Modeling the piano. Numerical Aspects.

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Modeling the grand piano. Numerical Aspects.

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Abstract

This paper deals with the discretization of the global piano model described in [2]. We have to solve a complex system of coupled equations, where each subsystem has different spatial dimensions, which poses specific difficulties. The hammer-strings part is a 1D system governed by nonlinear equations. The soundboard is a 2D system with diagonal damping. The acoustic field is a 3D problem in an unbounded domain. Energy based methods allow to build an accurate and *a priori* stable scheme.

1 Introduction

The nonlinear parts of the problem (hammer-strings interaction, string vibration), the couplings between the subsystems and, more generally, the size of the problem in terms of computational burden, requires to guarantee the long-term numerical stability. In the context of wave equations, and in musical acoustics particularly, a classical and efficient technique to achieve this goal is to design numerical schemes based on the formulation of a discrete energy which is either constant or decreasing with time (see [7], [5]). Ensuring the positivity of the discrete energy, consistent with the continuous energy of the physical system, yields to a priori estimates for the unknowns of the problem, leading to the stability of the method. For most numerical schemes this imposes a restriction on the discretization parameters, as, for example, an upper bound for the time step.

In the discrete formulation, the coupling terms need a specific handling in order to guarantee a simple energy transfer, without any artificial introduction of dissipation or instabilities. Our choice here is to consider discrete coupling terms that cancel each other when computing the complete energy. In total, this method yields centered implicit couplings between the unknowns of the subsystems. The order of accuracy of the method is preserved, compared to the order of each subsystem taken independently, with no additional stability condition.

In view of the diversity of the various problems encountered in the full piano model, different discretization methods are chosen for each subsystem and for the coupling terms. We focus in the presentation on a general survey on the numerical resolution and on its main difficulties.

2 Strings

Standard high-order finite elements are used for the space discretization of the nonlinear system of equations that govern the vibrations of the strings. The spatial discretization parameters (mesh size and polynomial order) are selected to ensure a small numerical dispersion in the audio range. The time discretization of the strings system is probably the most novel and innovative method used in our piano numerical formulation. It combines a new scheme for nonlinear systems developped in [3], based on the expression of a discrete gradient, which ensures the conservation of an energy and an improved time discretisation for Timoshenko systems developped in [1]. A 1D nonlinear system must be solved at each time step. The solution is computed via an iterative modified Newton-Raphson method which needs the evaluation of both the scheme and its Jacobian with respect to the unknowns. It can be shown that a discrete energy is decaying, after extinction of the source. The stability of the numerical scheme can be derived from this property, with condition on the time step.

3 Hammer-strings coupling

Since the displacement of the hammer is a scalar function of time, we choose to solve the hammer-strings system by considering all together the unknowns of every strings belonging to the considered note, plus the hammer scalar unknown.The nonlinear hammer-strings interacting force is treated in a centered conservative way. A global discrete energy is shown to be decaying with respect to time when the hammer is given with an initial velocity.

4 Soundboard

The soundboard model assumes a diagonal damping in the modal basis. Its motion is first decomposed onto the modes of the undamped Reissner-Mindlin system belonging to the audio range, after semi-discretization in space with high-order finite elements as in [5]. These modes are only computed once for all, before starting the time iterations. This procedure yields decoupled equations which can be solved analytically in time, without introducing any additional approximation or numerical dispersion. The energy identity over time of the semi-discrete problem is also exactly satisfied with this method. However, one drawback of this choice is the loss of the local nature of the couplings with strings and air.

5 Strings-soundboard coupling at the bridge

The discrete formulation of the strings-soundboard continuity equations must ensure the stability of the resulting scheme, which couples the implicit three points nonlinear strings scheme described in 2 with the time semi-analytic soundboard model described in 4. New variables are introduced that represent the coupling forces associated to the conditions between strings and soundboard expressing the velocity continuity at the bridge. The strings and soundboard unknowns are evaluated on interleaved time grids : $\{n\,\Delta t\}$ for the strings, and $\{(n+1/2)\,\Delta t\}$ for the soundboard. The forces at the bridge are considered to be constant on time intervals of the form $[(n-1/2)\Delta t,\,(n+1/2)\Delta t]$. The discrete coupling condition is implicit and centered on times $n\,\Delta t$. Due to the linearity of the soundboard model, it is possible to express the soundboard unknowns as linear functions of the forces at the bridge. Thanks to this property, it is possible to perform Schur complements on the system which, originally, is globally implicit. An algorithm is then writen which updates first the unknowns of the strings and the forces at the bridge, and, in a second step, updates the unknowns of the soundboard.

6 Acoustic propagation and structural acoustics

The artifical truncation of the acoustic domain is done with with Perfectly Matched Layers [6]. The acoustical problem is solved in space with high-order finite elements and in time with an explicit leap-frog scheme, in view of the large number of degrees of freedom to consider. The acoustic velocity and pressure unknowns are calculated at times $\{n\,\Delta t\}$ and $\{(n+1/2)\,\Delta t\}$, respectively. In the variational formulation, the coupling between soundboard and air appears as source terms for the soundboard and the sound pressure equations. These terms are constructed in the discrete scheme so that they vanish when computing the energy, centered at times $n\,\Delta t$. An implicit coupling exists between the soundboard displacement and the acoustic pressure in the vicinity of the plate, which implies a change of basis between the physical and the modal representations of the soundboard. Due to the linearity of the equations, it is possible here to perform Schur complements, and to write an efficient algorithm that updates separately the plate (with a semi-analytic method) and the air variables (with the leap-frog scheme).

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