Modeling the grand piano

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Abstract

A global model of a piano is presented. Its aim is to reproduce the main vibratory and acoustic phenomena involved in the generation of a piano sound from the initial blow of the hammer against the strings to the radiation from soundboard to the air. One first originality of the work is due to the string model which takes both geometrical nonlinear effects and stiffness into account. Other significant improvements are due to the combined modeling of the three main couplings between the constitutive parts of the instrument: hammer-string, string-soundboard and soundboard-air coupling.

1 Introduction

Simplifying assumptions were made in the model. The hammer is supposed to be perfectly aligned with the strings. The agraffe is assumed to be rigidly fixed. Both the string-soundboard and soundboard-air couplings are lossless. The soundboard is considered as simply supported along its edge, and the "listening" room is anechoic with no obstacle except the piano itself. The action of the mechanism prior to the shock of the hammer against the strings is ignored: the tone starts when the hammer hits the strings with an imposed velocity.

The physical parameters of hammers, strings and soundboards included in the model are obtained from standard string scaling and geometrical data from manufacturers, and complemented with our own measurements. For the losses in materials, approximate models based on experimental data are used. The numerical formulation of the model is based on a discrete formulation of the global energy of the system, which ensures stability (see [1]). This requires that the continuous energy of the problem is decaying with time. The global model of the piano is thus written according to this condition.

2 Strings

The string model accounts for large deformations, inducing geometrical nonlinearities, and intrinsic stiffness. The governing equations correspond to those of a nonlinear Timoshenko beam under axial tension. For the end conditions, we assume zero displacement (in both transverse and longitudinal directions) and zero moment at the agraffe. At the bridge, the end conditions are consecutive to coupling with the soundboard. The string is considered at rest at the origin of time. A source term accounts for the action of the hammer against the strings. A simple viscoelastic model accounts globally and approximately for the damping effects. The coefficients of this model are determined from measured sounds for each string, through comparisons between simulated and measured spectrograms. The global energy of this string model is preserved, under the condition $EA > T_0$ where E is the Young's modulus, A is the cross-sectional area of the string and T_0 its tension. This condition is always fulfilled in piano strings.

3 Hammer

The hammer's center of gravity is supposed to be moving along a straight line orthogonal to the strings at rest. The interaction force between the hammer and one string of a given note is distributed on a small portion of the string, through a spreading function localized around the impact point, and oriented in the transversal direction. The interaction force depends on the distance $d(t)$ between the hammer and the string: if $d(t)$ is larger than the mean hammer displacement $\overline{\xi}$, there is no contact and the force is zero. If $d(t) \leq \overline{\xi}$, the force is a function of the distance. According to previous studies, we define the function:

$$\Phi(d) = \left[\left(\overline{\xi} - d \right)^{+} \right]^{p} \tag{1}$$

where $(\cdot)^{+}$ means "positive part of", and where p is a real positive nonlinear exponent. In practive, this coefficient varies between 1.5 and 3.5. In order to account further for the observed hysteretic behavior of the felt, a dissipative term is added in the expression of the force [2].

4 Soundboard

It is assumed that the only vibrating element is the soundboard, all other parts (rim, keybed, lid, iron frame...) being assumed to be perfectly rigid. A bidimensional Reissner-Mindlin plate model is considered. The bridge and ribs are considered as heterogeneities, and the orientation of the orthotropy axes can be space dependent. As a consequence, the density, thickness and elastic coefficients are functions of space. The soundboard is assumed to be simply supported on its edge. Finally, a source term is imposed in the transverse vertical direction. This term accounts for both the string's tension at the bridge and the air pressure jump. A modal approach has been adopted where the modal damping can be adjusted, mode by mode. This method is justified as long as the damping factor is small compared to the eigenfrequency and requires also that the modes are sufficiently well separated, a condition that is only strictly valid for the piano below 1 to 2 kHz [3]. The modal amplitudes are then solution of second-order uncoupled damped oscillators equations. Again, it is possible to exhibit an energy decaying with time for this part of the system.

5 Strings-soundboard coupling

A plausible, though not fully validated, model is used for the transformation of string longitudinal motion to bridge transverse motion. It is based on the observation that the strings form a slight angle with the horizontal plane due to both bridge height and soundboard curvature. It is assumed that the bridge moves in the vertical direction. When the hammer strikes the strings, it gives rise to a transversal wave which, in turn, induces a longitudinal wave, because of nonlinear geometrical coupling. The longitudinal wave travels 10 to 20 times faster than the transverse one, and comes first at the bridge. The resulting variation of tension is oriented in the direction of the string. Because of the angle formed by the string with the horizontal plane, this induces a vertical component of the longitudinal force at the bridge, in addition to the transverse force. The total bridge force is distributed in space in the soundboard by means of a rapidly decreasing regular function centered on the point where the string is attached on the soundboard. The associated kinematic boundary conditions are the continuity of string and soundboard velocities in the vertical direction, and the nullity of the velocity in the horizontal direction.

6 Soundboard-air coupling

For the propagation of piano sounds in free space, the rim is considered to be a rigid obstacle. The acoustic velocity and pressure are solutions of the linearized Euler equations in the unbounded domain which excludes the rim and the plate. Viscothermal losses in the air are ignored. The normal component of the acoustic velocity vanishes on the rim. The coupling between the 3D sound field and the vibrating soundboard obeys to the condition of continuity of the velocity normal components. Finally, the soundboard force is the pressure jump across both sides the plate. Again, this vibroacoustic system satisfies a property of energy decay.

Conclusion

This piano model accounts for the phenomena consecutive to amplitude dependence of string vibrations: presence of precursors [4], time-evolution of eigenfrequencies, transverse-longitudinal coupling and phantom partials. Due to the string-coupling, the presence of soundboard modes in the transients are reproduced in a natural way. Finally, due to the integration of ribs and bridges, the influence of soundboard modifications on the radiated sound can be investigated systematically.

References

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