State Feedback Control Design Using Eigenstructure Decoupling

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Abstract: In this paper the design of controlling a class of linear systems via state feedback eigenstructure assignment is investigated. The design aim is to synthesize a state feedback control law such that for prescribed eigenvalues of the closed-loop control system corresponding eigenvectors are as close to decoupled ones as possible. The set of parametric vectors and the set of closed-loop eigenvalues represent the degrees of freedom existing in the control design, and can be further properly chosen to meet some desired specification requirement, such as mode decoupling and robustness. An illustrative example and the simulation results show that the proposed parametric method is effective and simple.

Keywords: Mode decoupling, singular value decomposition, state feedback, linear control systems, eigenstructure assignment.

1. INTRODUCTION

The static and the dynamic pole placement belongs to the prominent design problems of modern control theory, and, although its practical usefulness has been continuously in dispute, it is one of the most intensively investigated in control system design. It seems that the state-feedback pole assignment in control system design is one from the preferred techniques. In the single-input case the solution to this problem, when it exists, is unique. In the multi-input multi output (MIMO) case various solutions may exist (Filasová (1999), Ipsen (2009)), and to determine a specific solution additional conditions have to be supplied in order to eliminate the extra degrees of freedom in design strategy.

In last significant progress has been achieved in this field, coming in its formulation closest to the algebraic geometric nature of the pole placement problem (Kautsky et al. (1985), Wonham (1985)). The reason for the discrepancy in opinions about the conditioning of the pole assignment problem is that one has to distinguish among three aspects of the pole placement problem, the computation of the memoryless feedback control law matrix gain, the computation of the closed loop system matrix eigenvalues spectrum and the suppressing of the cross-coupling effect (Wang (2003)), where one manipulated input variable cause change in more outputs variables .

Thus, eigenstructure assignment seems to be a powerful technique concerned with the placing of eigenvalues and their associated eigenvectors via feedback control laws, to meet closed-loop design specifications. The eigenvalues are the principal factors that govern the stability and the rates of decay or rise of the system dynamic response. The right and left eigenvectors, on the other hand, are dual factors that together determine the relative shape of the system dynamic response (Kocsis and Krokavec (2008), Sobel and Lallman (1989)).

The general problem of assigning the system matrix eigenstructure using the state feedback control is considered in this paper. Based on the classic algebraic methods (Golub and Van Loan (1989), Datta (2004), Poznyak (2008)), as well as on the algorithms for pole assignment using Singular Value Decomposition (SVD) (Filasová (1997), Krokavec and A. Filasová (2006)) the exposition of the pole eigenstructure assignment problem is generalized here to handle the specified structure of the left eigenvector set in state feedback control design for MIMO linear systems. Extra freedom, which makes dependent the closed-loop eigenvalues spectrum, is used for closed-loop state variables mode decoupling.

The integrated procedure provides a straightforward methodology usable in linear control system design techniques when the memory-free controller in the state-space control structures takes the standard form. Presented application for closed-loop state variables mode decoupling is relative simple and its worth can help to disclose the continuity between eigenstructure assignment and system variable dominant dynamic specification.

2. PROBLEM STATEMENT

Linear dynamic systems with n degree of freedom can be modelled by the state-space equations

$$\dot{\boldsymbol{q}}(t) = \boldsymbol{A}\boldsymbol{q}(t) + \boldsymbol{B}\boldsymbol{u}(t) \tag{1}$$

$$\boldsymbol{y}(t) = \boldsymbol{C}\boldsymbol{q}(t) \tag{2}$$

with constant matrices $\boldsymbol{A} \in \mathbb{R}^{n\times n}$, $\boldsymbol{B} \in \mathbb{R}^{n\times r}$, and $\boldsymbol{C} \in \mathbb{R}^{m\times n}$. Generally, to the controllable time-invariant linear MIMO system (1) a linear state feedback regulator control law, defined generally as

$$\boldsymbol{u}(t) = -\boldsymbol{K}\boldsymbol{q}(t) + \boldsymbol{L}\boldsymbol{w}(t) \tag{3}$$

with $\boldsymbol{K} \in \mathbb{R}^{r\times n}$, $\boldsymbol{L} \in \mathbb{R}^{r\times m}$ gives rise to the closed-loop system

$$\dot{\boldsymbol{q}}(t) = \boldsymbol{A}_c\boldsymbol{q}(t) + \boldsymbol{B}\boldsymbol{L}\boldsymbol{w}(t) \tag{4}$$

which closed loop poles are eigenvalues of matrix $\boldsymbol{A}_c = (\boldsymbol{A} - \boldsymbol{B}\boldsymbol{K})$ and $\boldsymbol{A}_c \in \mathbb{R}^{n\times n}$.

Throughout the paper it is assumed the pair $(\boldsymbol{A}, \boldsymbol{B})$ is controllable.

