Stabilization of an Exploited Fish Population

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Abstract. The purpose of the paper is to design a fishing strategy in order to regulate an exploited fish population. To this end, we use a stage-structured model of a harvested fish population that has been derived in [5]. We give a formula for the fishing effort as a feedback control that allows to stabilize the system.

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1 Introduction

The use of dynamic population models has become common in the study of exploited fish populations. These models are used to provide qualitative and quantitative descriptions of events in various fisheries, to give predictions of the future evolution of a given population and they are also useful for making policy decisions about fisheries.

In this paper we consider the following stage-structured model that has been built in [5]. It includes $(n+1)$ stages represented by their abundance $X_i(t)$, stage 0 being the pre-recuits stage. Each stage is characterized by its fecundity, mortality and predation rates (for more details, see [5]):

$$\left\{\begin{array}{rcl} \dot{X}_0(t) & = & -\alpha X_0(t) - m_0 X_0(t) + \sum_{i=1}^{n} f_i l_i X_i(t) - \sum_{i=0}^{n} p_i X_i(t) X_0(t), \\ \dot{X}_1(t) & = & \alpha X_0(t) - \alpha X_1(t) - m_1 X_1(t), \\ \vdots & & \\ \dot{X}_n(t) & = & \alpha X_{n-1}(t) - \alpha X_n(t) - m_n X_n(t), \end{array}\right. \tag{1}$$

where

- m_i: linear mortality rate.
- α: linear aging coefficient.
- p_0: juvenile competition parameter.
- p_i: predation rate of class i on class 0.

- f_i: fecundity rate of class i.
- l_i: reproduction efficiency of class i.

As suggested in [5], the mortality coefficient can be written as a sum of the natural mortality rate M_i and the fishing mortality coefficient F_i. Hence one can write:

$$m_i = M_i + q_i E$$

where q_i is the catchability of stage i and E is the fishing effort that can be seen as a control term. The goal of this paper is to show how to control this system by acting on the fishing effort E. To this end, we rewrite it as follows:

$$\left\{\begin{array}{lcl} \dot{X}_0(t) & = & -\alpha_0 X_0(t) + \sum_{i=1}^{n} f_i l_i X_i(t) - \sum_{i=0}^{n} p_i X_i(t) X_0(t) \\ \dot{X}_1(t) & = & \alpha X_0(t) - (\alpha_1 + q_1 E(t)) X_1(t) \\ \vdots & & \\ \dot{X}_n(t) & = & \alpha X_n(t) - (\alpha_n + q_n E(t)) X_n(t) \end{array}\right. \tag{2}$$

where

- M_i is the natural mortality rate.
- $\alpha_i = \alpha + M_i$
- q_i is the relative catchability coefficient of class i.
- E is the fishing effort.

Our aim is to construct a feedback control $E = u(X)$ to ensure the survival of the population. More precisely, we shall compute $u(X)$ in such a way that the closed loop system (2) with this feedback has a nontrivial equilibrium state which is globally asymptotically stable.

In the following we shall use these notations:

$\mathbb{R}_+^{n+1} = \{X = (X_0, X_1, \ldots, X_n) \in \mathbb{R}^{n+1} \mid X_i \geq 0 \ , \ i = 0 \ldots n+1\}$.

$\Omega = Int(\mathbb{R}_+^{n+1})$ is the interior of $\mathbb{R}_+^{n+1}$, that is $\Omega = \{X \in \mathbb{R}^{n+1} \mid X_i > 0 \ , \ i = 0 \ldots n+1\}$.

$X(t, X^0)$ is the solution of (2) with initial state X^0, that is $X(0, X^0) = X^0$.

