)

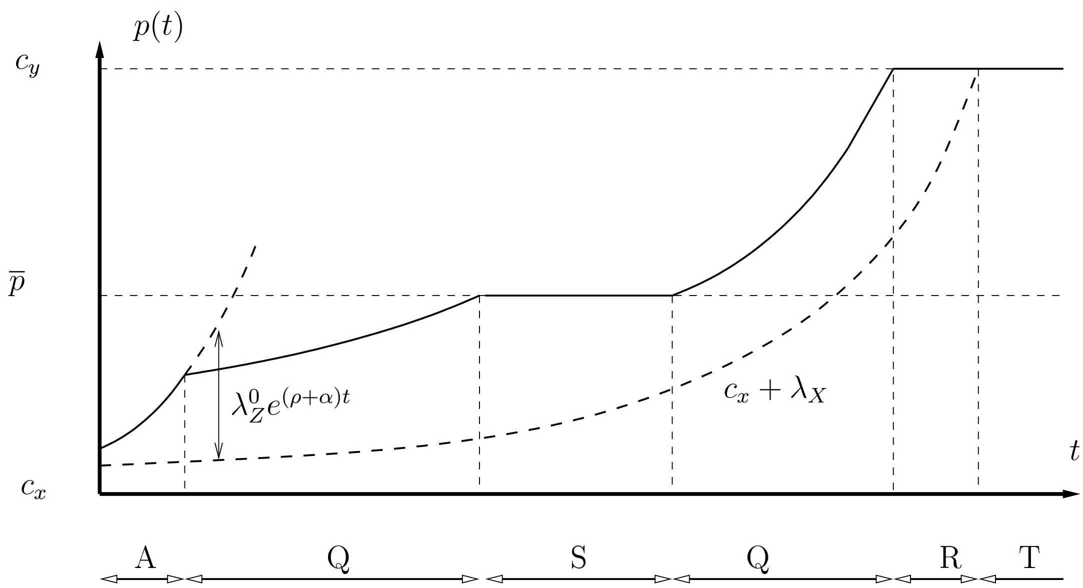

Figure 4: Trajectory from a large initial stock X^0

given by (41) and (42):

$$\begin{aligned} x &= q^d(c_x + \lambda_X - \zeta\lambda_Z) \\ s &= \zeta(x - \overline{x}) + \beta S \;=\; \zeta x - \beta(S_m - S)\ . \end{aligned}$$

According to Equation (44), the price of energy $p(t) = u'(x(t))$ can be written as:

$$p(t) \;=\; c_x + \lambda_X - \zeta\lambda_Z \;=\; c_x + \frac{\rho+\beta}{\rho}\zeta c_s \;+\; \zeta\left(\frac{\rho+\beta}{\rho}(\hat{c}_s - c_s) - \lambda_Z^0\right) e^{\rho(t-t0)}\ , \qquad (26)$$

so that the growth is now of order $e^{\rho t}$, that is, the same as $\lambda_X(t)$. Since $p(t) < \overline{p}$, then $x(t) > \overline{x}$ and $\dot{S} > 0$, so that $S(t)$ eventually reaches S_m. The trajectory then stays in the state $(S_m, \overline{Z})$ (labeled as phase “S”, see Section 5.3.3) with the constant consumption $x(t) = \overline{x}$, and capture at its maximum $s(t) = \zeta\overline{x}$. According to Equation (48), the multiplier γ_{sx} is constant over time, and is positive under the condition $c_s \leq \hat{c}_s$. The price $p(t)$ is constant at $\overline{p}$. The situation persists until $X(t)$ reaches the point labeled as Θ in Figure 3. Then the trajectory enters phase “Q” again, but with a consumption smaller than $\overline{x}$, so that the sequestered stock $S(t)$ decreases.

Both the consumption x and the capture s decrease. Eventually, either x will become less than $\widetilde{y}$, or s will become 0. Figure 4 and Figure 3 illustrate the first situation. In that case, the phase “Q” is followed by a phase “R” (Section 5.3.1) in which capture is 0 and consumption is given by Equations (39) and (40), that is:

$$\begin{aligned} x &= \overline{x} - \frac{\beta}{\zeta}S \;=\; \frac{\beta}{\zeta}(S_m - S) \\ y &= \widetilde{y} - x\ . \end{aligned}$$

The total consumption is therefore $\widetilde{y}$, and the adjoint variables are such that $\lambda_X - \zeta\lambda_Z = c_y - c_x$. The consumption of nonrenewable resource is less than $\overline{x}$ and can be arbitrarily small: the larger $S(t)$ is, the smaller is the consumption. At the limit $S(t) = S_m$, there is no consumption of nonrenewable resource. At the other limit $S(t) = S_t$, there is no consumption of the renewable resource. Both S and X are decreasing. The trajectory may either reach $S(t) = S_t$ first, in which case it enters Phase “P” (see below), or reach $X(t) = 0$, in which case it enters Phase “T” (Section 5.2). In this latter case, both adjoint variables λ_Z and λ_S become null simultaneously.

