Mobility constraints on the legs of a parallel robot to improve the kinematic calibration.

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Mobility Constraints on the Legs of a Parallel Robot to Improve Kinematic Calibration.

David Daney
David.Daney@sophia .inria.fr
INRIA Sophia-Antipolis - Projet SAGA
2004, Route des Lucioles - BP 93 - 06902 Sophia Antipolis Cedex France

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1 Introduction

Due to errors such as manufacturing or assembly defects, it is well known that the geometry of robotics manipulators does not exactly match the design goals. A direct drawback lies in a reduced accuracy of the manipulator, which turns out to be a problem since robot control requires accurate kinematic models. One way to tackle this problem consists in improving the knowledge of the kinematic model using kinematic calibrations. This is the problem we address in this contribution for parallel manipulator.

1.1 Unified formulation for calibration problem

According to the general paradigm of [15], a unified calibration formulation can be stated as follows. First, given the unknown kinematic parameters of the manipulator x and the measurements m provided by sensors, some loop equations $f(x, m) = 0$ have to be figured out. The measurements m can be of two types : either external measurements of the position/orientation of the robot's end-effector, with its articular coordinates ; or only internal measurements of the articular coordinates and redundant sensors —see [18], [19] (in particular, the knowledge of this particular information in the case where the direction of the segments is also known makes the forward kinematic problem simpler.) To obtain the loop equations, one can use forward, inverse kinematic, closing loops, mobility constraints [1], [3], [11] on the legs or on the end effector. Second, these loop equations can be solved using some optimization [8], linearization [4] or resolution [6] machinery.

1.2 Leg mobility constraints

The simplest ways to calibrate a parallel manipulator without any constraint are directly derivate from the serial robot calibration methods. The loop equations can be provided by either the forward kinematic (FK) or the inverse kinematic (IK).

- For the FK methods, the kinematic parameters are provided by minimization of the difference between the position/orientation measurements and the position/orientation calculated from the FK which is function of the leg length measurements and the kinematic parameters unknowns. But for a Gough platform, there is no closed form for FK, so numerical methods are used to solve this problem. In this case, there are uncertainties on the convergence of the FK in presence of noise measurement, and more on the calibration convergence. These methods are slow due to the high number of iterations.

- For the IK method, the kinematic parameters are provided by minimization of the difference between the leg length measurements and the leg length calculated from the IK as function of the position/orientation measurements and the kinematic parameters unknowns. Due to the formal form of IK, these methods are simpler and faster than FK methods.

These basic methods can be improved by imposing constraints on the robot and one possibility is to use constraints on the robot legs. For example Zhuang [17] fixes the length of one leg for each measurement configurations, to remove a kinematic parameter (offset on the leg length) and to lower the degrees of the equations. Murrarecci [10] fixes the direction of a leg for a set of measurement and then determine the kinematic parameter which minimize the changes in the U-ball angles calculated from two measurement configurations. We shall use these ideas thereafter.

1.3 Optimization Method

Most of algorithms use iterative methods to solve the non-linear loop equations see–[7]. Innocenti [5] uses the same constraints than Zhuang [17], but solves the equations by a dialytic method and finds 21 solutions for one leg kinematic parameters. As the problem may have multiple solutions, the use of numerical algorithms is problematic, specially in presence of noise measurement.

To take these problems into account we use leg mobility constraints to obtain some linear loop equations.

2 Robot model

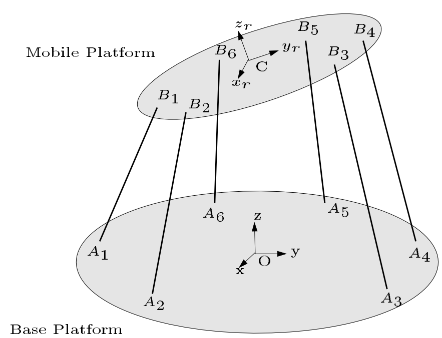


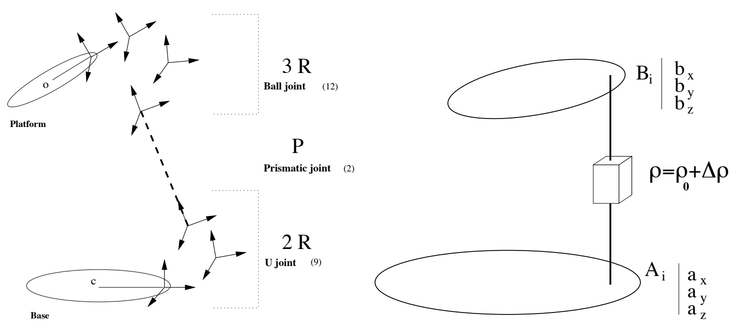
Figure 1: Gough platform

We perform the calibration for a Gough platform 6 DOF (figure 1).