3. BASIS PRELIMINARIES

3.1 *Orthogonal Complement*

Definition 1. (Null space) Let $\boldsymbol{E}$, $\boldsymbol{E} \in \mathbb{R}^{h\times h}$, $\mathrm{rank}(\boldsymbol{E}) = k < h$ be a rank deficient matrix. Then the null space $\mathcal{N}_{\boldsymbol{E}}$ of $\boldsymbol{E}$ is the orthogonal complement of the row space of $\boldsymbol{E}$.

Proposition 1. Let $\boldsymbol{E}$, $\boldsymbol{E} \in \mathbb{R}^{h\times h}$, $\mathrm{rank}(\boldsymbol{E}) = k < h$ be a rank deficient matrix. Then an orthogonal complement $\boldsymbol{E}^{\perp}$ of $\boldsymbol{E}$ is

$$\boldsymbol{E}^{\perp} = \boldsymbol{D}\boldsymbol{U}_2^T \tag{5}$$

where $\boldsymbol{U}_2^T$ is the null space of $\boldsymbol{E}$ and $\boldsymbol{D}$ is an arbitrary matrix of appropriate dimension.

Proof. (Filasová and Krokavec (2010b)) The SVD of $\boldsymbol{E}$, $\boldsymbol{E} \in \mathbb{R}^{h\times h}$, $\mathrm{rank}(\boldsymbol{E}) = k < h$ gives

$$\boldsymbol{U}^T\boldsymbol{E}\boldsymbol{V} = \begin{bmatrix}\boldsymbol{U}_1^T\\ \boldsymbol{U}_2^T\end{bmatrix}\boldsymbol{E}\begin{bmatrix}\boldsymbol{V}_1 & \boldsymbol{V}_2\end{bmatrix} = \begin{bmatrix}\boldsymbol{\Sigma}_1 & \boldsymbol{0}_{12}\\ \boldsymbol{0}_{21} & \boldsymbol{0}_{22}\end{bmatrix} \tag{6}$$

where $\boldsymbol{U}^T \in \mathbb{R}^{h\times h}$ is the orthogonal matrix of the left singular vectors, $\boldsymbol{V} \in \mathbb{R}^{h\times h}$ is the orthogonal matrix of the right singular vectors of $\boldsymbol{E}$ and $\boldsymbol{\Sigma}_1 \in \mathbb{R}^{k\times k}$ is the diagonal positive definite matrix

$$\boldsymbol{\Sigma}_1 = \mathrm{diag}\begin{bmatrix}\sigma_1 & \cdots & \sigma_k\end{bmatrix}, \quad \sigma_1 \geq \cdots \geq \sigma_k > 0 \tag{7}$$

which diagonal elements are the singular values of $\boldsymbol{E}$. Using orthogonal properties of $\boldsymbol{U}$ and $\boldsymbol{V}$, i.e. $\boldsymbol{U}^T\boldsymbol{U} = \boldsymbol{I}_h$, $\boldsymbol{V}^T\boldsymbol{V} = \boldsymbol{I}_h$, $\boldsymbol{U}_2^T\boldsymbol{U}_1 = \boldsymbol{0}$, then

$$\begin{aligned}\boldsymbol{E} = \boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{V}^T = \begin{bmatrix}\boldsymbol{U}_1 & \boldsymbol{U}_2\end{bmatrix}\begin{bmatrix}\boldsymbol{\Sigma}_1 & \boldsymbol{0}_{12}\\ \boldsymbol{0}_{21} & \boldsymbol{0}_{22}\end{bmatrix}\begin{bmatrix}\boldsymbol{V}_1^T\\ \boldsymbol{V}_2^T\end{bmatrix} = \\ = \begin{bmatrix}\boldsymbol{U}_1 & \boldsymbol{U}_2\end{bmatrix}\begin{bmatrix}\boldsymbol{S}_1\\ \boldsymbol{0}_2\end{bmatrix} = \boldsymbol{U}_1\boldsymbol{S}_1\end{aligned} \tag{8}$$

where $\boldsymbol{S}_1 = \boldsymbol{\Sigma}_1\boldsymbol{V}_1^T$. Thus, (8) implies

$$\boldsymbol{U}_2^T\boldsymbol{E} = \boldsymbol{U}_2^T\begin{bmatrix}\boldsymbol{U}_1 & \boldsymbol{U}_2\end{bmatrix}\begin{bmatrix}\boldsymbol{S}_1\\ \boldsymbol{0}_2\end{bmatrix} = \boldsymbol{0} \tag{9}$$

