Fault Estimation in a Class of First Order Nonlinear Systems

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***Abstract*—Reformulated principle of fault estimation design for one class of first order continuous-time nonlinear system is treated in this paper, where a neural network is regarded as model-free fault approximator. The problem addressed is presented as approach based on sliding mode methodology with combination of radial basis function neural network to design robust nonlinear fault estimation. The method utilizes Lyapunov function and the steepest descent rule to guarantee the convergence of the estimation error asymptotically. Simulation results show the feasibility of the proposed approach.**

***Index Terms* — Sliding mode state estimator, radial basis function neural network, Lyapunov function, steepest descent rule, linear matrix inequalities.**

I. Introduction

The theory of sliding mode has emerged as a method capable of use in given robust control systems, as well as state estimation [15], [19]. The main advantage of sliding mode in state estimation exists in the sliding where the state error trajectories are constrained on the predetermined equivalent system. The sliding mode observer design can be then divided into two phases. First, in the reaching phase, give an error switching law such that the estimated error trajectory is trapped on a switching surface and remain on it thereafter. And second, in the sliding phase, determine the switching surface such that the error dynamics in the sliding mode have good performance.

It should be pointed out that the robustness of a variable structure resides in its sliding phase, but not in its reaching phase. There is no easy way to shape the error dynamics of the reaching phase. In fact, the error dynamic is not completely robust over all time. In addition, if the system suffers from uncertainties or faults, then the error behavior in the sliding mode is not only governed by the switching surface but also determined by faults. In this case, the state estimator stability may not be assured [4], [11]. However, in many cases sliding mode control is supposed [16], [17], [18], and different observer structures used to estimate sensor, actuator, and system faults [3], [7], [10].

Recently, neural networks has been successfully used in model-free fault estimation in nonlinear systems. The known supports of these methods give their ability to good performance for approximation of nonlinear functions. Combining sliding mode observer technology with neural networks to design fault estimation structures offer new opportunity in design [6].

Radial basis function (RBF) neural networks seems to be one of the neural networks with high approximation and regularization capability [2], where the essential phenomenological rationale for the use of RBF rest in the realm of the purpose of feed-forward networks and feature extraction possibility. Gaussian RBF are employed most frequently, since it is bounded, strictly positive and continuous on $\mathbb{R}$, and the optimization of RBF networks provides better approximation and interpolation capability as compared to the sigmoid functions. However, the performance of RBF neural networks depends on the number of neurons in hidden units, and on methods used for determining the output neuron weights, where training of a RBF neural network is, in general, a challenging nonlinear optimization problem.

The paper is focused on robust system fault estimation and isolation (FDI) approach for one class of nonlinear single input/output (SISO) systems. The FDI scheme is based on sliding mode neural observer [9], which is robust against system uncertainty, and fault estimation be realized using the sliding boundary size. When a system fault is occurred the estimate part in the observer for faults is enabled. When a RBF neural network is used to approximate a system fault the fault estimation can be formulated in a simpler way.

The main contribution of the paper is to present a reformulated design method for continuous-time nonlinear SISO systems to solve system fault estimation problem using a sliding mode observer. Used structure is motivated by the need for robustness in model-free fault estimation. Under defined conditions, the stability of the observer is assured, where the state observer error is bounded. In comparison with methods based on analytical models (see e.g. [5], [8], [13], [14]) design conditions for specified system class are here derived using radial basis function neural network.

II. Problem Description

The system under consideration is a first order SISO nonlinear dynamic systems, which is given as

$$\dot{q}(t) = aq(t) + bu(t) + f(q(t)) \tag{1}$$

$$y(t) = q(t) \tag{2}$$

where $q(t) \in I\!R$, $u(t) \in I\!R$, and $y(t) \in I\!R$ are the state, input and output variables, respectively, and scalars $a \in I\!R$, $b \in I\!R$. A system fault is modelled by an unknown additive function $f(q(t)) \in I\!R$.

The problem of interest is to design sliding-mode observer based estimation of system faults.