The following notation will be used throughout this paper :

- i leg index.
- A_i, B_i attachment points of the leg i with the base (U joint) and with the platform (ball joint).
- $R_{ab} = (O, x, y, z)$ an absolute frame and $R_{ee} = (C, x_r, y_r, z_r)$ an end effector frame.
- $\mathcal{P}$ vector (3×1) defines an end-effector position in R_{ab}.
- $\mathcal{R}$ rotation matrix (3×3) or $\mathcal{A} = (\psi, \theta, \phi)^T$ Euler's angles vector defines a platform orientation.
- For a simplify notation : $b_i = \mathbf{CB_i}$ in R_{ee}, $(b_i)_o = \mathbf{CB_i}$ in R_{ab}, $a_i = \mathbf{OA_i}$ in R_{ab}

Each leg is a RRPRRR chain.



(a) Complete model (b) Reduced model

Figure 2: Model of a parallel manipulator leg

Wang [16] has shown that, a complete model (figure 2(a)) of one leg need 23 parameters (9 for the U-joint, 2 for the prismatic joint and its offset, 12 for the ball joint). But he shows that the contribution of joint manufacturing tolerances have a minor effect on the platform pose error. Under the assumption that prismatic joints are perfectly assembled and U-joints and ball-joints are ideal, one leg can be considered as a SS chain (figure 2(b)).

So we have to identify 7 kinematic parameters per leg : 3 coordinates of the base attachment point ax_k, ay_k, az_k, 3 coordinates of the mobile attachment point bx_k, by_k, bz_k, 1 offset on the leg length $\Delta\rho_k$. In total, 42 kinematic parameters have to be found to calibrate the robot (Model 42). For this model the vector of kinematic parameters is $P_k = [bx_k, by_k, bz_k, ax_k, ay_k, az_k, \Delta\rho_k]^T$.

In a first part of this contribution, we consider the offset on the leg length as perfectly known (the sensor leg have been already calibrate), so we want only to determine the coordinate attachment points, (6 legs $\times$ 3 coordinates $\times$ 2 attachment points) parameters (Model 36). For this model, $P_k = [bx_k, by_k, bz_k, ax_k, ay_k, az_k]^T$.

3 New methods

The idea of this contribution is to fix the direction of one leg by a clamping mechanism (figure 3(b)) in order to obtain some linear loop equations function of measurements and our unknowns. An additional constraint on the fixed leg length (figure 3(a)) can be used to remove some unknowns. We impose these kind of constraints on one leg at a time. The obtained measurement equations are fully decoupled for each leg. So this type of algorithm can calibrate only one leg of the robot (k : index of fixed leg). The same operation have to be process 6 time for each leg of the robot for a complete calibration of the Gough platform.

Two methods are shown :

- The first method is used to calibrate in one step Model 36. For all measurement configurations ($i = 1..N$), the direction of the leg k (to be calibrated) is fixed. The position, the orientation of the mobile and the leg length are free in order to verify the direction constraint (figure 3(b)). The direction of the leg provide us a 3D linear loop equations function of ours unknowns.

- The second method take into account the segment offset and calibrate as a Model 42 in three steps. First, for all measurement poses, the direction and the length of the leg k are fixed (figure 3(a)). We will obtain some linear loop equations only function of the coordinates of mobile attachment point. Then, (similarly to Zhuang method [17]) we fix the leg k length but we let free the segment direction to determine the coordinate of the base attachment point (figure 3(c)). Finally the length leg offset is easily determinable with the knowledge of b_k, a_k.

For all measurement poses, the orientation, the position of the platform and the leg length are measured to provide us enough data for calibration.

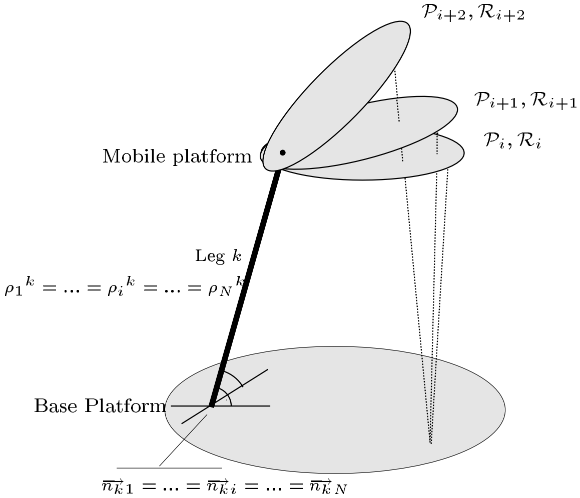
3.1 Constrained measurement poses

We simulate three of measurement poses set denoted Type I, Type II and Type III for each type of constraints on the leg direction and/or the leg length :