It is evident that for an arbitrary matrix $\boldsymbol{D}$ is

$$\boldsymbol{D}\boldsymbol{U}_2^T\boldsymbol{E} = \boldsymbol{E}^{\perp}\boldsymbol{E} = \boldsymbol{0} \tag{10}$$

respectively, which implies (5). ■

3.2 *System Model Canonical Form*

Proposition 2. If $\mathrm{rank}(\boldsymbol{C}\boldsymbol{B}) = m$ then there exists a coordinates change in which $(\boldsymbol{A}^{\circ}, \boldsymbol{B}^{\circ}, \boldsymbol{C}^{\circ})$ takes the structure

$$\boldsymbol{A}^{\circ} = \begin{bmatrix}\boldsymbol{A}_{11}^{\circ} & \boldsymbol{A}_{12}^{\circ}\\ \boldsymbol{A}_{21}^{\circ} & \boldsymbol{A}_{22}^{\circ}\end{bmatrix}, \ \boldsymbol{B}^{\circ} = \begin{bmatrix}\boldsymbol{0}\\ \boldsymbol{B}_2^{\circ}\end{bmatrix}, \ \boldsymbol{C}^{\circ} = \begin{bmatrix}\boldsymbol{0} & \boldsymbol{I}_m\end{bmatrix} \tag{11}$$

where $\boldsymbol{A}_{11}^{\circ} \in \mathbb{R}^{(n-m)\times(n-m)}$, $\boldsymbol{B}_2^{\circ} \in \mathbb{R}^{m\times m}$ is a non-singular matrix, and $\boldsymbol{I}_m \in \mathbb{R}^{m\times m}$ is identity matrix.

Proof. (Filasová and Krokavec (2010a)) Considering the state-space description of the system (1), (2) with $r = m$ and defining the transform matrix $\boldsymbol{T}_1^{-1}$ such that

$$\boldsymbol{C}_1 = \boldsymbol{C}\boldsymbol{T}_1 = \begin{bmatrix}\boldsymbol{0} & \boldsymbol{I}_m\end{bmatrix}, \ \boldsymbol{T}_1^{-1} = \begin{bmatrix}\boldsymbol{I}_{n-m} & \boldsymbol{0}\\ \multicolumn{2}{c}{\boldsymbol{C}}\end{bmatrix} \tag{12}$$

then

$$\boldsymbol{B}_1 = \boldsymbol{T}_1^{-1}\boldsymbol{B} = \boldsymbol{T}_1^{-1}\begin{bmatrix}\boldsymbol{B}_1\\ \boldsymbol{B}_2\end{bmatrix} = \begin{bmatrix}\boldsymbol{B}_1\\ \boldsymbol{CB}\end{bmatrix} = \begin{bmatrix}\boldsymbol{B}_{11}\\ \boldsymbol{B}_{12}\end{bmatrix} \quad (13)$$

If $\boldsymbol{CB} = \boldsymbol{B}_{12}$ is a regular matrix (in opposite case the pseudoinverse of $\boldsymbol{B}_{12}$ is possible to use), then the second transform matrix $\boldsymbol{T}_2^{-1}$ can be defined as follows

$$\boldsymbol{T}_2^{-1} = \begin{bmatrix}\boldsymbol{I}_{n-m} & -\boldsymbol{B}_{11}\boldsymbol{B}_{12}^{-1}\\ \boldsymbol{0} & \boldsymbol{I}_m\end{bmatrix} \quad (14)$$

$$\boldsymbol{T}_2 = \begin{bmatrix}\boldsymbol{I}_{n-m} & \boldsymbol{B}_{11}\boldsymbol{B}_{12}^{-1}\\ \boldsymbol{0} & \boldsymbol{I}_m\end{bmatrix} \quad (15)$$

This results in

$$\boldsymbol{B}^{\circ} = \boldsymbol{T}_2^{-1}\boldsymbol{B}_1 = \begin{bmatrix}\boldsymbol{0}\\ \boldsymbol{B}_2^{\circ}\end{bmatrix} \quad (16)$$

where

$$\boldsymbol{B}_{11} = \boldsymbol{B}_1, \quad \boldsymbol{B}_2^{\circ} = \boldsymbol{B}_{12} = \boldsymbol{CB} \quad (17)$$

and

$$\boldsymbol{C}^{\circ} = \boldsymbol{C}_1\boldsymbol{T}_2 = \begin{bmatrix}\boldsymbol{0} & \boldsymbol{I}_m\end{bmatrix}\boldsymbol{T}_2 = \begin{bmatrix}\boldsymbol{0} & \boldsymbol{I}_m\end{bmatrix} \quad (18)$$

Finally, with $\boldsymbol{T}_c^{-1} = \boldsymbol{T}_2^{-1}\boldsymbol{T}_1^{-1}$ it yields

$$\boldsymbol{A}^{\circ} = \boldsymbol{T}_c^{-1}\boldsymbol{A}\boldsymbol{T}_c = \boldsymbol{T}_2^{-1}\boldsymbol{T}_1^{-1}\boldsymbol{A}\boldsymbol{T}_1\boldsymbol{T}_2 \quad (19)$$

Thus, (16), (18), and (19) implies (11). This concludes the proof. ■

Note, the structure of $\boldsymbol{T}_1^{-1}$ is not unique and others can be obtained by permutations of the first $n-m$ rows in the structure defined in (12).