III. Observer Structure

Since the calculation of $f(q(t))$ may be not accurate enough, in order to rectify this kind of errors as well as to reduce the number of sensors a sliding mode observer can be used to estimate $f(q(t))$, i.e. the sliding mode observer of system (1), (2) can be designed e.g. as

$$\dot{q}_e(t) = aq_e(t) + bu(t) + j(y(t) - y_e(t)) + s(t) \tag{3}$$

$$y_e(t) = q_e(t) \tag{4}$$

where $q_e(t) \in I\!R$ is the state observer variable, $y_e(t) \in I\!R$ is the estimated output, and j is an observer gain. The term $s(t) \in I\!R$, which represents the unknown $f(q(t))$, is defined as a switching function of the tracking error of the observer, i.e.

$$s(t) = \varrho\,\mathrm{sign}(e(t)) \tag{5}$$

where $e(t)$ is the observer estimation error defined as

$$e(t) = q(t) - q_e(t) \tag{6}$$

and ϱ is a constant factor. The constant ϱ has to be chosen in such way that sliding mode is enforced in the manifold $\nu_e = q(t) - q_e(t) = 0$. Once sliding mode is enforced, the differential equations (1) and (3) have the same solution and the terms $f(q(t))$ and $s(t)$ have to be equivalent. During application a low pass filter has to be used to gain the average value $s(t)$ of the discontinuous time function (5).

Model-free sliding mode observer does not require the model of $f(q(t))$, but it cannot arrive finite time convergence, the sliding mode gain should be bigger than an upper bound, and incomplete information about the rest nominal parameters causes chattering phenomenon. It seems to be natural to construct its estimate $f(q(t)|\boldsymbol{w}(t))$ depending on a parameter $\boldsymbol{w}(t)$, which can be adjusted online by means of an updating law. In the next, a neural network is used to approximate $f(q(t))$ and construct a neural observer of the form

$$\begin{aligned}\dot{q}_e(t) &= aq_e(t) + bu(t) + \\ &+ j(y(t) - y_e(t)) + f_a(q_e(t)|\boldsymbol{w}_a(t)) + (1 - r(t))s(t)\end{aligned} \tag{7}$$

$$y_e(t) = q_e(t) \tag{8}$$

$$f_a(q_e(t)|\boldsymbol{w}_a(t)) = \boldsymbol{w}_a^T(t)\boldsymbol{\varphi}(q_e(t)) \tag{9}$$

Here $s(t) \in I\!R$ is an external feed-forward compensation signal and $r(t)$ is a switch function. The switch function switch between the neural estimator and the sliding mode observer in the dependence on the output error (6), i.e.

$$r(t) = \begin{cases} 1 & \text{if } e^2(t) \geq \delta \\ 0 & \text{if } e^2(t) < \delta \end{cases} \tag{10}$$

where δ is known upper bound of the neural modelling error.

If $r(t) = 1$, the observer is pure neural observer, and (7), (8) becomes

$$\begin{aligned}\dot{q}_e(t) &= aq_e(t) + bu(t) + \\ &+ j(y(t) - y_e(t)) + f_a(q_e(t)|\boldsymbol{w}_a(t))\end{aligned} \tag{11}$$

$$y_e(t) = q_e(t) \tag{12}$$

If after time t_0 $e^2(t) < \delta$, i.e. $r(t) = 0$ then $\boldsymbol{w}_a^T(t)$ is a constant vector $\boldsymbol{w}_a^T = \boldsymbol{w}_a^T(t_0)$, and the observer (7), (8) becomes pure sliding mode observer

$$\begin{aligned}\dot{q}_e(t) &= aq_e(t) + bu(t) + \\ &+ j(y(t) - y_e(t)) + f_a(q_e(t)|\boldsymbol{w}_a) + s(t)\end{aligned} \tag{13}$$

$$y_e(t) = q_e(t) \tag{14}$$

IV. RBF Neural Network

The model-free fault estimation employed in the next use a radial basis function neural network (RBFNN) to approximate an unknown fault combining with the sliding mode observer.

Supposing that there are p receptive field units in the neural network hidden layer than the output of the RBFNN is

$$f_a(q_e(t)) = \sum_{h=1}^{p} w_h(t)\varphi_h(q_e(t)) = \boldsymbol{w}_a^T(t)\boldsymbol{\varphi}(q_e(t)) \tag{15}$$

$$\varphi_h(q_e(t)) = \exp\left(-\frac{(q_e(t) - c_h)^2}{\sigma_h^2}\right) \tag{16}$$

$$\boldsymbol{w}_a^T(t) = \begin{bmatrix} w_1(t) & w_2(t) & \cdots & w_p(t) \end{bmatrix} \tag{17}$$

$$\boldsymbol{\varphi}^T(q_e(t)) = \begin{bmatrix} \varphi_1(q_e(t)) & \varphi_2(q_e(t)) & \cdots & \varphi_p(q_e(t)) \end{bmatrix} \tag{18}$$

where c_h, σ_h are the center and width (spread factor) of the neural cell of the h-th neuron in the hidden layer, respectively, and $\boldsymbol{w}_h(t) \in I\!R$ is the weight connecting the h-th hidden layer neuron and the network output.