2 Main result

2.1 Stability with a constant fishing effort

Consider the system with a constant fishing effort $\bar{E}$:

$$\begin{cases} \dot{X}_0(t) = -\alpha_0 X_0 + \sum_{i=1}^n f_i l_i X_i(t) - \sum_{i=1}^n p_i X_i(t) X_0(t) - p_0 X_0^2(t) \\ \dot{X}_i(t) = \alpha X_{i-1}(t) - (\alpha_i + q_i \bar{E}) X_i(t) \quad \text{for } i = 1, \ldots, n. \end{cases} \tag{3}$$

The origin is an equilibrium point which corresponds to an extinct population and is therefore not very interesting. Under the following nonlinearity and survival conditions ([5]):

(H1) $\sum_{i=0}^n p_i \neq 0$

(H2) $\sum_{i=1}^n f_i l_i \pi_i > \alpha_0$

where

$$\pi_i = \frac{\alpha^i}{\prod_{j=1}^i (\alpha_i + q_i \bar{E})}$$

system (3) has an other equilibrium X^*

$$\begin{cases} X_0^* = \dfrac{\sum_{i=1}^n f_i l_i \pi_i - \alpha_0}{p_0 + \sum_{i=1}^n p_i \pi_i} \\ X_i^* = \pi_i X_0^*. \end{cases} \tag{4}$$

In ([6], pp 65-74), it has been shown that the positive orthant $\Omega = \{X \in \mathbb{R}^{n+1} \mid X_i > 0 \ , \ i = 0 \dots n+1\}$ is invariant and the following stability results for system (1) has been derived. It has been proved that if assumptions **H1**, **H2** are satisfied and if moreover

(H3) $\quad f_n l_n \neq 0$

(H4) $\quad X_0^* < \mu = \min\limits_{i=1,\dots,n} \left(\dfrac{f_i l_i}{p_i}\right) \quad$ for $\; f_i l_i p_i \neq 0$

then:

(i) the origin is unstable.

(ii) there exists an invariant domain $\mathcal{D} = \prod_{i=1}^n [a_i, b_i]$, with $0 < a_i < b_i$, such that the nontrivial equilibrium X^* belongs to $\mathcal{D}$, X^* is asymptotically stable and its attraction domain contains $\mathcal{D}$.

The proof of **(i)** uses the properties of positives matrices in order to show that the linearized system around the origin has a positive eigenvalue associated to a positive eigenvector. The proof of **(ii)** uses the properties of cooperative systems [4]. The numbers a_i can be chosen as small as one needs but the numbers b_i are bounded by some functions of the parameters f_i, l_i and π_i (see [6] for more details).

Here we shall prove that the attraction domain of X^* is the whole positive orthant Ω. Our proof uses the Lyapunov theory. The advantage of this method is that we get a global result and the Lyapunov function allows us to construct a variable fishing effort as a function of the state of the system and that stabilizes the population at its nontrivial equilibrium.

Proposition 2.1 *Let $\bar{E}$ be any positive constant fishing effort. If assumptions* **(H1)**, **(H2)**, **(H3)** *and* **(H4)** *are satisfied then the state X^* is a globally asymptotically stable equilibrium state for system (3) defined on Ω.*

Proof: Let us consider the following Lyapunov function

$$V(X) = \frac{1}{2}\left((X_0 - X_0^*)^2 + \sum_{i=1}^{n} \Big(\frac{\sum_{j=i}^{n} k_j \pi_j}{\alpha_i + q_i \bar{E}} \Big) \Big(\frac{X_i - X_i^*}{\pi_i} \Big)^2 \right)$$

where

$$k_i = f_i l_i - p_i X_0^* \qquad i = 1, \dots, n$$

V is a positive definite and proper function of X

- $V(X) > 0$ for $X \neq X^*$, and $V(X^*) = 0$
- $\lim_{\|X\| \to +\infty} V(X) = +\infty$

The derivative of V along the solutions of system (3) is

$$\begin{aligned} \dot{V}(X) \quad = \quad & \left(-\alpha_0 X_0 + \sum_{i=1}^{n} f_i l_i X_i - \sum_{i=1}^{n} p_i X_i X_0 - p_0 X_0^2 \right) (X_0 - X_0^*) \\ & + \sum_{i=1}^{n} \Big(\frac{\sum_{j=i}^{n} k_j \pi_j}{\alpha_i + q_i \bar{E}} \Big) \Big(\frac{\alpha X_{i-1} - (\alpha_i + q_i \bar{E}) X_i}{\pi_i} \Big) \Big(\frac{X_i - X_i^*}{\pi_i} \Big) \end{aligned}$$