As the trajectory enters in phase “R”, the value of $x(t)$ is *discontinuous*, while the value of $q(t) = x(t) + y(t)$ is continuous. Indeed, the value of $x + y$ is $\widetilde{y}$ throughout Phase “R”. It also has this value in Phase “T”. When the trajectory enters Phase “R” coming from Phase “Q”, the value of x given by (42) is $\widetilde{y}$ because, by continuity, the value of $\lambda_X - \zeta\lambda_Z$ is $c_y - c_x$.

On the other hand, the value of x inside Phase “R” is given by (39), so that $x(t)$ passes from $\widetilde{y}$ before entering the phase (say, at time $\overline{t}_r$), to some lower value $\overline{x} - \frac{\beta}{\zeta}S(\overline{t}_r)$, then increases as S decreases, up to $\overline{x} - \frac{\beta}{\zeta}S(T)$ (T is introduced in Section 4.1 as the time at which the nonrenewable resource is exhausted), then decreases to 0 in Phase “T”.

When compared to the optimal path when no leaks occur (which is the topic of Lafforgue *et alii* (2008-b), and which corresponds to letting $\beta \to 0$ in our model), several qualitative and quantitative differences occur. The first difference is the presence of phase "S", which represents the blocking of consumption caused by the fact that the atmospheric pollution stock is saturated, and that the sequestered stock is producing a maximal flow of leaked CO_2.

The fact that both resources are consumed simultaneously is another specificity of the present model. The price path happens to reach its maximum value of c_y *before* the non-renewable resource is exhausted, and it does *not* join the Hotelling path $c_x + \lambda_X(t)$. This phenomenon never occurs with non-leaking reservoirs.

Moderate initial stock of carbon, small sequestered stock. If the initial stock X^0 is not large enough, then the trajectory may not reach $S = S_m$. Figure 5 illustrates the price along such a trajectory. With respect to the previous case, the sojourn in phase "S" has disappeared. The optimal path stays in phase "Q", and $S(t)$ increases as long as $x(t) \geq \overline{x}$, then decreases as the price $p(t)$ becomes larger than $\overline{p}$ and $x(t)$ becomes less than $\overline{x}$.

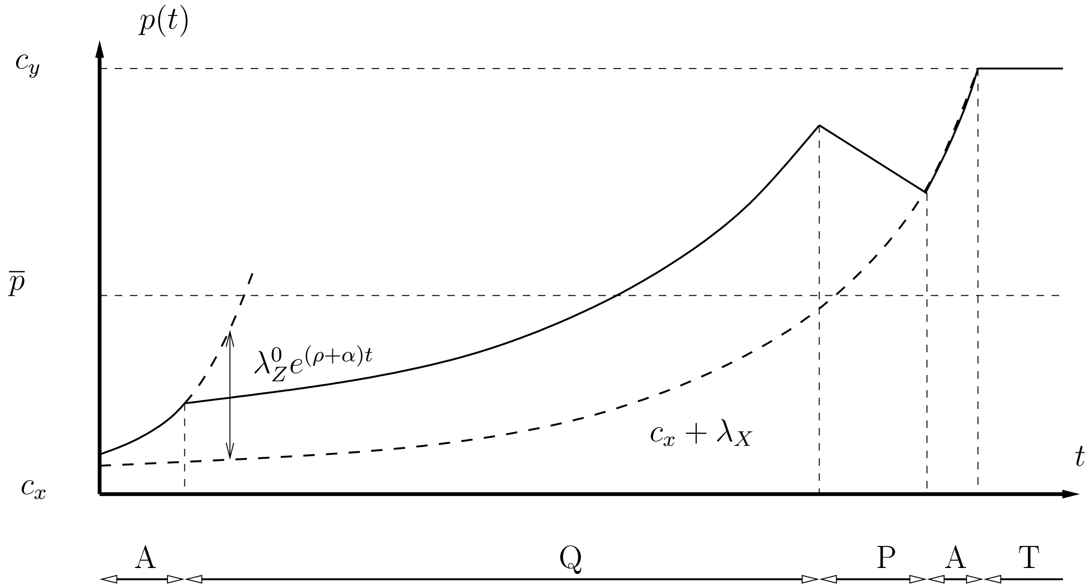

Figure 5: Trajectory from a small initial stock X^0, small sequestered stock

When the nonrenewable resource becomes rare, the possibility arises that the capture $s(t)$ becomes equal to 0. Then the optimal path enters in phase "P" (Section 5.3.1). The capture is $s = 0$, and the consumption is:

$$x \;=\; \overline{x} - \frac{\beta}{\zeta} S \;=\; \frac{\beta}{\zeta}(S_m - S)\ . \tag{27}$$

In this phase, both X and S are decreasing. This implies that the consumption is *increasing*, while staying below $\overline{x}$ (equivalently, the price decreases while staying above $\overline{p}$). The phase ends when the values of $\lambda_S(t)$ and $\lambda_Z(t)$ become null simultaneously. At this instant, the trajectory finishes in phase "A" again, then phase "T". In this second phase "A", the adjoint variables λ_Z and λ_S are null. Consequently, the consumption is simply given by

$$x = q^d(c_x + \lambda_X) ,$$

that is, the price joins the standard Hotelling price path $p(t) = c_x + \lambda_X(t)$, and there is no capture. This consumption decreases with time. The values of both S and Z decrease ($Z(t)$ leaves the ceiling). The phase ends when, simultaneously, the consumption reaches the lowest level $\widetilde{y}$ and the resource is exhausted. The system then follows the terminal phase "T".

When compared with the case with no leakage, one observe a difference in phase "P", which does occur also in that case. Here, the price of energy in this phase is decreasing over time and would converge to $\overline{p}$ if this phase were to last long enough so that the leakage $\beta S(t)$ would become negligible. In the non-leaking case, the price is constant and precisely equal to $\overline{p}$. The price path is therefore non-monotonous: in increases, then decreases when the pressure of the leaking reservoir on the atmosphere becomes smaller, then increases again when the nonrenewable resource is almost depleted.

Moderate initial stock of carbon, large sequestered stock. Finally, consider a third case, where the initial stock of coal is moderate, but there is already a large stock of sequestered pollution. The optimal path is then similar to that of Figure 4, except that there is no phase "S".

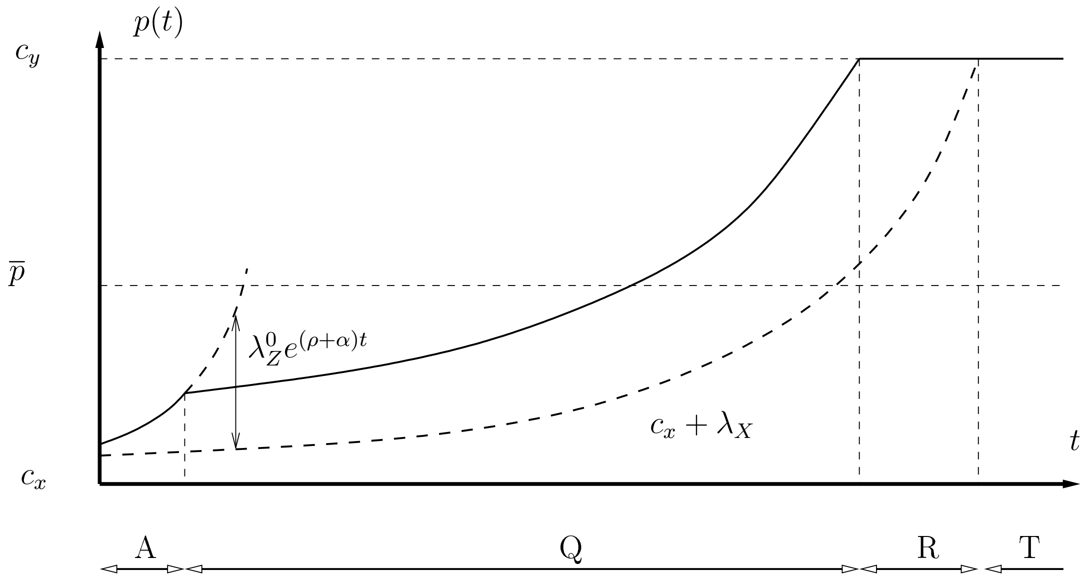

Figure 6: Trajectory from a small initial stock X^0, large sequestered stock

4.3 Optimal paths for intermediate capture costs

The "intermediate case" is when $\hat{c}_s < c_s < \bar{c}_s$. This case is summarized in Figure 7.