Thus, the optimal weight values of RBF neural network can be considered as follows

$$\boldsymbol{w}^\star = \arg \min_{\boldsymbol{w}_a \in \Omega_f} \left(\sup_{q_e \in S_q} |f(q_e(t)|\boldsymbol{w}_a) - f(q_e(t))| \right) \tag{19}$$

where

$$\Omega_f = \{\boldsymbol{w}_a : \|\boldsymbol{w}_a\| \leq \delta_w\} \tag{20}$$

is a valid field of $\boldsymbol{w}_a(t)$, δ_w is a parameter, and $S_q \in I\!R$ is a variable space of the state observer variable.

Because radial basis function $\varphi(q_e(t))$ satisfies Lipschitz condition

$$\|\boldsymbol{\varphi}(q(t)) - \boldsymbol{\varphi}(q_e(t))\|^2 \leq \kappa_\varphi (q(t) - q_e(t))^2 \tag{21}$$

where κ_φ can be selected by users, then according to the Stone-Weierstrass theorem, the smooth function $f(q(t))$ can be written as

$$f(q(t)) = \boldsymbol{w}^{\star T}\boldsymbol{\varphi}(q(t)) + \varepsilon(t) \tag{22}$$

where $\boldsymbol{w}^{\star T}$ is a fixed weight matrix of the neural network, and $\varepsilon(t)$ is is the smallest approximation (modelling) error satisfying condition

$$\varepsilon^2(t) \leq \delta \tag{23}$$

Then, according to (15) is

$$\begin{gathered} f(q(t)) - f_a(q_e(t)|\boldsymbol{w}_a(t)) = \nu(t) = \\ = \boldsymbol{w}^{\star T}\boldsymbol{\varphi}(q(t)) + \varepsilon(t) - \boldsymbol{w}_a^T(t)\boldsymbol{\varphi}(q_e(t)) = \\ = \boldsymbol{w}^{\star T}(\boldsymbol{\varphi}(q(t)) - \boldsymbol{\varphi}(q_e(t))) + \varepsilon(t) + \\ +(\boldsymbol{w}^{\star T} - \boldsymbol{w}_a^T(t))\boldsymbol{\varphi}(q_e(t)) \end{gathered} \tag{24}$$

where with notations

$$\boldsymbol{w}^{\star T}\Delta\boldsymbol{\varphi}(t) = \boldsymbol{w}^{\star T}(\boldsymbol{\varphi}(q(t)) - \boldsymbol{\varphi}(q_e(t))) \tag{25}$$

$$\boldsymbol{w}_e^T(t) = \boldsymbol{w}^{\star T} - \boldsymbol{w}_a^T(t) \tag{26}$$

it yields

$$\nu(t) = \varepsilon(t) + \boldsymbol{w}^{\star T}\Delta\boldsymbol{\varphi}(t) + \boldsymbol{w}_e^T(t)\boldsymbol{\varphi}(q_e(t)) \tag{27}$$

V. Sliding Mode Neural Observer Stability

The neural sliding mode observer design method given by [9] is represented as follows. Assuming that the sliding mode neural observer switches between two models then if $r(t) = 1$ then (1), (11), (6), and (27) implies

$$\begin{gathered} \dot{e}(t) = aq(t) + bu(t) - aq_e(t) - bu(t) - \\ -j(y(t) - y_e(t)) + f(q(t)) - f_a(q_e(t)|\boldsymbol{w}_a(t)) = \\ = (a-j)e(t) + \nu(t) \end{gathered} \tag{28}$$

$$\begin{gathered} \dot{e}(t) = \\ = (a-j)e(t) + \varepsilon(t) + \boldsymbol{w}^{*T}\Delta\boldsymbol{\varphi}(t) + \boldsymbol{w}_e^T(t)\boldsymbol{\varphi}(q_e(t)) \end{gathered} \tag{29}$$

respectively, where

$$\Delta\boldsymbol{\varphi}(t) = \boldsymbol{\varphi}(q_e(t) + e(t)) - \boldsymbol{\varphi}(q_e(t)) \tag{30}$$

Since neural networks have to be discrete updated and $e(i)$ be used in the updating law, then

$$\begin{gathered} \boldsymbol{w}_e(i+1) = \boldsymbol{w}_e(i) - r(i)\mu(i)\boldsymbol{\varphi}(q_e(i))e(i) = \\ = \boldsymbol{w}_e(i) - \mu(i)\boldsymbol{\varphi}(q_e(i))e(i) \end{gathered} \tag{31}$$

where $0 < \mu(i) < 1$.