Since

$$\alpha X_{i-1}^* - (\alpha_i + q_i \bar{E}) X_i^* = 0 \;\; \text{for all} \quad i = 1, \dots, n \tag{5}$$

and

$$-\alpha_0 X_0^* + \sum_{i=1}^{n} f_i l_i X_i^* - \sum_{i=1}^{n} p_i X_i^* X_0^* - p_0 {X_0^*}^2 = 0, \tag{6}$$

we get

$$\begin{aligned}\dot{V}(X) = \; & \left(\sum_{i=0}^{n} k_i (X_i - X_i^*) - \sum_{i=0}^{n} p_i (X_i - X_i^*)(X_0 - X_0^*) \right) (X_0 - X_0^*) \\ & + \sum_{i=1}^{n} \Big(\frac{\sum_{j=i}^{n} k_j \pi_j}{\alpha_i + q_i \bar{E}} \Big) \Big(\frac{\alpha (X_{i-1} - X_{i-1}^*)}{\pi_i} - (\alpha_i + q_i \bar{E}) (\frac{X_i - X_i^*}{\pi_i}) \Big)\end{aligned}$$

where

$$k_0 = -(\alpha_0 + p_0 (X_0 + X_0^*) + \sum_{i=1}^{n} p_i X_i^*).$$

Thus

$$\begin{aligned}&\dot{V}(X) = \Big(k_0 - \sum_{i=1}^{n} p_i (X_i - X_i^*) \Big) (X_0 - X_0^*)^2 + \sum_{i=1}^{n} k_i (X_i - X_i^*)(X_0 - X_0^*) \\ & + \sum_{i=1}^{n} \sum_{j=i}^{n} k_j \pi_j \left(\frac{\alpha}{\alpha_i + q_i \bar{E}} \Big(\frac{X_{i-1} - X_{i-1}^*}{\pi_i} \Big) - \Big(\frac{X_i - X_i^*}{\pi_i} \Big) \right) \Big(\frac{X_i - X_i^*}{\pi_i} \Big)\end{aligned}$$

Taking into account that $\dfrac{\alpha}{(\alpha_i + q_i \bar{E}) \pi_i} = \dfrac{1}{\pi_{i-1}}$, and remarking that

$$k_0 - \sum_{i=1}^{n} p_i (X_i - X_i^*) \leq -\alpha_0 - p_0 X_0^*,$$

it follows

$$\begin{aligned}\dot{V}(X) \;\; \leq \;\; & -(\alpha_0 + p_0 X_0^*)(X_0 - X_0^*)^2 + \sum_{i=1}^{n} k_i (X_i - X_i^*)(X_0 - X_0^*) \\ & + \sum_{i=1}^{n} \left[(\sum_{j=i}^{n} k_j \pi_j) \Big((\frac{X_{i-1} - X_{i-1}^*}{\pi_{i-1}})(\frac{X_i - X_i^*}{\pi_i}) - (\frac{X_i - X_i^*}{\pi_i})^2 \Big) \right]\end{aligned}$$

The inequality $ab \leq \frac{1}{2}(a^2+b^2)$ that holds for any pair of numbers a and b allows to write the inequalities ($\pi_0 = 1$ by convention)

$$
\begin{aligned}
\dot{V}(X) \;\leq\; & -(\alpha_0 + p_0 X_0^*)(X_0 - X_0^*)^2 + \sum_{i=1}^{n} k_i (X_i - X_i^*)(X_0 - X_0^*) \\
& + \sum_{i=1}^{n} \left[\sum_{j=i}^{n} k_j \pi_j \Big(\frac{1}{2} (\frac{X_{i-1} - X_{i-1}^*}{\pi_{i-1}})^2 - \frac{1}{2} (\frac{X_i - X_i^*}{\pi_i})^2 \Big) \right] \\
\leq\; & (-\alpha_0 - p_0 X_0^* + \frac{1}{2} \sum_{i=1}^{n} k_i \pi_i)(X_0 - X_0^*)^2 \\
& + \sum_{i=1}^{n} k_i (X_i - X_i^*)(X_0 - X_0^*) \\
& + \frac{1}{2} \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} k_j \pi_j \Big(\frac{X_i - X_i^*}{\pi_i} \Big)^2 - \frac{1}{2} \sum_{i=1}^{n} \sum_{j=i}^{n} k_j \pi_j \Big(\frac{X_i - X_i^*}{\pi_i} \Big)^2 \\
\leq\; & \Big(-\alpha_0 - p_0 X_0^* + \frac{1}{2} \sum_{i=1}^{n} k_i \pi_i \Big)(X_0 - X_0^*)^2 \\
& + \sum_{i=1}^{n} k_i \pi_i \Big(\frac{X_i - X_i^*}{\pi_i} \Big)(X_0 - X_0^*) - \sum_{i=1}^{n} k_i \pi_i \Big(\frac{X_i - X_i^*}{\pi_i} \Big)^2
\end{aligned}
$$