3.3 *System Modes Properties*

Proposition 3. Given system eigenstructure with distinct eigenvalues then for $j, k \in \{1, 2, \dots n\}, l \in \{1, 2, \dots m\}$, $m = r$

i. the k-th mode $(s-s_k)$ is unobservable from the l-th system output if the l-th row of matrix $\boldsymbol{C}$ is orthogonal to the k-th eigenvector of the closed-loop system matrix $\boldsymbol{A}_c$, i.e. with $j \neq k$

$$\boldsymbol{c}_l^T\boldsymbol{n}_k = \boldsymbol{n}_j^T\boldsymbol{n}_k = 0, \quad \boldsymbol{C}^T = \begin{bmatrix}\boldsymbol{c}_1 & \cdots & \boldsymbol{c}_m\end{bmatrix} \quad (20)$$

ii. the k-th mode $(s-s_k)$ is uncontrollable from the l-th system input if the l-th column of matrix $\boldsymbol{B}$ is orthogonal to the k-th eigenvector of the closed-loop system matrix $\boldsymbol{A}_c$, i.e. with $j \neq k$

$$\boldsymbol{n}_k^T\boldsymbol{b}_l = \boldsymbol{n}_k^T\boldsymbol{n}_j = 0, \quad \boldsymbol{B} = \begin{bmatrix}\boldsymbol{b}_1 & \cdots & \boldsymbol{b}_r\end{bmatrix} \quad (21)$$

Proof. (Krokavec and A. Filasová (2006)) Let $\boldsymbol{n}_k$ is the k-th right eigenvector corresponding to the eigenvalue s_k, i.e.

$$\boldsymbol{A}_c\boldsymbol{n}_k = (\boldsymbol{A} - \boldsymbol{BK})\boldsymbol{n}_k = s_k\boldsymbol{n}_k \quad (22)$$

By definition, the closed-loop system resolvent kernel is

$$\boldsymbol{\Upsilon} = (s\boldsymbol{I}_n - \boldsymbol{A}_c)^{-1} \quad (23)$$

If the closed-loop system matrix is with distinct eigenvalues, (22) can be written in the compact form

$$\boldsymbol{A}_c\begin{bmatrix}\boldsymbol{n}_1 & \cdots & \boldsymbol{n}_n\end{bmatrix} = \begin{bmatrix}\boldsymbol{n}_1 & \cdots & \boldsymbol{n}_n\end{bmatrix}\begin{bmatrix}s_1 & & \\ & \ddots & \\ & & s_n\end{bmatrix} \quad (24)$$

$$\boldsymbol{A}_c\boldsymbol{N} = \boldsymbol{NS}, \quad \boldsymbol{N}^{-1} = \boldsymbol{N}^T \quad (25)$$

respectively, where

$$\boldsymbol{S} = \operatorname{diag}\begin{bmatrix}s_1 & \cdots & s_n\end{bmatrix}, \quad \boldsymbol{N} = \begin{bmatrix}\boldsymbol{n}_1 & \cdots & \boldsymbol{n}_n\end{bmatrix} \quad (26)$$

Using the property of orthogonality given in (25), the resolvent kernel of the system takes form

$$\begin{aligned}\boldsymbol{\Upsilon} &= \left(s\boldsymbol{N}\boldsymbol{N}^{-1} - \boldsymbol{NS}\boldsymbol{N}^{-1}\right)^{-1} = \\ &= \boldsymbol{N}(s\boldsymbol{I} - \boldsymbol{S})^{-1}\boldsymbol{N}^T\end{aligned} \quad (27)$$

$$\boldsymbol{\Upsilon} = \begin{bmatrix}\boldsymbol{n}_1 & \cdots & \boldsymbol{n}_n\end{bmatrix}\begin{bmatrix}\frac{1}{s-s_1} & & \\ & \ddots & \\ & & \frac{1}{s-s_n}\end{bmatrix}\begin{bmatrix}\boldsymbol{n}_1^T\\ \vdots\\ \boldsymbol{n}_n^T\end{bmatrix} \quad (28)$$

$$\boldsymbol{\Upsilon} = \sum_{h=1}^{n}\frac{\boldsymbol{n}_h\boldsymbol{n}_h^T}{s-s_h} \quad (29)$$

respectively. Thus, the closed loop transfer functions matrix takes form

$$\begin{aligned}\boldsymbol{G}(s) &= \\ &= \boldsymbol{C}\left(s\boldsymbol{I} - \boldsymbol{A}_c\right)^{-1}\boldsymbol{BL} = \sum_{h=1}^{n}\frac{\boldsymbol{C}\boldsymbol{n}_h\boldsymbol{n}_h^T\boldsymbol{B}}{s-s_h}\boldsymbol{L}\end{aligned} \quad (30)$$