With respect to the case of small c_s, the behavior of trajectories is different. On the one hand, phase "S" cannot exist: indeed, in this case the multiplier γ_{sx} given by Equation (48) is negative. Therefore, trajectories beginning in some state $(X^0, \overline{Z}, S_m)$ must leave $S = S_m$ immediately and enter phase "Q" or "R", depending on the value of X^0.

On the other hand, the price of energy in phase "Q" (Equation (26)), is always larger than $\overline{p}$ and the consumption smaller than $\overline{x}$. Since $\dot{S} = \zeta(x - \overline{x})$ in this phase, this means that the sequestered stock S always decreases. This is consistent with the behavior on the line $S = S_m$.

The rest of the trajectories are similar to the case "c_s small": either through phases "P-A-T", "R-P-A-T" or "R-T". When X_0 is large, the trajectory will use the first alternative, that is, enter phase "P" and stay there for a long period of time, before terminating in phases "A" and "T".

4.4 Optimal paths for large capture costs

When $c_s > \hat{c}_s$, the cost of capture is so large that sequestrating carbon is never economically optimal, even when $Z = \overline{Z}$. Depending on the value of the current stock of sequestered carbon, the consumption is mixed renewable/nonrenewable (phase "R", the price of energy being equal to c_y), or is given by phase "P". Consumption remains below $\overline{x}$ all the time. If the initial state has $S^0 = S_m$, then the curve leaves instantaneously the line $S = S_m$, perpendicularly since $\dot{X} = (\beta/\zeta)(S_m - S) = 0$.

4.5 Observations on the threshold values for the capture cost

As a conclusion, we comment on the limiting values defined in (23) and (24). Observe that when $\beta = 0$ (that is, when there are no leaks from the reservoir, the situation previously studied in the literature), these thresholds reduce to, respectively, $(\overline{p} - c_x)/\zeta$ and $(c_y - c_x)/\zeta$. The comparisons $c_s < \hat{c}_s$ and $c_s < \bar{c}_s$ are respectively equivalent to $c_x + \zeta c_s < \overline{p}$ and $c_x + \zeta c_s < c_y$. In other words, comparing c_s to the threshold values amounts to comparing the combined unit cost of extraction and capture of the polluting resource to, respectively, that of maintaining a sustained extraction with no capture and pollution kept at the ceiling, and turning to the renewable resource.