Defining Lyapunov function candidate as

$$v(\boldsymbol{w}_e(i)) = \|\boldsymbol{w}_e(i)\|^2 = \text{trace}\left(\boldsymbol{w}_e(i)\boldsymbol{w}_e^T(i)\right) \tag{32}$$

then

$$\begin{gathered} \Delta v(\boldsymbol{w}_e(i)) = \|\boldsymbol{w}_e(i+1)\|^2 - \|\boldsymbol{w}_e(i)\|^2 = \\ = \|\boldsymbol{w}_e(i) - \mu(i)\boldsymbol{\varphi}(q_e(i))e(i)\|^2 - \|\boldsymbol{w}_e(i)\|^2 = \\ = \mu^2(i)\|\boldsymbol{\varphi}(q_e(i))\|^2\|e(i)\|^2 - \\ -2\mu(i)\|\boldsymbol{w}_e(i)\|\|\boldsymbol{\varphi}(q_e(i))\|\|e(i)\| \end{gathered} \tag{33}$$

respectively. Inserting (29) it yields [12]

$$\begin{gathered} 2\mu(i)\|\boldsymbol{w}_e(i)\|\|\boldsymbol{\varphi}(q_e(i))\| \geq \\ \geq 2\mu(i)\|\boldsymbol{w}_e^T(i)\boldsymbol{\varphi}(q_e(i))\| = \\ = 2\mu(i)\|\dot{e}(i) - (a-j)e(i) - \varepsilon(i) - \boldsymbol{w}^{*T}\Delta\boldsymbol{\varphi}(i)\| \geq \\ \geq \mu(i)2\|e(i)\boldsymbol{w}^{*T}\Delta\boldsymbol{\varphi}(i)\| - \\ -2e(i)(a-j)e(i) - 2\|e(i)\dot{e}(i)\| - 2\|e(i)\varepsilon(i)\| \end{gathered} \tag{34}$$

and it is obvious that (21) implies

$$\begin{gathered} 2\|e(i)\boldsymbol{w}^{*T}\Delta\boldsymbol{\varphi}(i)\| \leq \|e(i)\|^2 + \|\boldsymbol{w}^{*T}\|\|\Delta\boldsymbol{\varphi}(i)\|^2 \leq \\ \leq (1 + \kappa_\varphi\|\boldsymbol{w}^{*T}\|)e^2(i) \end{gathered} \tag{35}$$

Since for a smooth stable system and any $i > 0$ yields

$$\frac{\|\dot{e}(i)\|}{\|e(i)\|} \leq i \tag{36}$$

then

$$-2\|e(i)\dot{e}(i)\| \geq -i\|e(i)\|^2 = -ie^2(i) \tag{37}$$

and

$$-2e(i)(a-j)e(i) = -2(a-j)e^2(i) \geq 0 \tag{38}$$

Because it yields, too

$$-2\|e(i)\varepsilon(i)\| \geq -\|e(i)\|^2 - \|\varepsilon(i)\|^2 = -\|e(i)\|^2 - \delta \tag{39}$$

then

$$\begin{gathered} 2\mu(i)\|\boldsymbol{w}_e^T(i)\boldsymbol{\varphi}(q_e(i))\| \geq -\mu(i)\delta + \\ +\mu(i)(1 + \kappa_\varphi\|\boldsymbol{w}^{*T}\| - i - 2(a-j) - 1)e^2(i) \end{gathered} \tag{40}$$

Subsequently, (33) implies

$$\begin{gathered} \Delta v(\boldsymbol{w}_e(i)) \leq \mu(i)\delta - \\ -\mu(i)\begin{pmatrix} -\mu(i)\|\boldsymbol{\varphi}(q_e(i))\|^2 + \\ +\kappa_\varphi\|\boldsymbol{w}^{*T}\| - 2(a-j) - i \end{pmatrix} e^2(i) < 0 \end{gathered} \tag{41}$$

Setting

$$\mu(i) = \frac{\kappa_\varphi\|\boldsymbol{w}^{*T}\| - 2(a-j) - i}{1 + \|\boldsymbol{\varphi}(q_e(i))\|^2} \tag{42}$$

where $\mu_i > 0$ is the learning rate, (33) takes the form

$$\Delta v(\boldsymbol{w}_e(i)) \leq -\mu(i)e^2(i) + \mu(i)\delta < 0 \tag{43}$$

Since in this case $e^2(i) \geq \delta$, then $\Delta v(\boldsymbol{w}_e(i)) < 0$, $v(\boldsymbol{w}_e(i))$ is bounded, and also $\|\boldsymbol{w}_e(i)\|$ is bounded.