It is obvious that (30) implies (20), (21). This concludes the proof. ■

4. EIGENSTRUCTURE ASSIGNMENT

In the pole assignment problem, a feedback gain matrix $\boldsymbol{K}$ is sought so that the closed-loop system has a prescribed eigenvalues spectrum $\Omega(\boldsymbol{A}_c) = \{s_h : \Re(s_h) < 0, \ h = 1, 2, ..., n\}$. Note, the spectrum $\Omega(\boldsymbol{A}_c)$ is closed under complex conjugation, and the observability and controllability of modes is determined by the closed-loop eigenstructure.

Considering the same assumptions as above then (22) can be rewritten as

$$\begin{bmatrix} s_h\boldsymbol{I}-\boldsymbol{A} & \boldsymbol{B}\end{bmatrix}\begin{bmatrix}\boldsymbol{n}_h\\ \boldsymbol{K}\boldsymbol{n}_h\end{bmatrix}=\boldsymbol{L}_h\begin{bmatrix}\boldsymbol{n}_h\\ \boldsymbol{K}\boldsymbol{n}_h\end{bmatrix}=\boldsymbol{0} \quad (31)$$

where $\boldsymbol{L}_h \in I\!R^{n\times(n+r)}$,

$$\boldsymbol{L}_h = \begin{bmatrix} s_h\boldsymbol{I}-\boldsymbol{A} & \boldsymbol{B}\end{bmatrix} \quad (32)$$

Subsequently, the singular value decomposition (SVD) of $\boldsymbol{L}_h$ gives

$$\begin{bmatrix}\boldsymbol{u}_{h1}^T\\ \vdots\\ \boldsymbol{u}_{hn}^T\end{bmatrix}\boldsymbol{L}_h\begin{bmatrix}\boldsymbol{v}_{h1}\cdot\boldsymbol{v}_{hn}\boldsymbol{v}_{h,(n+1)}\cdot\boldsymbol{v}_{h,(n+r)}\end{bmatrix}= = \begin{bmatrix}\sigma_{h1} & & & \\ & \ddots & & \boldsymbol{0}_{n+1}\cdot\boldsymbol{0}_{n+r}\\ & & \sigma_{hn} & \end{bmatrix} \quad (33)$$

$\{\boldsymbol{u}_{hl}^T, l=1,2,\ldots,n\}$, $\{\boldsymbol{v}_{hk}, k=1,2,\ldots,n+r\}$ are sets of the left and right singular vectors of $\boldsymbol{L}_h$ associated with the set of singular values $\{\sigma_{hl}$, $l=1,2,\ldots,n\}$

It is evident that vectors $\{\boldsymbol{v}_{hj}, j = n+1, n+2,\ldots,n+r\}$ satisfy (31), i.e.

$$\boldsymbol{L}\boldsymbol{v}_{hj} = \begin{bmatrix} s_h\boldsymbol{I}-\boldsymbol{A} & \boldsymbol{B}\end{bmatrix}\boldsymbol{v}_{hj} = \boldsymbol{0} \quad (34)$$

The set of vectors $\{\boldsymbol{v}_{hj}, j=n+1,n+2,\ldots,n+r\}$ is a non-trivial solution of (32), and results the null space of $\boldsymbol{L}_h$, $h=1,2,\ldots,n$

$$\begin{bmatrix}\boldsymbol{n}_h\\ \boldsymbol{K}\boldsymbol{n}_h\end{bmatrix}\in\mathcal{N}\begin{bmatrix} s_h\boldsymbol{I}-\boldsymbol{A} & \boldsymbol{B}\end{bmatrix} \quad (35)$$

The null space (35) consists of the normalized orthogonal set of vectors. Any combination of these vectors (the span of null space) will provide a vector $\boldsymbol{n}_h$ which used as an eigenvector produces the desired eigenvalue s_h in the closed-loop system matrix.

Proposition 4. The canonical form eigenstructure optimization provides optimal eigenstructure also for that model from which the canonical form was derived.