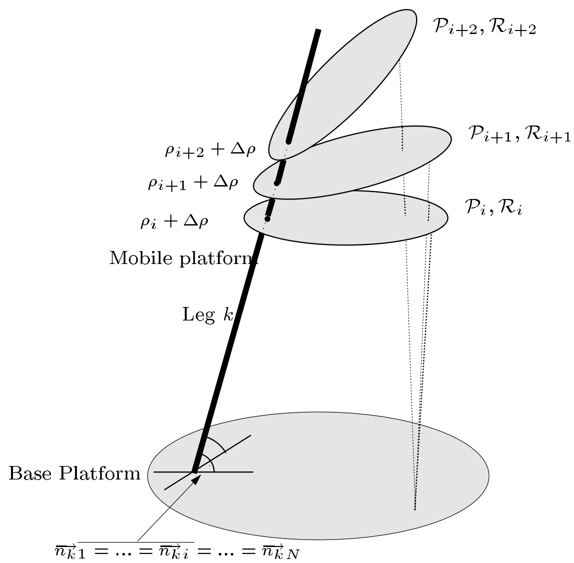
- Type I :We compute random positions $\mathcal{P}_i$ and orientation $\mathcal{R}_i, i = 1..N$ of the platform such as the direction of the leg k (denoted $\overrightarrow{n_k}_i$) is the same for all measurement poses $i = 1..N$: $\overrightarrow{n_k}_1 = ... = \overrightarrow{n_k}_i = ... = \overrightarrow{n_k}_N$ see–fig3(b). The leg lengths are free : ${\rho_1}^k \neq ... \neq {\rho_i}^k \neq ... \neq {\rho_N}^k$.

- Type II : We compute random positions $\mathcal{P}_i$ and orientations $\mathcal{R}_i, i = 1..N$ of the platform such as the directions and the length of leg k (note $\overrightarrow{n_k}_i$ and ${\rho_i}^k$) are the same for all measurement poses $i = 1..N$: $\overrightarrow{n_k}_1 = ... = \overrightarrow{n_k}_i = ... = \overrightarrow{n_k}_N$ and ${\rho_1}^k = ... = {\rho_i}^k = ... = {\rho_N}^k$ see–fig3(a).

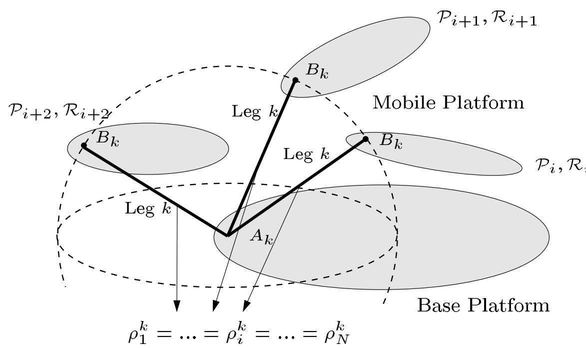
- Type III : We compute random positions $\mathcal{P}_i$ and orientations $\mathcal{R}_i, i = 1..N$ of the platform such as the length of leg k (denoted ${\rho_i}^k$) is the same for all measurement poses ${\rho_1}^k = ... = {\rho_i}^k = ... = {\rho_N}^k$ see–fig3(c) but the direction of segment k is free : $\overrightarrow{n_k}_1 \neq ... \neq \overrightarrow{n_k}_i \neq ... \neq \overrightarrow{n_k}_N$.



(a) The direction and the length of one leg are fixed simultaneously



(b) The direction of one leg is fixed



(c) The length of one leg is fixed

Figure 3: Constrained measurement poses

3.2 The direction of one leg is fixed fig3(b)

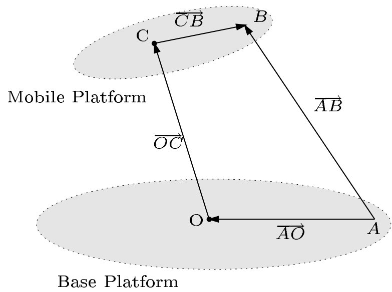


Figure 4: Inverse Kinematic.

For each leg of the parallel manipulator (fig4), we have:

$$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OC} + \mathcal{R}\overrightarrow{CB_r} \tag{1}$$

For any measurement configurations of the platform $i = 1..N$ with N number of measures, for the segment K :

$$\overrightarrow{a_k(b_k)_o} = \mathcal{P}_i + \mathcal{R}_i b_k - a_k \tag{2}$$

In the case where all measurement configurations ($i = 1..N$) have been computed for a fixed direction of the leg k (Type I), we get, for all measurement pose ($i = 1..N$) :

$$\frac{1}{\|\overrightarrow{a_k(b_k)_o}\|}\overrightarrow{a_k(b_k)_o} = \frac{1}{\rho_1^k}(\mathcal{P}_1 + \mathcal{R}_1 b_k - a_k) = ... = \frac{1}{\rho_i^k}(\mathcal{P}_i + \mathcal{R}_i b_k - a_k)... = \frac{1}{\rho_N^k}(\mathcal{P}_N + \mathcal{R}_N b_k - a_k) \tag{3}$$

where, ρ_i^k is the length measurement of the fixed segment k for the pose i.
We have the loop equation by writing that the unit vector of the leg k is constant for measurement pose i and $i+1$:

$$\frac{1}{\rho_i^k}(\mathcal{P}_i + \mathcal{R}_i b_k - a_k) - \frac{1}{\rho_{i+1}^k}(\mathcal{P}_{i+1} + \mathcal{R}_{i+1} b_k - a_k) = 0 \tag{4}$$

with $i = 1..N-1$.