Here the initial condition $\|\boldsymbol{w}^{*T}\|$ can be selected as

$$\kappa_\varphi\|\boldsymbol{w}^{*T}\| < 1 + i + 2(a-j) \tag{44}$$

which gives

$$0 < \kappa_\varphi\|\boldsymbol{w}^{*T}\| - i - 2(a-j) < 1 \tag{45}$$

and it is evident that (42) implies $0 < \mu(i) < 1$.

If at time $t = t_0$ be $r(t) = 0$ the weights become constants, the observer (7), (8) becomes pure sliding mode taking form (13), (14). The error dynamics obtained from (1), (2), (13), and (14) is

$$\begin{gathered} \dot{e}(t) = aq(t) + bu(t) - aq_e(t) - bu(t) - s(t) - \\ -j(y(t) - y_e(t)) + f(q(t)) - f_a(q_e(t)|\boldsymbol{w}_a) = \\ = (a-j)e(t) + \Delta f(t) - s(t) \end{gathered} \tag{46}$$

$$e_y(t) = e(t) \tag{47}$$

where $\Delta f(t)$ is neural modelling error when the weight of the neural networks is fixed as $\boldsymbol{w}_a$.

Defining the Lyapunov function of the form

$$v(e(t)) = \frac{1}{2}pe^2(t) \tag{48}$$

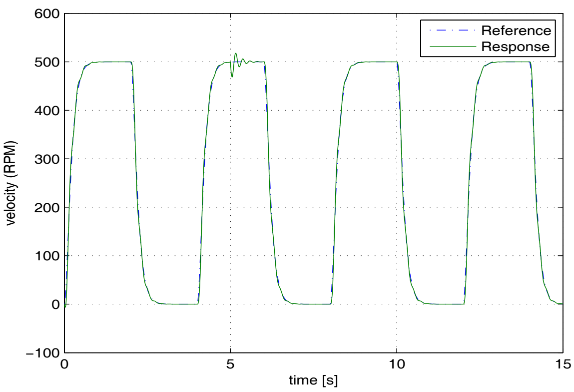

Fig. 1. Responses of the controlled system

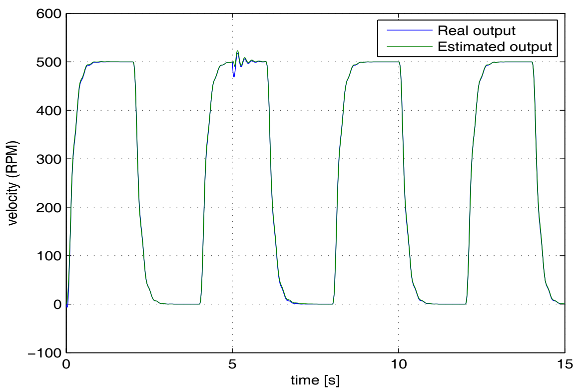

Fig. 2. The corresponding observer output response

with $p > 0 \in I\!R$, and taking the time derivative of $v(e(t))$ results in

$$\dot{v}(e(t)) = pe(t)\dot{e}(t) = e(t)p(aq(t) + bu(t) + \Delta f(t) - \\ -s(t) - aq_e(t) - bu(t) - j(q(t) - q_e(t)) \tag{49}$$

$$\dot{v}(e(t)) = e(t)p(a_e e(t) - s(t) + \Delta f(t)) < 0 \tag{50}$$

where

$$a_e = a - j \tag{51}$$

Supposing $\Delta f(t) = s(t)$ and setting $pj = 1$ (50) implies

$$p(a - j) = pa - 1 < 0 \tag{52}$$

Thus, for any p satisfying (52) be $j = p^{-1}$. To proceed $\Delta f(t) = s(t)$, a function

$$s(e(t)) = \frac{p^{-1}e(t)}{\|e(t)\|} \|e(t)p\Delta f(t)\| = \\ = \varrho(e(t), \Delta f(t), p)\frac{p^{-1}e(t)}{\|e(t)\|} \tag{53}$$

be introduced conditioned by $e(t) \neq 0$, and $s(e(t)) = 0$ if $e(t) = 0$. This function characterizes the sliding mode in state estimation, and it is evident that with a positive p satisfying (52) the observer error is bounded.