¿From the expression (4) we check that

$$
-\alpha_0 - p_0 X_0^* + \frac{1}{2} \sum_{i=1}^{n} k_i \pi_i = -\frac{1}{2} \sum_{i=1}^{n} k_i \pi_i .
$$

Thus

$$
\begin{aligned}
\dot{V}(X) \leq & -\frac{1}{2} \sum_{i=1}^{n} k_i \pi_i \left((X_0 - X_0^*)^2 + 2 (\frac{X_i - X_i^*}{\pi_i})(X_0 - X_0^*) - (\frac{X_i - X_i^*}{\pi_i})^2 \right) \\
\leq & -\frac{1}{2} \sum_{i=1}^{n} k_i \pi_i \left((X_0 - X_0^*) - (\frac{X_i - X_i^*}{\pi_i}) \right)^2 \\
\leq & \; 0
\end{aligned}
$$

The equilibrium X^* is then Lyapunov stable for system (3). Note that all solutions of the system are bounded because V is proper and its derivative is non positive. It remains to show the attraction of the equilibrium. Let

$$W = \{X \in \mathbb{R}_+^{n+1} \mid \dot{V}(X) = 0\}$$

According to LaSalle theorem [3], all the solutions starting in Ω tend to L the largest invariant set contained in W. Let $X \in L$, we have

$$\dot{V}(X) = 0 \Leftrightarrow X_0 - X_0^* = \frac{X_1 - X_1^*}{\pi_1} = ... = \frac{X_n - X_n^*}{\pi_n} \tag{7}$$

Substituting (5) in (7) and by using the equalities $\pi_i = \dfrac{\alpha}{\alpha_i + q_i \bar{E}} \pi_{i-1}$, it follows that

$$\alpha X_{i-1} - (\alpha_i + q_i \bar{E}) X_i = 0 \quad \text{for all} \quad i = 1, ..., n.$$

Which implies that for all $i = 1, ..., n$, $\dot{X}_i = 0$ and

$$X_i = \pi_i X_0. \tag{8}$$

Since L is invariant, we get $\dot{X}_0 = 0$, so

$$\dot{X}_0 = -\alpha_0 X_0 + \sum_{i=1}^{n} f_i l_i \pi_i (\frac{X_i}{\pi_i}) - \sum_{i=1}^{n} p_i \pi_i (\frac{X_i}{\pi_i}) X_0 - p_0 X_0^2 = 0.$$

Exploiting equality (8), we deduce that

$$\left(-\alpha_0 + \sum_{i=1}^{n} f_i l_i \pi_i - \sum_{i=1}^{n} p_i \pi_i X_0 - p_0 X_0 \right) X_0 = 0.$$

This implies that $X_0 = 0$ or $X_0 = X_0^*$ and so, by (8) we deduce that $L \subset \{0,\ X^*\}$.

But the origin can not be an omega-limit point: suppose there exists a state $X^0 \in \Omega$ such that the solution $X(t, X^0)$, issued from X^0, tends to the origin as t tends to $+\infty$, then by continuity of V, $V(X(t, X^0)) \longrightarrow V(0)$ as $t \longrightarrow +\infty$. Since V is nonincreasing along the solutions of the system, we have