For a set of N measurement, we can put equation (4) in linear form :

$$\begin{pmatrix} \rho_1^k\mathcal{R}_2 - \rho_2^k\mathcal{R}_1 & (\rho_1^k - \rho_2^k)I \\ ... & ... \\ \rho_i^k\mathcal{R}_{i+1} - \rho_{i+1}^k\mathcal{R}_i & (\rho_i^k - \rho_{i+1}^k)I \\ ... & ... \\ \rho_{N-1}^k\mathcal{R}_N - \rho_N^k\mathcal{R}_{N-1} & (\rho_{N-1}^k - \rho_N^k)I \end{pmatrix} \begin{pmatrix} b_k \\ \\ a_k \end{pmatrix} = \begin{pmatrix} \rho_2^k\mathcal{P}_1 - \rho_1^k\mathcal{P}_2 \\ ... \\ \rho_{i+1}^k\mathcal{P}_i - \rho_i^k\mathcal{P}_{i+1} \\ ... \\ \rho_N^k\mathcal{P}_{N-1} - \rho_{N-1}^k\mathcal{P}_N \end{pmatrix} \tag{5}$$

where I a 3×3 unit matrix,$\mathcal{R}_i$ a 3×3 matrix,$\mathcal{P}_i$ a 3×1 vector.
Equation (5) may also be written as :

$$\mathcal{M}_{3(N-1)\times 6}(P_k)_{6\times 1} = \mathcal{N}_{3(N-1)\times 1} \tag{6}$$

with $P_k = [bx_k, by_k, bz_k, ax_k, ay_k, az_k]^T$.

For N measurement poses, we get $3(N-1)$ linear equations. We have to identify 6 kinematic parameters per leg, $3(N-1) \geqslant 6$ so $N \geqslant 3$.
The least squares solution for these equations is :

$$P_k = (\mathcal{M}^T\mathcal{M})^{-1}\mathcal{M}^T\mathcal{N} \tag{7}$$

3.3 The direction and the length of one leg are fixed simultaneously

We want now calibrate Model 42. We take into account the offset of the legs.

Equation (4) can be rewritten, in the case where the direction of leg k is fixed as :

$$\frac{\mathcal{P}_i + \mathcal{R}_i b_k - a_k}{\rho_i^k + \Delta\rho_k} - \frac{\mathcal{P}_{i+1} + \mathcal{R}_{i+1} b_k - a_k}{\rho_{i+1}^k + \Delta\rho_k} = 0 \tag{8}$$

3.3.1 Determination of b_k

For a set of measurement configurations, we fix simultaneously the leg length and the direction of the leg k, (Type II). So $\rho_i^k + \Delta\rho_k = \rho_{i+1}^k + \Delta\rho_k$.

The equation (8) becomes :

$$(\mathcal{P}_{i+1} + \mathcal{R}_{i+1} b_k) - (\mathcal{P}_i + \mathcal{R}_i b_k) = 0 \tag{9}$$

The 3D equation (9) is linear in b_k term of a hence for N measurement poses :

$$\begin{pmatrix} \mathcal{R}_2 - \mathcal{R}_1 \\ \dots \\ \mathcal{R}_{i+1} - \mathcal{R}_i \\ \dots \\ \mathcal{R}_N - \mathcal{R}_{N-1} \end{pmatrix}_{3(N-1)\times 3} \begin{pmatrix} b_k \end{pmatrix}_{3\times 1} = \begin{pmatrix} \mathcal{P}_1 - \mathcal{P}_2 \\ \dots \\ \mathcal{P}_i - \mathcal{P}_{i+1} \\ \dots \\ \mathcal{P}_{N-1} - \mathcal{P}_N \end{pmatrix}_{3(N-1)\times 1} \tag{10}$$

with $i = 1..N$, $\mathcal{R}_i$ a 3×3 matrix,$\mathcal{P}_i$ a 3×1 vector.
We put equation (10) in a linear form :

$$\mathcal{A}_{3(N-1)\times 3}(b_k)_{3\times 1} = \mathcal{B}_{3(N-1)\times 1} \tag{11}$$

Equation (10) provide us $3(N-1)$ equations. To determinate the 3 coordinates of b_k we need at minimum 3 independent equations. As the rank of the matrix $(\mathcal{R}_{i+1} - \mathcal{R}_i)$ is 2, equation (10) provides us $2(N-1)$ independent equations. Hence to solve this problem, we need $N \geqslant 3$.

The least squares solution for these equations is :

$$b_k = (\mathcal{A}^T\mathcal{A})^{-1}\mathcal{A}^T\mathcal{B} \tag{12}$$

3.3.2 Determination of a_k

Now, we want to determine a_k with the knowledge of b_k (determined in 3.3.1).
In the sequel vector $\mathcal{P}_i + \mathcal{R}_i b_k$ for pose i will be denoted $\mathcal{V}_i$ (dim 3×1).
For all configuration poses, the leg length is fixed $\rho_i^k + \Delta\rho_k = \rho_{i+1}^k + \Delta\rho_k$ but the direction of the leg k is free (Type III).