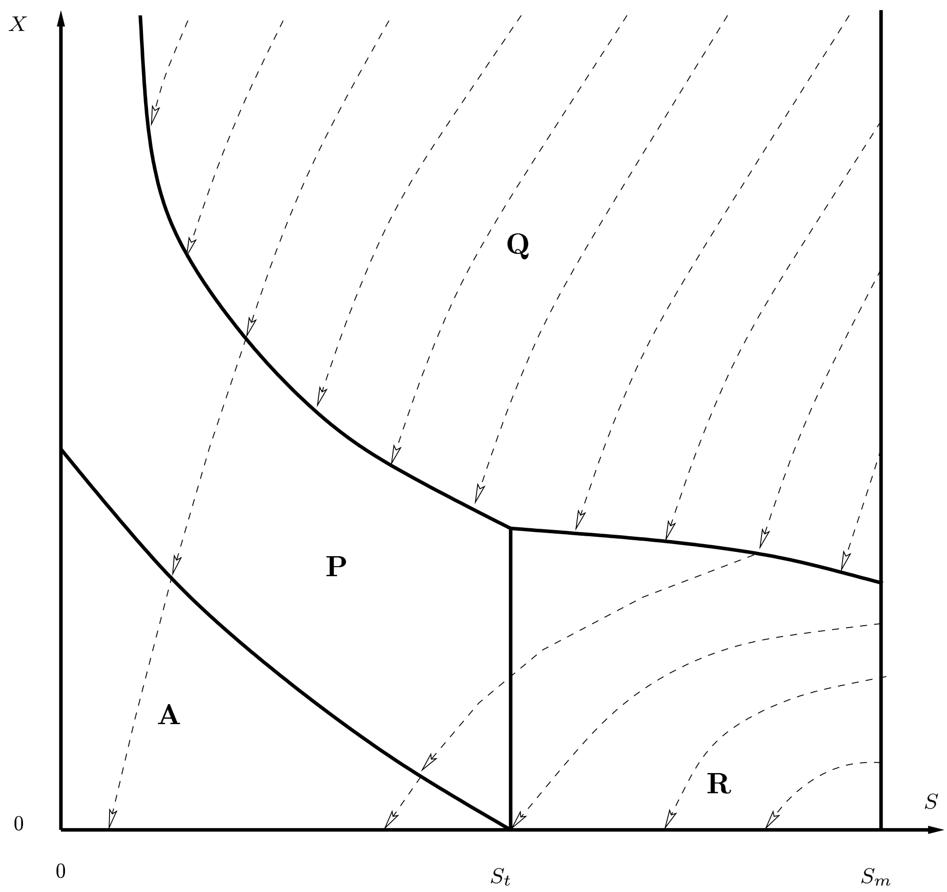

Figure 7: Phases (case $\hat{c}_s < c_s < \bar{c}_s$)

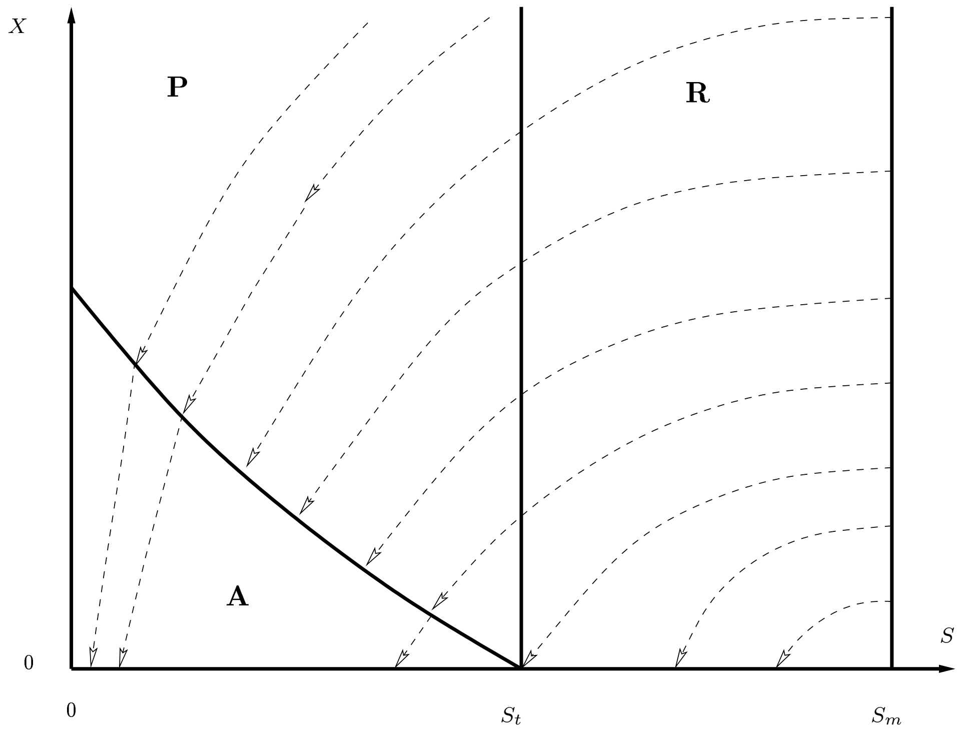

Figure 8: Phases (case $\bar{c}_s < c_s$)

On the other hand, seen as functions of β, both these thresholds are decreasing. When $\beta \to \infty$ (that is, when the reservoir does not retain anything and the captured carbon goes actually immediately into the atmosphere), they both become equal to 0.

5 Dynamics in Phases

This section provides some technical developments necessary to justify the preceding analysis, phase by phase. Recall that we view a "phase" as a piece of optimal path for which the set of active constraints on states or controls is constant. We begin with studying phases which are "interior" with respect to state constraints, next "terminal" phases, and finally phases such that the atmospheric stock has reached its ceiling.

The detailed justification of the consistency of optimal trajectories made of successions of phases is left out this preliminary report. It involves a backwards reasoning, starting from terminal phases and applying continuity properties of adjoint variables where appropriate, see *e.g.* Seierstad and Sydsæter (1999).