Proof. Using (16), (18), (19) and (22) it can be written

$$\begin{aligned}&(\boldsymbol{A}-\boldsymbol{B}\boldsymbol{K})\boldsymbol{n}_h =\\ &= (\boldsymbol{T}_c\boldsymbol{A}^\circ\boldsymbol{T}_c^{-1}-\boldsymbol{T}_c\boldsymbol{B}^\circ\boldsymbol{K}\boldsymbol{T}_c\boldsymbol{T}_c^{-1})\boldsymbol{n}_h=\\ &= \boldsymbol{T}_c(\boldsymbol{A}^\circ-\boldsymbol{B}^\circ\boldsymbol{K}^\circ)\boldsymbol{T}_c^{-1}\boldsymbol{n}_h = s_h\boldsymbol{n}_h\end{aligned} \quad (36)$$

$$s_h\boldsymbol{T}_c^{-1}\boldsymbol{n}_h = s_h\boldsymbol{n}_h^\circ = (\boldsymbol{A}^\circ-\boldsymbol{B}^\circ\boldsymbol{K}^\circ)\boldsymbol{n}_h^\circ \quad (37)$$

respectively, where

$$\boldsymbol{K}^\circ = \boldsymbol{K}\boldsymbol{T}_c, \quad \boldsymbol{n}_h = \boldsymbol{T}_c\boldsymbol{n}_h^\circ, \quad (38)$$

$$\boldsymbol{N} = \boldsymbol{T}_c\boldsymbol{N}^\circ, \quad \boldsymbol{N}^{-1} = \boldsymbol{N}^{\circ T}\boldsymbol{T}_c^{-1} \quad (39)$$

and subsequently using (27) it yields

$$\begin{aligned}\boldsymbol{G}(s) &= \boldsymbol{C}\boldsymbol{N}(s\boldsymbol{I}-\boldsymbol{S})^{-1}\boldsymbol{N}^{-1}\boldsymbol{B}\boldsymbol{L} =\\ &= \boldsymbol{C}\boldsymbol{T}_c\boldsymbol{N}^\circ(s\boldsymbol{I}-\boldsymbol{S})^{-1}\boldsymbol{N}^{\circ T}\boldsymbol{T}_c^{-1}\boldsymbol{B}\boldsymbol{L} =\\ &= \boldsymbol{C}^\circ\boldsymbol{N}^\circ(s\boldsymbol{I}-\boldsymbol{S})^{-1}\boldsymbol{N}^{\circ T}\boldsymbol{B}^\circ\boldsymbol{L}\end{aligned} \quad (40)$$

$$\begin{aligned}\boldsymbol{G}(s) &= \boldsymbol{C}(s\boldsymbol{I}-\boldsymbol{A}_c)^{-1}\boldsymbol{B}\boldsymbol{L} =\\ &= \boldsymbol{C}^\circ(s\boldsymbol{I}-\boldsymbol{A}^\circ)^{-1}\boldsymbol{B}^\circ\boldsymbol{L}\end{aligned} \quad (41)$$

$$\begin{aligned}&\boldsymbol{G}(s) =\\ &=\sum_{h=1}^{n}\frac{\boldsymbol{C}\boldsymbol{n}_h\boldsymbol{n}_h^T\boldsymbol{B}}{s-s_h}\boldsymbol{L} = \sum_{h=1}^{n}\frac{\boldsymbol{C}^\circ\boldsymbol{n}_h^\circ\boldsymbol{n}_h^{\circ T}\boldsymbol{B}^\circ}{s-s_h}\boldsymbol{L}\end{aligned} \quad (42)$$

respectively. It is obvious that optimizing $\boldsymbol{C}^\circ\boldsymbol{n}_h^\circ$ is optimized $\boldsymbol{C}\boldsymbol{n}_h$. This concludes the proof. ■

5. PARAMETER DESIGN

Using eigenvector orthogonal properties, (22) can be rewritten for $h=1,2,\ldots,n$ as follows

$$(s_h\boldsymbol{I}-\boldsymbol{A}^\circ)\boldsymbol{n}_h^\circ = -\boldsymbol{B}^\circ\boldsymbol{K}^\circ\boldsymbol{n}_h = -\boldsymbol{B}^\circ\boldsymbol{r}_h^\circ \quad (43)$$

$$\boldsymbol{n}_h^\circ = -(s_h\boldsymbol{I}-\boldsymbol{A}^\circ)^{-1}\boldsymbol{B}^\circ\boldsymbol{r}_h^\circ = \boldsymbol{V}_h^\circ\boldsymbol{r}_h^\circ \quad (44)$$

respectively, where

$$\boldsymbol{r}_h^\circ = \boldsymbol{K}^\circ\boldsymbol{n}_h^\circ, \quad \boldsymbol{V}_h^\circ = -(s_h\boldsymbol{I}-\boldsymbol{A}^\circ)^{-1}\boldsymbol{B}^\circ \quad (45)$$

Subsequently, it can be obtained

$$\boldsymbol{r}_h^\circ = \boldsymbol{V}_h^{\circ\,\ominus 1}\boldsymbol{n}_h^\circ \quad (46)$$

where

$$\boldsymbol{V}_h^{\circ\,\ominus 1} = (\boldsymbol{V}_h^{\circ T}\boldsymbol{V}_h^\circ)^{-1}\boldsymbol{V}_h^{\circ T} \quad (47)$$

is Moore-Penrose pseudoinverse of $\boldsymbol{V}_h^\circ$.