The IK provides us :

$$\|\mathcal{P}_i + \mathcal{R}_i b_k - a_k\|^2 = (\rho_i^k + \Delta\rho_k)^2 \tag{13}$$

With the simplification and with leg length constraints $\rho_i^k + \Delta\rho_k = \rho_{i+1}^k + \Delta\rho_k$, we get :

$$\|\mathcal{V}_i - a_k)\|^2 = \|\mathcal{V}_{i+1} - a_k\|^2 = (\rho_i^k + \Delta\rho_k)^2 = (\rho_{i+1}^k + \Delta\rho_k)^2 \tag{14}$$

or in linear form for N measurement configurations :

$$\begin{pmatrix} (V_2 - V_1)^T \\ \dots \\ (V_{i+1} - V_i)^T \\ \dots \\ (V_N - V_{N-1})^T \end{pmatrix}_{N-1\times 3} \begin{pmatrix} a_k \end{pmatrix}_{3\times 1} = \frac{1}{2} \begin{pmatrix} \|V_2\|^2 - \|V_1\|^2 \\ \dots \\ \|V_{i+1}\|^2 - \|V_i\|^2 \\ \dots \\ \|V_N\|^2 - \|V_{N-1}\|^2 \end{pmatrix}_{(N-1)\times 1} \tag{15}$$

In matrix form :

$$\mathcal{C}_{(N-1)\times 3} \begin{pmatrix} a_k \end{pmatrix}_{3\times 1} = \mathcal{D}_{(N-1)\times 1} \tag{16}$$

The least squares solution for these equations is :

$$\begin{pmatrix} a_k \end{pmatrix} = (\mathcal{C}^T\mathcal{C})^{-1}\mathcal{C}^T\mathcal{D} \tag{17}$$

The Equation (15) provide $N-1$ equations. Hence to determinate the 3 coordinate of a_k, we need $N - 1 \geqslant 3$, so $N \geqslant 4$.

3.3.3 $\Delta\rho_k$ determination

With the knowledge of b_k (3.3.1), a_k (3.3.2) and with one position/orientation $\mathcal{P}, \mathcal{R}$ measurement pose, we want to determine $\Delta\rho_k$.

For any leg length measurement, the IK equation provides us :

$$\|\mathcal{P} + \mathcal{R}b_k - a_k\| = (\rho^k + \Delta\rho_k) \tag{18}$$

We get :

$$\Delta\rho_k = \|\mathcal{P} + \mathcal{R}b_k - a_k\| - \rho^k \tag{19}$$

4 Simulation

We want to simulate a calibration on the robot "Left Hand" built at INRIA. The coordinates of the attachment points are (in cm) :

Attachment points	Coordinates in cm X	Y	Z
a_1	-9.7	9.1	0.00
a_2	9.7	9.1	0.00
a_3	12.76	3.9	0.00
a_4	3	-13	0.00
a_5	-3	-13	0.00
a_6	-12.76	3.9	0.00
b_1	-3	7.3	0.00
b_2	3	7.3	0.00
b_3	7.822	-1.052	0.00
b_4	4.822	-6.248	0.00
b_5	-4.822	-6.248	0.00
b_6	-7.822	-1.052	0.00

This data are assumed to be the real parameters P_r of the robot (P_{kr} for the leg k). The real offset on the leg lengths is $\Delta\rho = 15.25cm$.

The algorithms are computed for k=3.

With this data, we compute some measurement poses of Type I, II and III. All simulations are done using Matlab.

Let P_{kc} be the kinematic parameters of leg k provided by the calibration algorithm. For evaluate the quality of the methods, we use the index error $\|P_{kc} - P_{kr}\|$.

To simulate measurement noise, we add on each measurement a random noise uniformly distributed. The amplitude of this error is err_p (in cm) for the position (i.e. we add a random number between $[-err_p...err_p]$ on the coordinate x,y,z of the position platform), err_a (in degree) for the orientation (i.e. we add a random number between $[-err_a...err_a]$ on the 3 Euler's angles describing the orientation platform) and err_l (in cm) for the leg length.

We have to take into consideration the influence of choice of the measurement poses on the calibration result. For ours methods this problem is easily understood : we rely on solving equation $\mathcal{A}x = \mathcal{B}$ to find a solution ; but the condition number of the matrix $\mathcal{A}$ (function of the measurement) determine the robustness of algorithms to the noise measurement ([2], [12]). In order to have a global evaluation of our algorithms independent of the choice of the measurement poses, we have to use each algorithm with a set of 1000 random measurements poses and computed an average index error as $\frac{1}{1000}\sum_1^{j=1000}\|P_{kc} - P_{kr}\|_j$ with j the index of a set of N measurement poses. To determinate the influence of the number of measurements, we process each algorithm for measurement pose number $N = 4..50$ for Type I, $N1 = 4..30$ for type II, and $N2 = 5..30$ for Type III (one more than the minimum required to get a robust least square solution).

- *For example for method 3.2 the test program is as follow :*
  For $N = 4$ to 50
  - *for $j = 1...1000$*
    - *1. We produce N simulation measurement poses of type I, we obtain $[\mathcal{P}_i, \mathcal{R}_i, \rho_i]_j$, we add error err_p, err_a, err_l on position orientation and leg length, we obtain $\mathcal{M}, \mathcal{N}$.*
    - *2. We process $P_{kc} = (\mathcal{M}^T\mathcal{M})^{-1}\mathcal{M}^T\mathcal{N}$*
    - *3. With the real kinematic parameters of the leg k : P_{kr}.*
      We process : $\|P_{kc} - P_{kr}\|_j = [\|a_{kc} - a_{kr}\|_j, \|b_{kc} - b_{kr}\|_j]$.
  - *end for*

- $||P_{kc} - P_{kr}|| = \frac{1}{1000} \sum_{i=j}^{1000} ||P_{kc} - P_{kr}||_j$

end For.