$$V(X^0) \geq V(X(t, X^0)) \geq V(0),\ \forall t \geq 0. \tag{9}$$

On the other hand, we have

$$\begin{aligned}
V(0) - V(X) &= \frac{1}{2}\left(X_0^{*2} + \sum_{i=1}^{n} \Big(\frac{\sum_{j=i}^{n} k_j \pi_j}{\alpha_i + q_i \bar{E}} \Big) \Big(\frac{X_i^*}{\pi_i} \Big)^2 \right) \\
& -\frac{1}{2}\left((X_0 - X_0^*)^2 + \sum_{i=1}^{n} \Big(\frac{\sum_{j=i}^{n} k_j \pi_j}{\alpha_i + q_i \bar{E}} \Big) \Big(\frac{X_i - X_i^*}{\pi_i} \Big)^2 \right) \\
&= \frac{1}{2}\left(X_0^{*2} + \sum_{i=1}^{n} \gamma_i X_i^{*2} - (X_0 - X_0^*)^2 - \sum_{i=1}^{n} \gamma_i (X_i - X_i^*)^2 \right) \\
&= \frac{1}{2}\left(X_0(2X_0^* - X_0) + \sum_{i=1}^{n} \gamma_i X_i (2X_i^* - X_i) \right).
\end{aligned}$$

where $\gamma_i = \dfrac{\sum_{j=i}^{n} k_j \pi_j}{(\alpha_i + q_i \bar{E})\pi_i^2}$. Hence,
$V(0) > V(X)$ for all $X \in U = \{X \in \mathbb{R}_+^{n+1}\ :\ 0 < X_i < 2X_i^*,\ i = 0, \dots, n\}$.
This is a contradiction to (9). So, we conclude that all the solutions tend to the nontrivial equilibrium X^*.

Remark 2.1 *A sufficient condition to get* $X_0^* < \mu$ *is*

$$\bar{E} \geq \frac{\alpha \sum_{i=i_0}^{n}(f_i l_i - p_i\mu)}{q_{i_0}(p_0\mu + \alpha_0 - \frac{\alpha}{\alpha_1}\sum_{i=1}^{i_0-1}(f_i l_i - p_i\mu))}.$$

with $i_0 = min\{i \in \{1, ...n\} \ / \ q_i \neq 0\}$

In fact for $\bar{E} \geq \frac{\alpha \sum_{i=i_0}^{n}(f_i l_i - p_i\mu)}{q_{i_0}(p_0\mu + \alpha_0 - \frac{\alpha}{\alpha_1}\sum_{i=1}^{i_0-1}(f_i l_i - p_i\mu))}.$ we have

$$(p_0\mu + \alpha_0) \quad \geq \quad \frac{\alpha}{q_{i_0}\bar{E}}\sum_{i=i_0}^{n}(f_i l_i - p_i\mu) + \frac{\alpha}{\alpha_1}\sum_{i=1}^{i_0-1}(f_i l_i - p_i\mu)$$

Taking into account that $\pi_i \leq \frac{\alpha}{q_{i_0}\bar{E}}$ for all $i = i_0, ...n$, and $\pi_i \leq \frac{\alpha}{\alpha_1}$ for all $i = 1, ...i_0 - 1$, we get

$$(p_0\mu + \alpha_0) \quad \geq \quad \sum_{i=1}^{n}(f_i l_i - p_i\mu)\pi_i$$

hence

$$\mu \geq \frac{\sum_{i=1}^{n} f_i l_i \pi_i - \alpha_0}{p_0 + \sum_{i=1}^{n} p_i \pi_i} = X_0^*$$

2.2 Stabilization with a variable fishing effort

Now we suppose that the fishing effort is time varying and we propose to construct it as a state feedback control that allows to stabilize the system around its nontrivial equilibrium. Thus we want to find a smooth function $u(x)$ in such a way that the state X^* is a globally asymptotically stable equilibrium point for the closed-loop system (2) with $E(t) = \bar{E} + u(X(t))$:

$$\left\{ \begin{array}{rcl} \dot{X}_0 & = & -\alpha_0 X_0 + \sum_{i=1}^{n} f_i l_i X_i - \sum_{i=0}^{n} p_i X_i X_0 \\ \dot{X}_1 & = & \alpha X_0 - (\alpha_1 + q_1 \bar{E}) X_1 - q_1 u X_1 \\ \vdots & & \\ \dot{X}_n & = & \alpha X_n - (\alpha_n + q_n \bar{E}) X_n - q_n u X_n \end{array} \right. \tag{10}$$