The sliding mode neural observer requires two design parameters: switch constant δ and the upper bound Δ of neural modelling error $\Delta f(t)$ when start the sliding mode compensation. Here δ decide when neural network learning is stopped and sliding mode observer is started. The bigger δ is, the shorter training time the neural observer has and since the neural modelling error is bigger, so Δ should be bigger. Usually $\Delta > \delta$, because δ corresponds to the modelling error with optimal synaptic weight, while Δ corresponds to the modelling error when $e^2(t) < \delta$. Note, this parameters are user-defined.

VI. Illustrative Example

Considering the synchronous reluctance motor drive system [4] described by the equation

$$\dot{q}(t) = aq(t) + bu(t) + f(q(t))$$

where $q(t) = \omega(t)$ is velocity, the input control $u(t)$ is the electromagnetic torque, $J = 0.00076\ kgm^2$ is the inertia moment of rotor, $m = 0.00012\ Nms/rad$ is the viscous friction coefficient, respectively, and

$$a = -\frac{m}{J}, \qquad b = \frac{1}{J}$$

A simple feedback controller has been introduced to stabilize the nominal system for demonstration purposes with the state control law of the form [12]

$$u(t) = -kq(t) + n\omega_r(t) + \frac{1}{T_I}\int_0^t (\omega_r(\tau) - \omega(\tau))\mathrm{d}\tau$$

where $k = 0.0075$, $n = 0.0076$, $T_I = 1.5sec.$

In this simulation was tested different numbers of hidden nodes with the nest result that after the hidden nodes number is more than 5 the estimation accuracy be not improved a lot.

There is consider no parameter variations except system fault at $t = 5\ sec$. The simulation results depicted in Figs. 1 and 2 represent the system velocity response and its estimation, and show that the proposed control approach is effective. Figs. 3 and 4 provide the fault estimation and synaptic weight adaptation obtained using the sliding mode neural observer.

VII. Concluding Remarks

This paper explores use of a sliding mode scheme for fault estimation in nonlinear system of the first order. Modified design method is presented where system fault estimation is achieved by the RBF neural network inclusion into the sliding mode observer. The stability analysis is given using the Lyapunov method and LMI observer condition to garanty observer error convergence, where LMI procedure give desirable observer dynamics. It is demonstrated that the sliding mode observer can be used to detection and estimate system faults. The ability to estimate the faults directly is very desirable for fault detection, and preferred to handle problem of fault occurrence and faults behavior.

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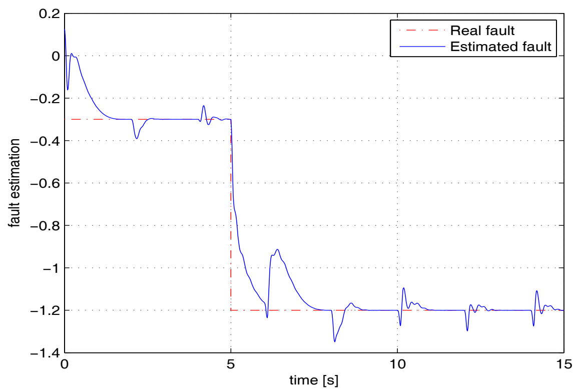

Fig. 3. System fault and its estimation

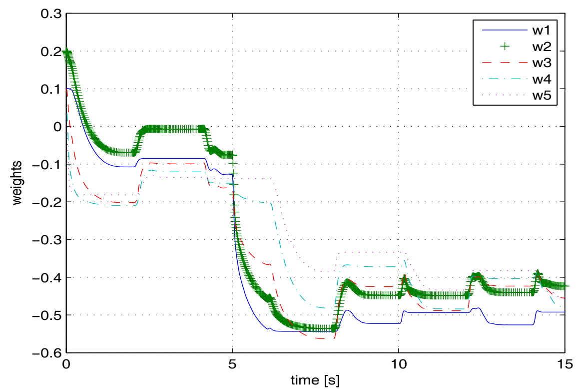

Fig. 4. Synaptic weights adaptation

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