5.1 Interior Phases

In this section, we study the dynamics in the interior of the domain, that is, when:

$$0 < X(t) , \quad 0 < S(t) , \quad 0 < Z(t) < \widetilde{Z}(S(t)) , \tag{28}$$

where the function $\widetilde{Z}$ has been defined in (4). For such time instants, the adjoint variables ν_X and ν_Z vanish, and the dynamics of state and adjoint variables reduce to (6) and

$$\left\{ \begin{array}{lcl} \dot{\lambda}_X & = & \rho\lambda_X \\ \dot{\lambda}_Z & = & (\rho+\alpha)\lambda_Z \\ \dot{\lambda}_S & = & (\rho+\beta)\lambda_S - \beta\lambda_Z \ . \end{array} \right. \tag{29}$$

Our first result is a sort of "bang-bang" principle for the capture control s in the interior of the domain.

Lemma 1. *Consider a piece of optimal trajectory located in the interior of the domain, such that $x(t) > 0$. Then for every time instant t, either $s(t) = 0$, or $s(t) = \zeta x(t)$.*

Proof. Assume by contradiction that $0 < s(t) < \zeta x(t)$. Then by (11) and (12), we have $\gamma_s(t) = \gamma_{sx}(t)$. Then, (8) reduces to:

$$-c_s - \lambda_Z(t) + \lambda_S(t) \ = \ 0 \ . \tag{30}$$

Differentiating, we must have, over some time interval, $\dot{\lambda}_Z(t) = \dot{\lambda}_S(t)$. Using (16) and (17), this implies in turn that

$$(\rho+\alpha)\lambda_Z \ = \ (\rho+\beta)\lambda_S - \beta\lambda_Z \tag{31}$$

because $\nu_Z = 0$. Finally, solving (30)–(31), we find that the adjoint variables are necessarily constant and equal to:

$$\lambda_Z \ = \ \frac{\rho+\beta}{\alpha}\, c_s \qquad \lambda_S \ = \ \frac{\rho+\beta+\alpha}{\alpha}\, c_s \ .$$

However, these functions do not solve the differential system (29), unless $c_s = 0$. This is excluded by Assumption 2, hence the contradiction. □

Next, we rule out the possibility that both the renewable resource and the non-renewable resource be used at the same time.

Lemma 2. *Consider a piece of optimal trajectory located in the interior of the domain. Then either $x(t) > 0$ or $y(t) > 0$ but not both.*

Proof. Assume by contradiction that $x(t) > 0$ and $y(t) > 0$. Then $\gamma_x(t) = \gamma_y(t) = 0$ and the first-order conditions (8)– (10) reduce to: $x + y = \widetilde{y}$ and

$$
\begin{aligned}
0 &= -c_s - \lambda_Z + \lambda_S + \gamma_s - \gamma_{sx} && (32)\\
0 &= c_y - c_x - \lambda_X + \zeta\lambda_Z + \zeta\gamma_{sx} \ . && (33)
\end{aligned}
$$

According to Lemma 1, either $s = 0$ and $\gamma_{sx} = 0$, or $s = \zeta x$ and $\gamma_s = 0$. In the first case, differentiating Equation (33) gives $\dot{\lambda}_X = \zeta\dot{\lambda}_Z$ or equivalently with (29): $\rho\lambda_X = \zeta(\rho + \alpha)\lambda_Z$. Then the adjoint variables are necessarily constant and equal to

$$
\lambda_Z = \frac{c_y - c_x}{\alpha\zeta} \qquad \lambda_X = \frac{\rho + \alpha}{\rho}\,\frac{c_y - c_x}{\alpha} \ .
$$

However, these functions do not solve the differential system (29): a contradiction.

In the second case, Equation (32) provides the identity $\lambda_Z + \gamma_{sx} = \lambda_S - c_s$, and replacing this into (33) yields:

$$
0 = c_y - c_x - \zeta c_s - \lambda_X + \zeta\lambda_S \ .
$$

Then the previous reasoning also leads to a contradiction. □

Given Lemmas 1 and 2, the optimal control on an interior piece of trajectory therefore reduces to one of the two alternatives: either $y = 0$, $s = 0$, $x > 0$, or $y = 0$, $s = \zeta x$, $x > 0$. The second situation does not appear in the results stated in the present paper.

We name the first situation Phase "A": it is characterized by the absence of constraints on the state, zero capture and exclusive consumption of nonrenewable energy. This consumption is directly given by the first order equations (9) and (10):

$$
x = q^d(c_x + \lambda_X - \zeta\lambda_Z) \qquad (34)
$$

and the value of the adjoint variable $\lambda_Z(t)$ is obtained integrating (16) with $\nu_Z = 0$, that is:

$$
\lambda_Z(t) = {\lambda_Z}^0 e^{(\rho+\alpha)(t-t^0)} \ .
$$

5.2 Terminal phases

A phase is called terminal if there are optimal trajectories which lie in it forever.

