Reversibility of Elementary Cellular Automata Under Fully Asynchronous Update

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Abstract

We investigate the dynamics of Elementary Cellular Automata (ECA) under fully asynchronous update with periodic boundary conditions. We tackle the reversibility issue, that is, we want to determine whether, starting from any initial condition, it is possible to go back to this initial condition with random updates. We present analytical tools that allow us to partition the ECA space into three classes: strongly irreversible, irreversible and recurrent.

keywords:
asynchronous cellular automata, reversibility, recurrence, Markov chain modelling, classification

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I Introduction

Cellular automata (CA) are spatially-extended dynamical systems which evolve in discrete time and space. They have been extensively studied as models of physical systems and as models of massively parallel computing devices.

Cellular automata are classically defined with a synchronous update, that is, all the cells simultaneously apply the local transition rule to produce the new state of the automaton. This definition has however been questioned in various works and different models of asynchronous cellular automata have been proposed. There are numerous reasons for studying asynchronism, such as: designing robust distributed algorithms (e.g. for self-stabilisation), studying the robustness of discrete models of natural phenomena, obtaining a better understanding of the dynamics of cellular automata, etc. Interested readers may refer to a recent survey paper for an overview of this field [5].

Our aim is to study how the notion of reversibility in the context of simple asynchronous CA with a *stochastic* updating. We focus on Elementary Cellular Automata (ECA), that is, binary, one-dimensional CA, where the next state of a cell after an update is determined by the current states of the left and right neighbors and the state of the cell itself.

Reversibility of synchronous deterministic cellular automata has been studied for decades [1,3,4,8,11]. However, the study of reversibility of asynchronous cellular automata has been only recently explored. Two different aspects have been studied: on the one hand, the question was asked as to how to update an asynchronous CA so that the system returns to its initial condition. It was shown that it is possible to find an answer for a given subset of one-dimensional asynchronous CA [2,9,10]. The construction of the arguments was possible under the hypothesis that the sequence of updates is *chosen*. This introduction of update patterns relies on the hypothesis that an external operator is allowed to choose the cells to update in order to return to a given initial condition.

On the other hand, given a CA rule and a type of updating, it was asked to which extent it is possible to construct another rule whose transition graph would be an "inverse" of the transition graph of the original rule. Formally, this means that, given a rule f, we want to know if there is a rule f' such that if for f a state y is reachable from x, then, for f', x is reachable from y [13].

We now tackle a different case: we consider that the ECA are updated in a (stochastic) fully asynchronous mode, that is, at each discrete time step, a single cell is chosen randomly and uniformly for update. In this context, as we will see below, studying reversibility amounts to answering the following question: can we decide whether an asynchronous cellular automaton is *recurrent*, that is, if the system will almost surely return to the initial condition?

Using the definitions from the theory of Markov chains, we propose a full characterisation of the ECA rules into three classes: the strongly irreversible, irreversible, and recurrent rules. Intuitively, these class respectively correspond to the following behaviours: no possibility to return to the initial condition, a possibility to return to the initial condition a finite number of times and, an infinite number of returns to the initial condition.

Table 1: Look-up table for rule 87, 99 and 110

x,y,z	111	110	101	100	011	010	001	000	*Rule*
RMT	(7)	(6)	(5)	(4)	(3)	(2)	(1)	(0)	
f(x,y,z)	0	1	0	1	0	1	1	1	87
f(x,y,z)	0	1	1	0	0	0	1	1	99
f(x,y,z)	0	1	1	0	1	1	1	0	110

II Definitions

The cellular automata we consider use periodic boundary conditions: cells are arranged as a *ring* and we denote by $\mathcal{L} = \mathbb{Z}/n\mathbb{Z}$ the set of cells. The global *state* of the system at a given time will be represented by an element of $\{\mathtt{0},\mathtt{1}\}^{\mathcal{L}}$; for example, for a ring of $n = 6$ cells, we will simply write $x = 011001$ a particular state and denote by x_i the state of a particular cell $i \in \mathcal{L}$. We denote by **0** and **1** the two homogeneous states with cell state 0 and 1, respectively. Similarly, **01** denote a state of even size in which cell states 0 and 1 alternate, **001** a state whose size is a multiple of three, where two 0s are followed by a 1, etc.

An ECA is defined by a local transition function $f : \{0,1\}^3 \mapsto \{0,1\}$; it is common to define such a function with a look-up table (see Table 1). There are $2^8 = 256$ ECA rules, each one referred to with the number that corresponds to the decimal equivalent of the binary number formed by the sequence of its transitions results [14]. Three such rules (87, 99 and 110) are shown in Table 1.