In a condensed form this system can be written:

$$\dot{X} = F(X) + uG(X), \tag{11}$$

where

$$F(X) = \begin{pmatrix} -\alpha_0 X_0 + \sum_{i=1}^{n} f_i l_i X_i - \sum_{i=0}^{n} p_i X_i X_0 \\ \alpha X_0 - (\alpha_1 + q_1 \bar{E}) X_1 \\ \vdots \\ \alpha X_n - (\alpha_n + q_n \bar{E}) X_n \end{pmatrix}, \quad G(X) = \begin{pmatrix} 0 \\ -q_1 X_1 \\ \vdots \\ -q_n X_n \end{pmatrix}.$$

To stabilize this system we shall use and adapt the Jurdjevic-Quinn [1] stabilization procedure:

A candidate stabilizer is $u = \Phi(X) = -\langle G(X), \nabla V(X) \rangle$ where $\langle ., . \rangle$ is a scalar product on $\mathbb{R}^{n+1}$ and

$$\nabla V(X) = \begin{pmatrix} \frac{\partial V}{\partial X_0} \\ \frac{\partial V}{\partial X_1} \\ \vdots \\ \frac{\partial V}{\partial X_n} \end{pmatrix}.$$

Using the Lyapunov function introduced above,we get:

$$\Phi(X) = \sum_{i=1}^{n} \gamma_i q_i X_i (X_i - X_i^*). \tag{12}$$

Where

$$\gamma_i = \frac{\sum_{j=i}^{n} k_j \pi_j}{(\alpha_i + q_i \bar{E})\pi_i^2}.$$

The derivative of V along the trajectories of the closed loop system is:

$$\begin{aligned}
\dot{V} &= \langle F(X), \nabla V(X)\rangle + u(X)\langle G(X), \nabla V(X)\rangle \\
&= \langle F(X), \nabla V(X)\rangle - \Phi(X)^2 \\
&\leq -\frac{1}{2}\sum_{i=1}^{n} k_i \pi_i \left((X_0 - X_0^*) - (\frac{X_i - X_i^*}{\pi_i}) \right)^2 - \Phi(X)^2 \\
&\leq 0
\end{aligned}$$

As in section 2.1, LaSalle Invariance Principle allows to conclude that X_0^* is globally asymptotically stable.

However, the function Φ is unbounded and takes positive as well as negative values, hence $E = \bar{E} + u(X)$ can take negative values which has no sense (E is a fishing effort). Thus, instead of using the feedback given by the formula (12), we use the following bounded function:

$$u(X) = \frac{2\Phi(X)}{1 + \Phi^2(X)} \bar{E}. \tag{13}$$

We then have $|u(X)| \leq \bar{E}$ which ensures that $0 \leq E(X) \leq 2\bar{E}$ for all $X \in \Omega$.