We begin with the identification of stationary points of the state space. According to the dynamics (6), if (X, Z, S) is such a point, then $x = 0$, $s = \beta S$ and $\zeta x = \alpha Z$. This in turn implies that $s = 0$, and therefore $S = Z = 0$.

The transversality condition (20) and the dynamics (15) of λ_X together imply that $X = 0$ because $\lambda_X \neq 0$. Actually, λ_X is constant and equal to $c_y - c_x$. The consumption of the renewable resource is also fixed at $\widetilde{y}$. We call this phase: "Phase T".

Apart from stationary points, one should consider the possibility that the trajectory stays forever in the interior of the domain (28) without stopping at a particular point. However, it can be proved that having $x(t) > 0$ forever is not possible. Indeed, in order not to exhaust the resource X, it is necessary that $x(t) \to 0$. But then $x(t)$ becomes eventually smaller than $\widetilde{y}$, so that consumption of the renewable resource becomes necessary, with $x(t) + y(t) = \widetilde{y}$. But this is not possible by Lemma 2. In conclusion, there do not exist such trajectories.

5.3 Phases with $Z = \overline{Z}$

When $Z(t) = \overline{Z}$ over some interval of time, the dynamics (6) imply that the control is constrained by

$$\zeta x - s \;=\; \alpha \overline{Z} - \beta S = \beta(S_m - S) \; . \tag{35}$$

We analyze the consequences in this section, depending on whether s is constrained at 0, interior ($0 < s < \zeta x$) or constrained at its maximum ($s = \zeta x$).

5.3.1 Constrained atmospheric stock and no capture

If capture is further constrained to be 0, this actually determines the consumption

$$x(t) \;=\; \beta(S_m - S(t)) \; . \tag{36}$$

We call this situation Phase "P". In such a phase, the values of the costate variables can be directly deduced from the first order conditions (8)–(10) and the dynamical system (16)–(17): t^0 being some time instant in the phase, $S^0 = S(t^0)$ and $\lambda_S^0 = \lambda_S(t^0)$,

$$\lambda_Z \;=\; \frac{1}{\zeta}\left(c_x + (c_y - c_x)e^{\rho(t-T)} - u'\left(\frac{\beta}{\zeta}(S_m - S^0 e^{-\beta(t-t^0)})\right)\right) \tag{37}$$

$$\lambda_S \;=\; \lambda_S^0 e^{(\rho+\beta)(t-t^0)} - \beta \int_{t^0}^{t} e^{(\rho+\beta)(t-u)} \lambda_Z(u) \mathrm{d}u \; . \tag{38}$$

Along every optimal path in this phase, the fact that $s(t) = 0$ must imply by (12) that $\gamma_s(t) = c_s + \lambda_Z(t) - \lambda_S(t) \geq 0$.

The first-order condition (10) implies that $x(t) + y(t) \geq \widetilde{y}$ at all times. However, if $x(t)$ is given by (36), this is not possible if $y(t) = 0$ and $S(t) > S_t$, where $S_t = (\zeta/\beta)(\overline{x} - \widetilde{y})$. When this last inequality occurs, we are in a situation where the the ceiling is reached ($Z(t) = \overline{Z}$), no sequestration occurs, but there is mixed consumption of the renewable *and* nonrenewable resource ($x(t) > 0$ and $y(t) > 0$). We call this situation Phase "R". Consumptions are given by:

$$x = \frac{\beta}{\zeta}(S_m - S) \tag{39}$$

$$y = \frac{\beta}{\zeta}(S - S_t) \ . \tag{40}$$

The dynamics of costate variables are integrated explicitly as:

$$\begin{aligned}
\lambda_Z &= \frac{1}{\zeta}(c_y - c_x)(e^{\rho(t-T)} - 1) \\
\lambda_S &= \lambda_S^0 e^{(\rho+\beta)(t-t^0)} - \beta \int_0^t e^{(\rho+\beta)(t-u)} \lambda_Z(u) \mathrm{d}u \\
&= \lambda_S^0 e^{(\rho+\beta)(t-t^0)} + \frac{1}{\zeta}(c_y - c_x) e^{\rho(t-T)}(1 - e^{\beta(t-t^0)}) - \frac{c_y - c_x}{\zeta} \frac{\rho+\beta}{\rho}(1 - e^{(\rho+\beta)(t-t^0)}) \\
\nu_Z &= -\alpha \lambda_Z + \frac{\rho}{\zeta}(c_y - c_x) \ .
\end{aligned}$$

The value of λ_Z is clearly negative for $t \leq T$, which implies that the value of ν_Z is positive.