Definition 1 *The association of the neighbourhood x, y, z to the value $f(x, y, z)$, which represents the result of the updating function, is called* Rule Min Term *(RMT). Each RMT is associated to a number $\mathtt{R}(x, y, z) = 4x+2y+z$. An RMT $\mathtt{R}(x, y, z)$ is active $f(x, y, z) \neq y$ and otherwise passive.*

For example, for rule 110, RMT 1 is active and RMT 6 is passive (see Table 1).

We now consider *fully asynchronous updating*, that is, the case where only a single cell is updated randomly and uniformly at each time step. While a synchronous CA is a deterministic system, in an asynchronous CA (ACA), the next state not only depends on the local rule but also on the cells which are updated.

We denote by u_t the cell that updated at time t ; the sequence $U = (u_t)_{t\in\mathbb{N}}$ is called an *update pattern*. For an initial condition x and an update pattern U, the evolution of the system is given by the sequence of states (x^t) obtained by successive applications of the updates of U. Formally, we have: $x^{t+1} = F(x^t, u_t)$ and $x^0 = x$, with:

$$x_i^{t+1} = \begin{cases} f(x_{i-1}^t, x_i^t, x_{i+1}^t) & \text{if } i = u_t \\ x_i^t & \text{otherwise.} \end{cases}$$

This evolution can be represented in the form of *a state transition diagram*. For example, Fig. 1 shows a partial state transition diagram of rule 110 with state

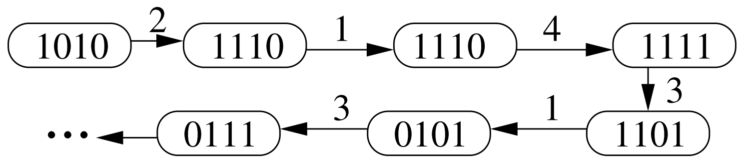

Figure 1: Partial state transition diagram of rule 110 with $n = 4$. The cells updated during evolution are noted over arrows (convention kept).

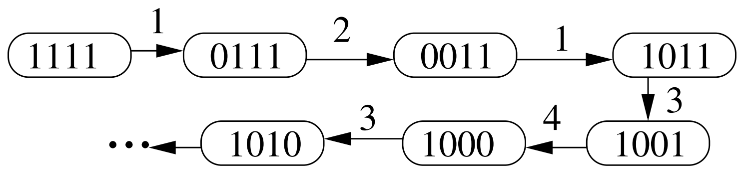

Figure 2: Partial state transition diagram of ECA 87 with $n = 4$.

$x = 1010$ and update pattern $U = (2, 1, 4, 3, 1, 3, \ldots)$. The index of the cell that is updated is noted over the arrows.

Definition 2 *A state x is* reachable *if it has at least one predecessor, that is, if there exists a CA state $y \in \mathcal{E}_n$ and an update position $u \in \mathcal{L}$ such that $F(y, u) = x$; otherwise the state is* non-reachable *(or a garden-of-Eden state).*

For instance, for ECA 110, the state 1110 is reachable as it has 1010 as a predecessor (see Fig. 1). By contrast, for ECA 87, 1111 is non-reachable (see Fig. 2). Indeed, if it had a predecessor, it would necessarily be equal to 1101, up to shifts, as only one cell can change at a time. But as the transition 101 (RMT 5) is passive, the last 0 can not disappear. Remark that a system may contain both types of states, reachable and non-reachable.

A state x is converted to an RMT sequence $\tilde{x}$ with: $\tilde{x_i} = \mathtt{R}(x_{i-1}, x_i, x_{i+1})$ for all $i \in \mathcal{L}$. For example, the state $x = 001010$ is associated to the RMT sequence $\tilde{x} = 012524$. RMT sequences will be used to establish the proofs of recurrence or irreversibility of the ECA rules.

III (Ir) reversibility of ACA

The issue of reversibility of CA has given rise to the use of various terms to name the same properties; for instance, the term "invertible" has been used as a synonymous of "reversible" [12]. This variety of terms comes from the proximity between the physical notion of reversibility and its equivalent in discrete dynamical systems. We emphasise that, in the CA context, reversibility informally denotes the possibility to "invert" the evolution of a cellular automaton, by using potentially *another* cellular automaton and *not* the fact that the evolution of the system is similar when it is run "backwards". The term *time-symmetric* has been recently used to qualify the rules whose evolution is similar if the arrow of time is "inverted" [7]. As there are multiple views on reversible CA, we note that in the deterministic synchronous case, the following statements are equivalent:

1. Each CA state has exactly one predecessor.
2. There exists no CA state that is non-reachable.
3. Each CA state lies on a cycle.
4. Each CA state is returned back in the course of dynamic evolution.