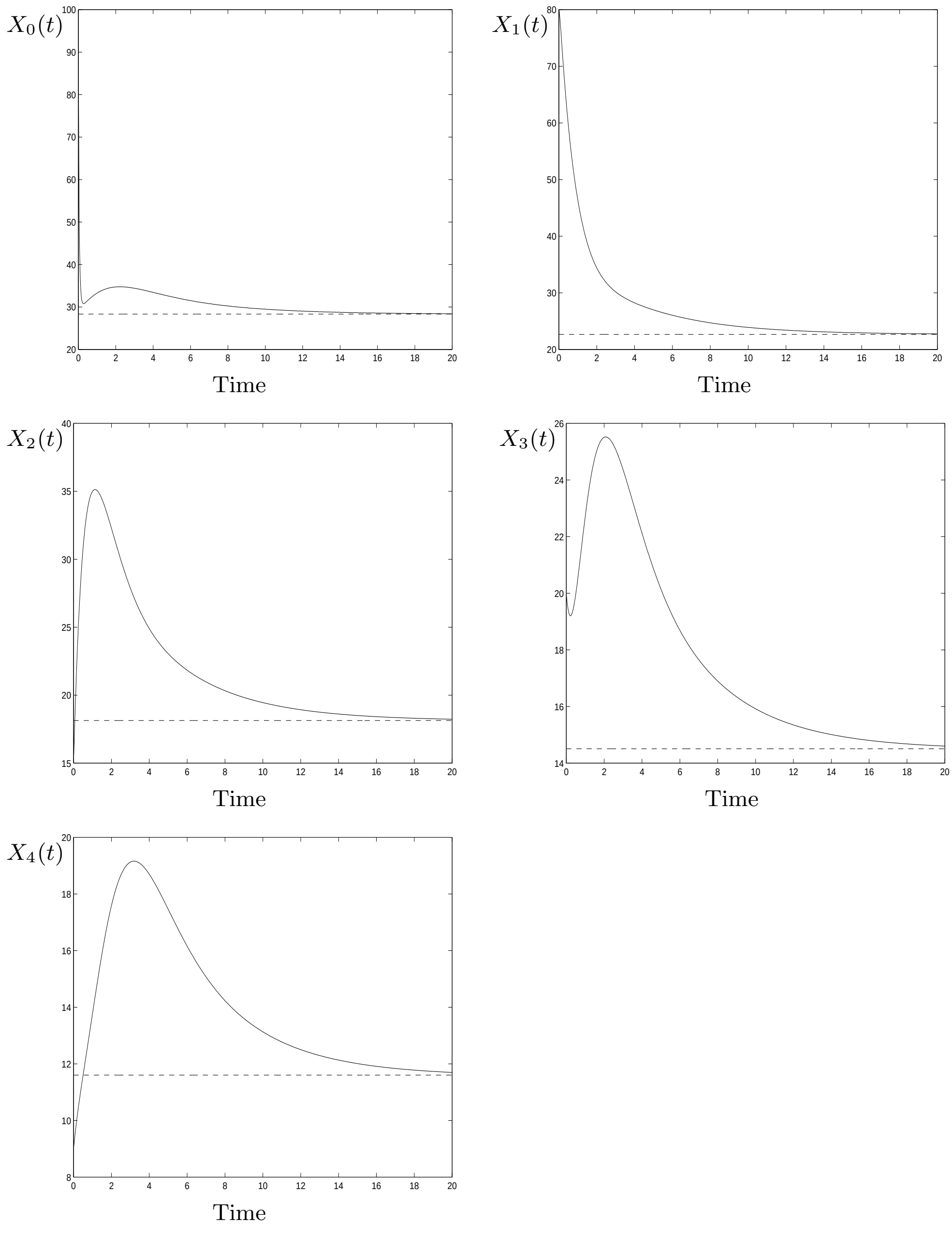
3 Remarks and simulations

1. The results of [5, 6] ensures the attraction of the equilibrium only for initial states that belong to $D = \prod_{i=0}^{n}[a_i, b_i]$. Our result proves that the attraction domain is the whole positive orthant. Here we give a simulation corresponding to a harvested fish population characterized by the parameter values given in table1, which are checked on from [5].

Stage i:	0	1	2	3	4
p_i	0.2	0	0.1	0.1	0.1
f_i		0	0.5	0.5	0.5
l_i		0	10	20	15
m_i	0.5	0.2	0.2	0.2	0.2
q_i	0	0	0	0.1	0.15
M_i	0.5	0.2	0.2	0.1	0.05
α	0.8				
α_i	1.3	1	1	0.9	0.85
$\bar{E}$	1				
$\alpha_i + q_i\bar{E}$	1.3	1	1	1	1

Table1: The parameter values of a harvested fish population.

It is clear that this parameter values satisfy hypotheses **(H1)**, **(H2)**, **(H3)** and **(H4)**. For these parameters we have $b_0 = 50$, $b_1 = 40$, $b_2 = 32$, $b_3 = 17.06$, $b_4 = 10.9$. For the initial state $X_{init} = (100, 80, 15, 20, 9)$ which is outside the domain D, we obtain the following curves that confirm our results.