Of interest are the eigenvectors of the closed-loop system which are as orthogonal as possible to rows of the orthogonal complement $\boldsymbol{C}^{\circ T\perp}$ of the output matrix $\boldsymbol{C}^{\circ T}$ and associated with the prescribed $m =$ rank$(\boldsymbol{C}^\circ)$ elements subset $\rho(\boldsymbol{A}^\circ)\subset\Omega(\boldsymbol{A}^\circ)$ of the desired closed-loop eigenvalues set $\Omega(\boldsymbol{A}^\circ) = \{s_h,\ \Re(s_h)<0,\ h=1,2,\ldots,n\}$, $\Omega(\boldsymbol{A}^\circ)=\Omega(\boldsymbol{A})$. The rest ($n$–$m$) eigenvalues can be associated with rows of the complement matrix $\boldsymbol{C}^\bullet$ obtained in such way that all zero elements in $\boldsymbol{C}^\circ$ be changed to ones, and all ones to zeros. Note, direct use of $\boldsymbol{C}^\circ$ maximize matrix weights of modes.

Let $\rho(\boldsymbol{A}^\circ) = \{s_h,\ \Re(s_h)<0,\ h=1,2,\ldots,n\}$,

$$\boldsymbol{r}_h^\diamond = \boldsymbol{V}_h^{\circ\,\ominus 1}\boldsymbol{c}_h^{\circ T\perp T},\ h=1,2,\ldots,m \quad (48)$$

$$\boldsymbol{r}_h^\bullet = \boldsymbol{V}_h^{\circ\,\ominus 1}\boldsymbol{c}_h^{\bullet T},\ h=m+1,\ldots n \quad (49)$$

Then, computing

$$\boldsymbol{n}_h^{\diamond} = \boldsymbol{V}_h^{\circ}\boldsymbol{r}_h^{\diamond}, \quad \boldsymbol{n}_h^{\bullet} = \boldsymbol{V}_h^{\circ}\boldsymbol{r}_h^{\bullet} \tag{50}$$

it is possible to construct and to separate the matrix $\boldsymbol{Q}^{\circ}$ of the form

$$\boldsymbol{Q}^{\circ} = \left[\boldsymbol{v}_1^{\diamond} \cdots \boldsymbol{v}_m^{\diamond}\boldsymbol{v}_{m+1}^{\bullet} \cdots \boldsymbol{w}_m^{\bullet}\right] = \begin{bmatrix}\boldsymbol{P}^{\circ}\\ \boldsymbol{R}^{\circ}\end{bmatrix} \tag{51}$$

with $\boldsymbol{P}^{\circ} \in I\!\!R^{n\times n}$, $\boldsymbol{R}^{\circ} \in I\!\!R^{r\times n}$ such that

$$\boldsymbol{K}^{\circ} = \boldsymbol{R}^{\circ}\boldsymbol{P}^{\circ-1}, \quad \boldsymbol{K} = \boldsymbol{K}^{\circ}\boldsymbol{T}_c^{-1} \tag{52}$$

6. ILLUSTRATIVE EXAMPLE

The system under consideration was described by (1), (2), where

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 & 0\\ 0 & 0 & 1\\ -5 & -9 & -5\end{bmatrix}, \boldsymbol{B} = \begin{bmatrix} 1 & 3\\ 2 & 1\\ 2 & 5\end{bmatrix}, \boldsymbol{C}^T = \begin{bmatrix} 1 & 1\\ 2 & 1\\ 1 & 0\end{bmatrix}$$

Constructing the transform matrices

$$\boldsymbol{T}_c^{-1} = \begin{bmatrix} 4.0 & 0.5 & -2.5\\ 1.0 & 2.0 & 1.0\\ 1.0 & 1.0 & 0.0\end{bmatrix}, \boldsymbol{T}_c = \begin{bmatrix} 1.0 & 2.5 & -5.5\\ -1.0 & -2.5 & 6.5\\ 1.0 & 3.5 & -7.5\end{bmatrix}$$

the system model canonical form parameters were computed as $\boldsymbol{C}^{\circ} = [\,\boldsymbol{0}\ \boldsymbol{I}_2\,]$

$$\boldsymbol{A}^{\circ} = \begin{bmatrix} -1 & 10.5 & 6\\ 0 & -3.0 & -2\\ 0 & 1.0 & -1\end{bmatrix} \qquad \boldsymbol{B}^{\circ} = \begin{bmatrix} 0 & 0\\ 7 & 10\\ 3 & 4\end{bmatrix}$$