5.3.2 Constrained atmospheric stock and intermediate capture

Consider the case where the atmospheric pollution ceiling is reached ($Z(t) = \overline{Z}$) and sequestration occurs, but not all emissions are sequestered ($0 < s(t) < \zeta x(t)$). We call this situation Phase "Q".

The condition (9) provides the value of $x(t)$:

$$x = q^d(c_x + \lambda_X - \zeta \lambda_Z) \ . \tag{41}$$

Then the constraint (35) provides the capture:

$$s = \zeta(x - \overline{x}) + \beta S = \zeta x - \beta(S_m - S) \ . \tag{42}$$

The use of the remaining first order condition (8) and the dynamical system leads to the following derivation. First, the first-order condition for s provides the identity:

$$\lambda_S(t) = \lambda_Z(t) + c_s \ . \tag{43}$$

Then, differentiating and using the dynamics on λ_Z, we obtain:

$$\dot{\lambda}_S = \dot{\lambda}_Z = \rho \lambda_Z + (\rho + \beta) c_s \ .$$

Integrating λ_Z, we get:

$$\lambda_Z = \lambda_Z^0 e^{\rho(t-t^0)} + \frac{\rho+\beta}{\rho} c_s (e^{\rho(t-t^0)} - 1) \ . \tag{44}$$

Replacing in $\lambda_S = \lambda_Z + c_s$, we also get:

$$\lambda_S \;=\; \lambda_S^0 e^{\rho(t-t^0)} + \frac{\beta}{\rho} c_s (e^{\rho(t-t^0)} - 1) \;.$$

Finally, we also have the following expressions for ν_Z:

$$\begin{aligned} \nu_Z &= (\rho+\beta)\lambda_S - (\rho+\alpha+\beta)\lambda_Z \\ &= (\rho+\beta)c_s - \alpha\lambda_Z \;=\; (\rho+\alpha+\beta)c_s - \alpha\lambda_S \end{aligned}$$

Let us focus on the form of the function λ_Z. According to (44), this function is increasing if

$$\lambda_Z^0 + \frac{\rho+\beta}{\rho} c_s \;>\; 0 \;. \tag{45}$$

In that case, it is always negative for $t \leq t^0$. In addition, whatever the value of λ_Z^0, we have:

$$\lim_{t\to-\infty} \lambda_Z(t) \;=\; -\frac{\rho+\beta}{\rho} c_s \;.$$

Consequently, assuming that Condition (45) holds, the function $\lambda_Z(t)$ is increasing from $-\frac{\rho+\beta}{\rho} c_s$ (a negative value) to $+\infty$. For each $c > \lambda_Z(-\infty)$, there exists a unique value t_c such that $\lambda_Z(t_c) = c$. If $c \leq \lambda_Z(-\infty)$, there exist no such value.

Under this same assumption, the function $\nu_Z(t)$ is always positive whenever λ_Z is negative: in particular for all $t \leq t^0$. The first-order optimality conditions are therefore all satisfied, and the trajectory is a consistent optimal path.

5.3.3 Constrained atmospheric stock and maximal capture

If one imposes that $Z = \overline{Z}$ and $s = \zeta\overline{x}$ over some time interval, then it follows necessarily that $S(t)$ is constant as well and $S(t) = S_m$. We call this situation Phase "S". The integration of the dynamics of the costate variables yields the following expressions:

$$\lambda_Z(t) \;=\; \frac{\rho+\beta}{\beta}(c_s - \frac{\overline{p}-c_x}{\zeta}) \;+\; \frac{c_y - c_x}{\zeta} e^{\rho(t-T)} \tag{46}$$

$$\lambda_S(t) \;=\; c_s \;-\; \frac{\overline{p}-c_x}{\zeta} \;+\; \frac{c_y - c_x}{\zeta} e^{\rho(t-T)} \;. \tag{47}$$

This in turn provides the value of the multipliers: from the first-order condition $\lambda_S = c_s + \lambda_Z + \gamma_{sx}$, we obtain

$$\gamma_{sx} \;=\; \frac{\rho}{\beta\zeta}(\overline{p} - c_x) \;-\; \frac{\rho+\beta}{\beta} c_s \;=\; \frac{\rho+\beta}{\beta}(\hat{c}_s - c_s) \;. \tag{48}$$

This value is constant over time. It is positive if and only if $c_s \leq \hat{c}_s$, the latter value being defined in (23).

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