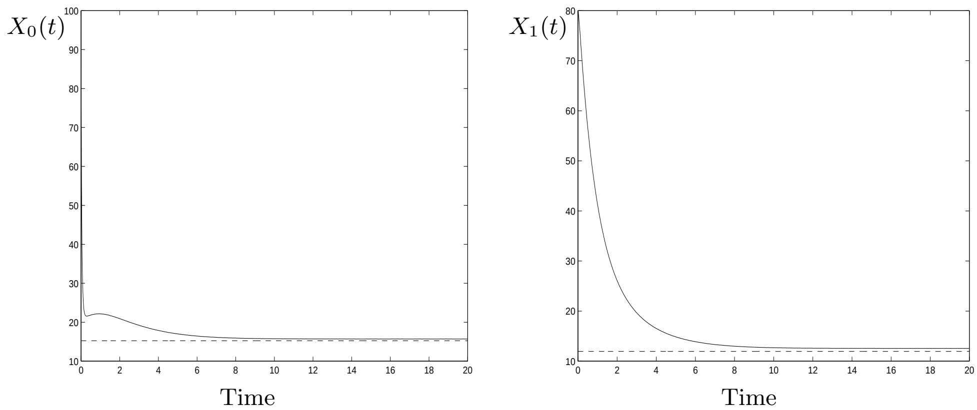


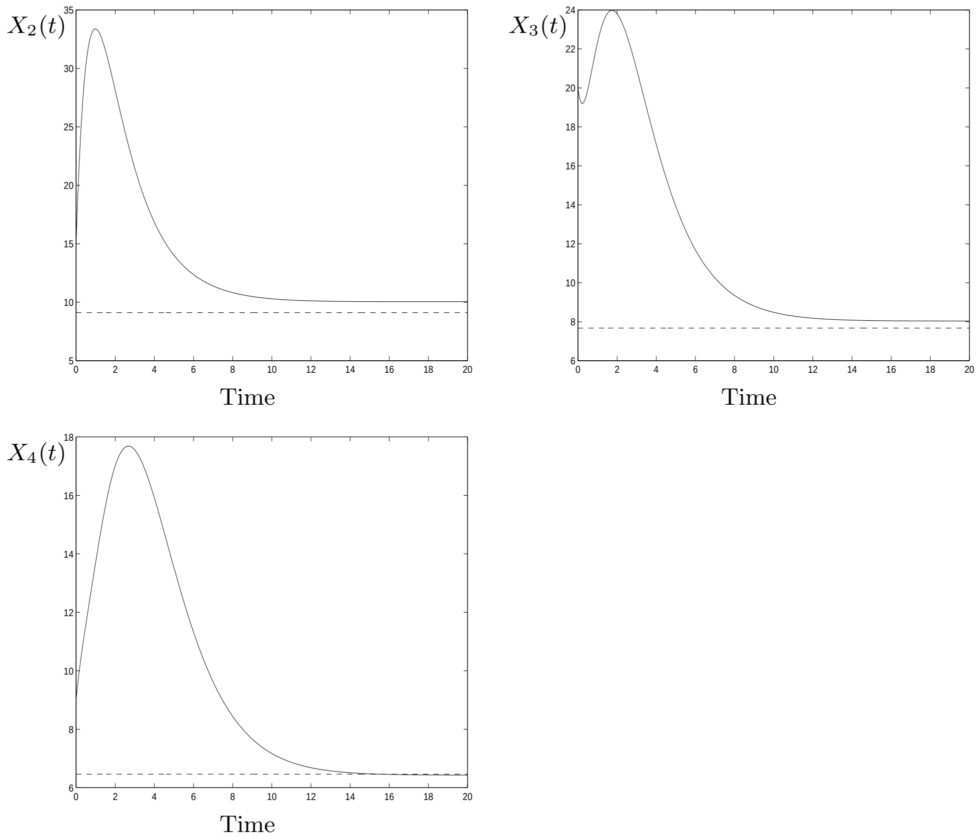
$X_0(t)$
Time
$X_1(t)$
Time
$X_2(t)$
Time
$X_3(t)$
Time
$X_4(t)$
Time

2. Our proof and the one of [5, 6] use the condition $X_0^* < \mu$. But we think that this condition is not necessary as it is suggested by the following example:

Stage i:	0	1	2	3	4
p_i	0.2	0	0.1	0.1	0.8
f_i		0	0.5	0.5	0.5
l_i		0	10	20	15
m_i	0.5	0.2	0.2	0.2	0.2
q_i	0	0.1	0.25	0.25	0.5
M_i	0.5	0.2	0.2	0.1	0.05
α	0.8				
α_i	1.3	0.98	0.95	0.95	0.9
$\bar{E}$	0.2				
$\alpha_i + q_i\bar{E}$	1.3	1	1	1	1

For these parameters $X_0^* = 15.2495$, $\mu = 9.375$. We have $X_0^* > \mu$. However the simulations show the attraction of the equilibrium X^*.





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