Thus, considering $\Omega(\boldsymbol{A}^{\circ}) = \{-0.5,\ -1.2,\ -6\}$ be

$$\boldsymbol{V}_1^{\circ} = \begin{bmatrix} -37.3846 & -54.4615\\ 0.7692 & 0.9231\\ -4.4615 & -6.1538\end{bmatrix}$$

$$\boldsymbol{V}_2^{\circ} = \begin{bmatrix} -10.0610 & -5.4878\\ 4.5122 & 6.0976\\ -7.5610 & -10.4878\end{bmatrix}$$

$$\boldsymbol{V}_3^{\circ} = \begin{bmatrix} -5.2059 & -7.3059\\ 2.4118 & 3.4118\\ 0.1176 & 0.0176\end{bmatrix}$$

and with $\boldsymbol{c}^{\circ T\perp} = \begin{bmatrix}1 & 0 & 0\end{bmatrix}$, $\boldsymbol{c}_1^{\bullet T} = \begin{bmatrix}1 & 0 & 1\end{bmatrix}$ yields

$$\boldsymbol{r}_1^{\circ} = \begin{bmatrix} 0.3891\\ -.0.2854\end{bmatrix}, \ \boldsymbol{r}_2^{\circ} = \begin{bmatrix} -0.1645\\ 0.1194\end{bmatrix}$$

$$\boldsymbol{r}_3^{\bullet} = \begin{bmatrix} 18.4978\\ -13.2737\end{bmatrix}$$

$$\boldsymbol{n}_1^{\circ T} = \begin{bmatrix}0.9983 \ 0.0358 \ 0.0205\end{bmatrix}$$

$$\boldsymbol{n}_2^{\circ T} = \begin{bmatrix}0.9997 \ -0.0144 \ -0.0082\end{bmatrix}$$

$$\boldsymbol{n}_3^{\bullet T} = \begin{bmatrix}0.6788 \ -0.6745 \ 0.6146\end{bmatrix}$$

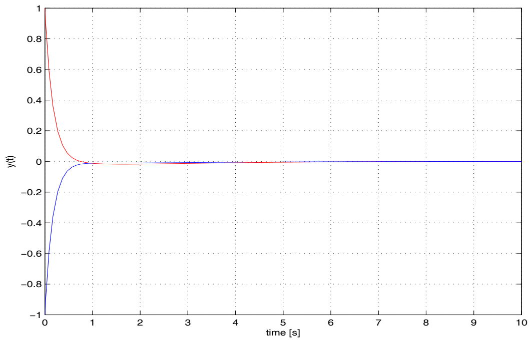

Fig. 1. System output response

Constructing the matrix $\boldsymbol{Q}^{\circ}$

$$\boldsymbol{Q}^{\circ} = \left[\begin{array}{rrr} 0.9983 & 0.9997 & 0.6788\\ 0.0358 & 0.0144 & -0.6745\\ 0.0205 & -0.0082 & 0.6146\\ \hline 0.3891 & -0.1645 & 18.4978\\ -0.2854 & 0.1194 & -13.2737\end{array}\right] = \begin{bmatrix}\boldsymbol{P}^{\circ}\\ \boldsymbol{R}^{\circ}\end{bmatrix}$$

the control law parameters satisfying (52) are

$$\boldsymbol{K}^{\circ} = \begin{bmatrix} -0.0062 & -3.7944 & 25.9402\\ 0.0036 & 2.6301 & -18.7151\end{bmatrix}$$

$$\boldsymbol{K} = \begin{bmatrix} 22.1212 & 18.3483 & -3.7990\\ -16.0707 & -13.4532 & 2.6211\end{bmatrix}$$

It is possible to verify that closed-loop system matrix eigenvalues belongs to the desired one.

In the presented Fig. 1 the example is shown of the unforced closed-loop system output response, where nonzero initial state was considered.

7. CONCLUDING REMARKS

This paper provides a design method for memory-free controllers where the general problem of assigning the eigenstructure for state variable mode decoupling in state feedback control design is considered. The method exploits standard numerical optimization procedures to manipulate the system feedback gain matrix as a direct design variable. The manipulation is accomplished in a manner that produces desired system global performance by pole placement and output dynamics by modification of the mode observability.

With generalization of the known algorithms for pole assignment the modified exposition of the problem is presented here to handle the optimized structure of the left eigenvector set in state feedback control design. Presented method makes full use of the freedom provided by eigenstructure assignment to find a controller which stabilizes the closed-loop system. Therefore, the feedback control law has a clear physical meaning and provides a valid design method of the controller for real systems. It is shown by appropriately assigning closed-loop eigenstructure in state feedback control the overall stability is achieved. Finally the design methodology is illustrated by an example.

ACKNOWLEDGMENT

The work presented in this paper was supported by Grant Agency of Ministry of Education and Academy of Science of Slovak Republic VEGA under Grant No. 1/0256/11, as well by Research & Development Operational Programme Grant No. 26220120030 realized in Development of Centre of Information and Communication Technologies for Knowledge Systems. These supports are very gratefully acknowledged.

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