- *Example with method 3.3 :*
  - *Determination of* b_k *:*
    For $N1 = 4$ *to 30*
    - *for* $j = 1...1000$
      - *1. We produce* $N1$ *simulation measurement poses of type II, we obtain* $[\mathcal{P}_i, \mathcal{R}_i]_j$*, we add error* err_p, err_a *on position and orientation, we obtain* $\mathcal{A}, \mathcal{B}$*.*
      - *2. We process* $b_{kc} = (\mathcal{A}^T\mathcal{A})^{-1}\mathcal{A}^T\mathcal{B}$
      - *3. With the real kinematic parameters of the leg* k *:* b_{kr}*. We process :* $||b_{kc} - b_{kr}||_j$*.*
    - *end for*
    - $||b_{kc} - b_{kr}|| = \frac{1}{1000} \sum_{i=j}^{1000} ||b_{kc} - b_{kr}||_j$

    end For.
  - *Determination of* a_k *: For* $N2 = 5$ *to 30*
    - *for* $j = 1...1000$
      - *1. We produce* $N2$ *simulation measurement poses of type III, we obtain* $[\mathcal{P}_i, \mathcal{R}_i]_j$*, we add error* err_p, err_a *on position,orientation and on* b_{kr} *(to simulate* b_{kc} *obtain by the method 3.3.1) we obtain* $\mathcal{C}, \mathcal{D}$*.*
      - *2. We process* $a_k = (\mathcal{C}^T\mathcal{C})^{-1}\mathcal{C}^T\mathcal{D}$
      - *3. With the real kinematic parameters of the leg* k*, We process :* $||a_{kc} - a_{kr}||_j$*.*
    - *end for*
    - $||a_{kc} - a_{kr}|| = \frac{1}{1000} \sum_{i=j}^{1000} ||a_{kc} - a_{kr}||_j$

    end For.
  - *Determination of* $\Delta\rho_k$ *:*
    - *for* $j = 1...1000$
      - *1. We produce one measurement pose, we obtain* $[\mathcal{P}, \mathcal{R}, \rho]_j$*, we add error* err_p, err_a, err_l *on position, orientation and on* b_{kr}, a_{kr}*.*
      - *2. We process* $(\Delta\rho_k)_j = ||\mathcal{P} + \mathcal{R}b_k - a_k|| - \rho^k$
      - *3. With the real kinematic parameters of the leg* k*, We process :* $||\Delta\rho_{kc} - \Delta\rho_{kr}||_j$*.*
    - *end for*
    - $||\Delta\rho_{kc} - \Delta\rho_{kr}|| = \frac{1}{1000} \sum_{i=j}^{1000} ||\Delta\rho_{kc} - \Delta\rho_{kr}||_j$

    end For.

5 Results

If we simulate these methods without measurement noise, the kinematic parameters are exactly determinated.

The following results show the influences of the noise on position, orientation and leg length measurements.

5.1 Method 3.2

The comparison between the calibration results for each platform show that the noise have a larger influence on the base point error determination (Fig: 5). For $N = 7$ the factor between the input error (error on measurement in cm and in degree) and the output error (error on kinematic parameters in cm) is around 13 for the base and 1.4 for the mobile (Fig: 5,6). This difference is due to the conditionement of the loop equations.

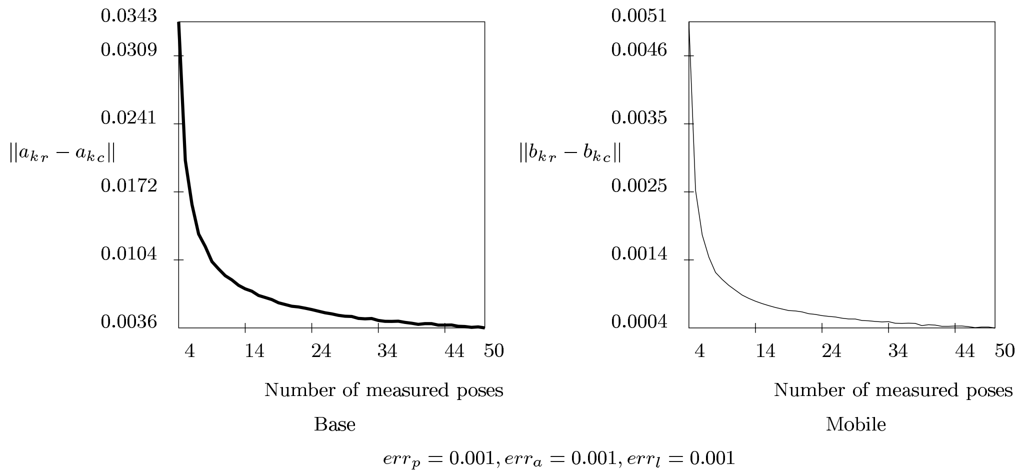


Figure 5:

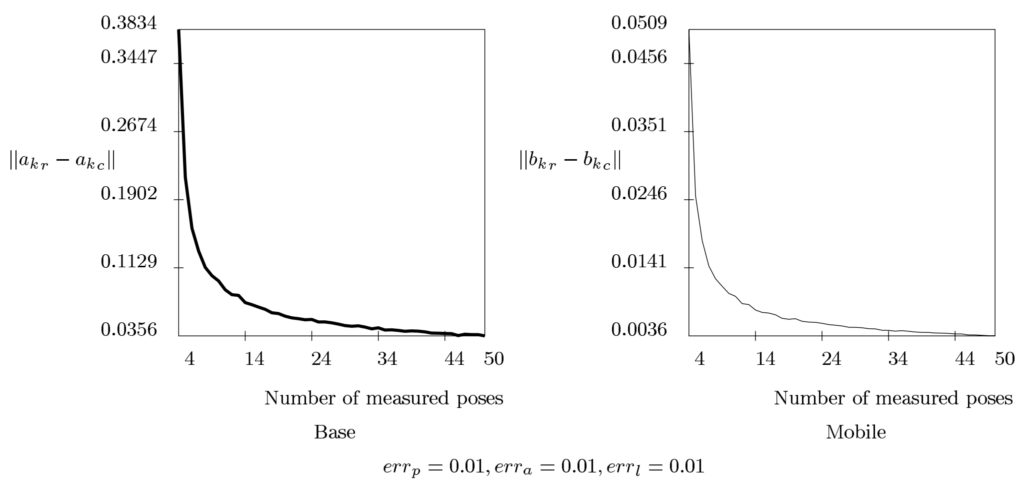


Figure 6:

To compare this algorithm with existing method, Innocenti's algorithm [6] has been used with the same type of noise and the same number of measurement poses. The result factor obtain for the base and for the mobile is near 15.

In order to compare the influences of orientation, position or leg length measurement error, we check the kinematic parameter error with different level of noise (Fig: 7,8).

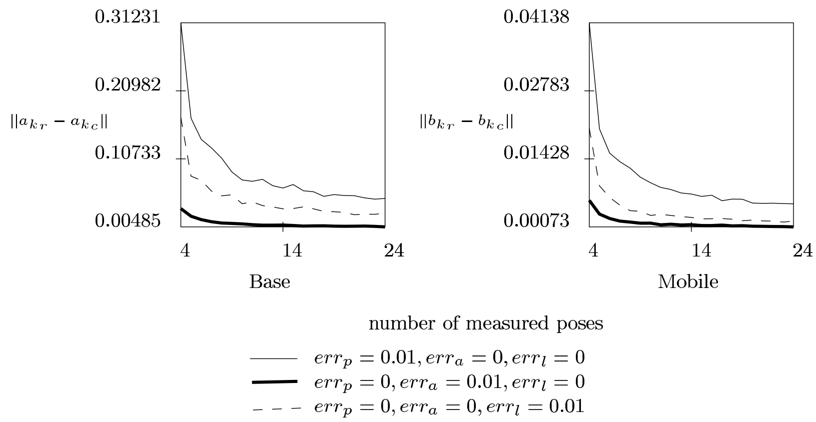


Figure 7:

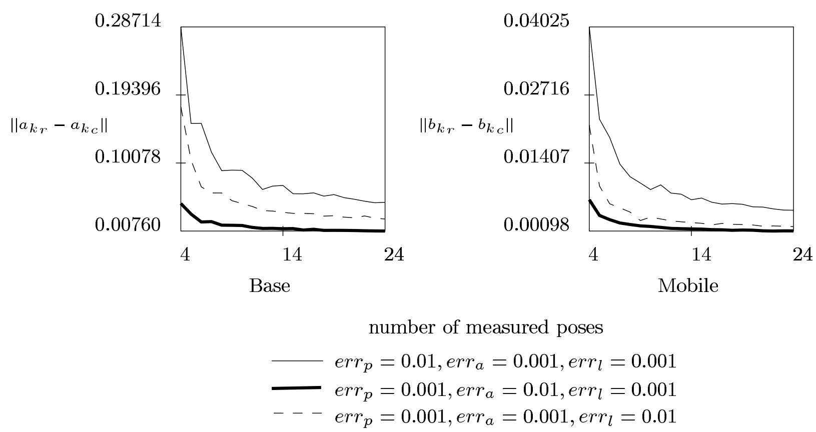


Figure 8:

5.2 Method 3.3

The base point determination algorithm (3.3.2) need the coordinates of mobile point process by algorithm (3.3.1). We use b_k determinate for $N1 = 7$ with a noise on position and orientation measurement :

- For $err_p = 0.01cm, err_a = 0.01degree$, we get $b_k = [7.8291, -1.05809, 0.007995]$.
- For $err_p = 0.001cm, err_a = 0.001degree$, we get $b_k = [7.82149, -1.052215, 0.0011]$.

It is interesting to note, we do not need leg length measurement to determine the attachment points. So we have not to take into account the leg measurement error.

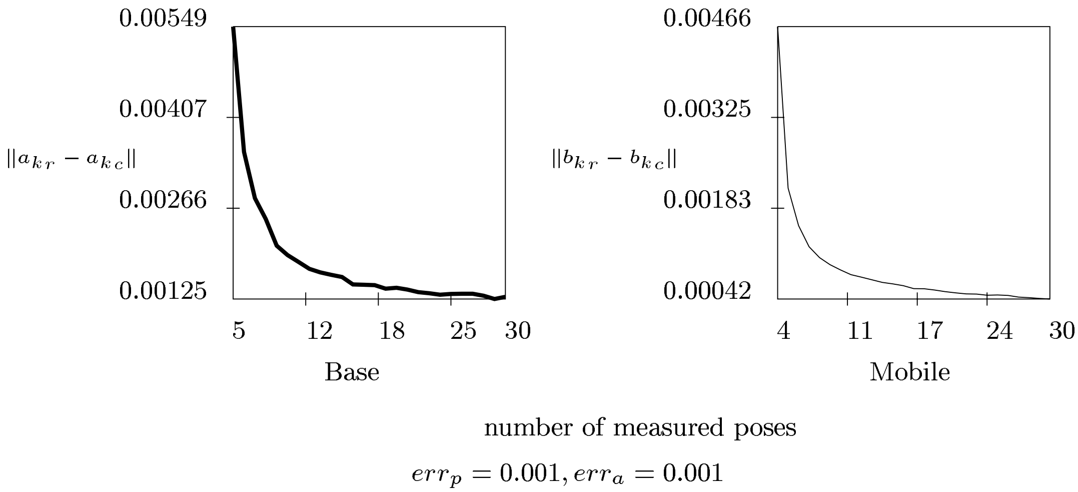


Figure 9:

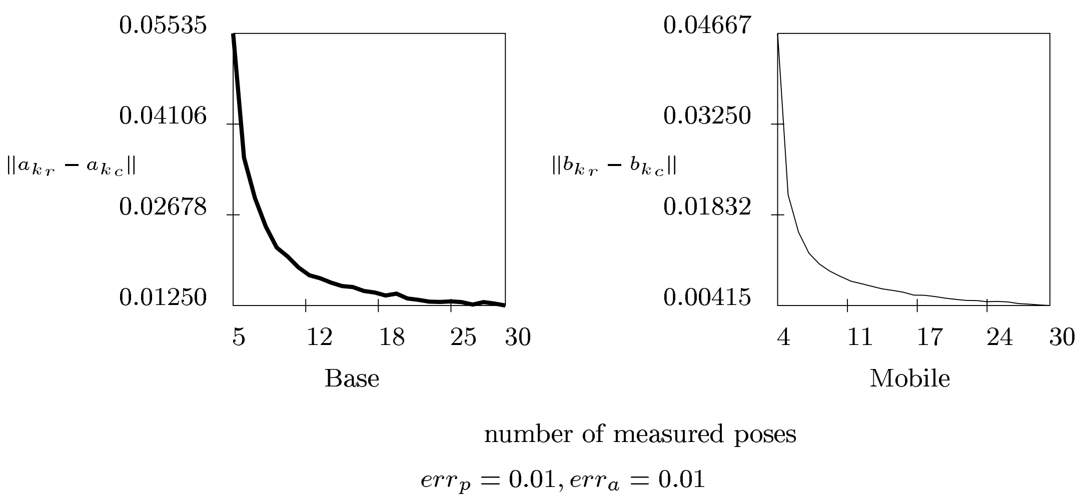


Figure 10:

For this method, the error on mobile kinematic parameters is as the same order as the previous method (figure 9,10)(output/input error factor = 1.4). But, if b_k is calculated with a small error (here $N1 = 7$), we get a better accuracy on a_k (of 2 $\times$ error on the mobile determination)(figure 9,10)(for $N1 = 7$, $N2 = 7$ output/input error factor = 2.8). We need here $N1 + N2$ independent location measurements instead of N for the first algorithm, so we have more information data to calibrate a robot leg. Conversely, the first algorithm is interesting for his low number of measurements.

The error on the linear transducers offset is dependent of the determination of attachment points. So the mean error on the offset is close to the biggest noise applied on measurement (position ,orientation, leg length) or processed through algorithm (b_k, a_k).

6 Conclusion

Two effective algorithms for identification of kinematic parameters have been presented and verified through simulations. The proposed calibration methods are based on data sets each composed of one platform location and corresponding leg length. The mobility constraints on leg provide us some linear equations function of unknown kinematic parameters and measurement. Neither algorithm requires initial estimation of the unknown. The error on kinematic parameters can be easily improved by additional measures. The first calibration algorithm unambiguously provides the coordinates of all spherical pair centers, and the second add the knowledge of the offset values of the manipulator linear transducers.

The advantages of these algorithms are the simplicity of the method due to the linear form of the loop equations, the velocity of these algorithms compared to non-linear numerical methods, a robust determination of mobile (3.2) or mobile/base (3.3) attachment points and the low number of minimum measurement configurations required.

An study of the condition number of the linear equations provided by these methods can be used to improve the kinematic parameters error by a choice of measurement configurations.

The paper show the interest to use constraints on the robot leg to simplify loop equations and/or to hide unknowns. Imposing constraints on the leg and on the location of the platform can be interesting to solve kinematic calibration problem.

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