However, these definitions can not be transposed in a straightforward way to asynchronous cellular automata and in that case, the *classical* definition of reversibility needs to be revisited. One solution was that proposed consisted in associating the notion of reversibility with a given update pattern, that is, to a sequence of updates decided in advance [10]. However, in the case where cells are updated *randomly*, new difficulties arise. For instance, in the ACA case, Statement 4 also implies Statement 2 and Statement 3, but does not imply Statement 1. This leads us to search for another definition of reversibility for an ACA. Here, we choose to start from Statement 4 for defining the reversibility of ACA: we require that in an asynchronous reversible CA, each state has to be returned back almost surely during the evolution of the system.

As we use the fully asynchronous updating, the evolution of our ACA is described by a Markov chain over the space of CA states $Q^{\mathcal{L}}$. We thus define the reversibility properties using the classical tools from Markov chain theory, which leads to identify reversibility and recurrence.

Definition 3 *For a couple of states $x, y \in Q^{\mathcal{L}}$, we say that y is reachable from x if there is a sequence of updates that leads from x to y, that is:*

$$\exists k \in N^*, U = (u_0, \ldots, u_{k-1}), x^0 = x, x^k = y$$

and

$$x^{i+1} = F(x^i, u_i) \text{ for all } i \in \{0, \ldots, k-1\}.$$

We now introduce the main tool of our study :

Definition 4 *A state $x \in Q^{\mathcal{L}}$ is recurrent if for every state y that is reachable for x, x is also reachable from y. A state that is not recurrent is transient.*

Intuitively, a transient state is such that a particular sequence of updates may bring into a particular state from which it will never be possible to return back to the initial state. More formally, if y is reachable from x and x is reachable from y, we say that x and y *communicate*. By convention, all states communicate with themselves. Clearly, the relationship "communicate" is an equivalence relation; this relation partitions the set of states into communication classes. In words, two major behaviours exist: for the transient states, the system remains for an

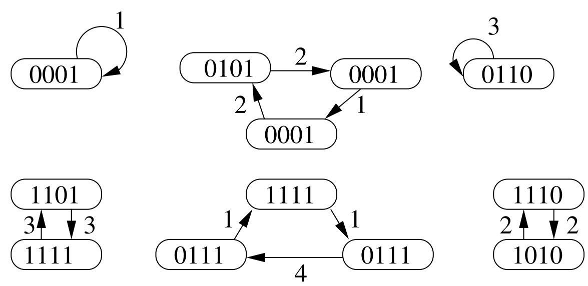

Figure 3: Transition diagram for ECA 99 with $n = 4$

almost surely finite time in the communication class, then "escapes" this class and never returns back to it. In contrast, when the system is in a recurrent state, it remains in the communication class for ever.

We can now define the (ir)reversible cellular automata:

Definition 5 *An ACA is recurrent if each CA state is recurrent, otherwise it is irreversible.*

The definitions above allow us to know if some irreversibility is present in the system but they do not say anything about the "degree of irreversibility" of the system. Indeed, it may well be that the system does possess a transient state but that the sequence of updates that leads to observe the irreversibility is never observed in practice when the updates are random. This is a difficult problem to tackle in all generality. As first step, we propose here to deal with the states where it is not possible to return back *whatever* the sequence of updates.

Definition 6 *A state x is evanescent if it is not reachable from itself. An ACA that possesses an evanescent state is strongly irreversible.*

It is interesting to remark that the set of evanescent and non-reachable states are equal. Indeed, by definition a non-reachable state is evanescent. To see why the converse is true, let us assume by contradiction that x is an evanescent state that is reachable from y. We say that the cell $i \in \mathcal{L}$ of a state $x \in Q^{\mathcal{L}}$ is *active* if the transition which applies in i is active, that is, if $f(x_{i-1}, x_i, x_{i+1}) \neq x_i$. Note that x is a *fully unstable state* (all its cells are active). It is then easy to see that y is also reachable from x (as the two states differ in only one cell) and thus, that x is reachable from itself, which contradicts the evanescence hypothesis.

As a consequence, if a rule is strongly irreversible it possesses at least one non-reachable state. However, the converse is not true: for instance, for rule 51 (the `NOT` rule), all states are fully unstable but the rule is reversible.

Fig. 3 depicts an example for a recurrent CA. In the state transition diagram of ECA 99, each state can be returned to with some given update of cells noted over the arrows. It can be shown (e.g., with an exhaustive search) that all the states of rule 99 ACA are recurrent.

IV Identifying the strongly irreversible rules

We first present the theorem that allows us to identify strongly irreversible ECA:

Theorem 1 *An ECA is strongly irreversible if and only if one of the following conditions is verified:*

1. *RMT 0 (resp. RMT 7) is active and RMT 2 (resp. RMT 5) is passive.*
2. *RMTs 2 and 5 are active and RMTs 0 and 7 are passive.*
3. *RMTs 1, 2 and 4 (resp. RMTs 3, 6 and 5) are active, and RMTs 0, 3 and 6 (resp. 1, 4 and 7) are passive.*

Proof :

First, let us prove the "if" part, that is, if one of the conditions is verified then the ECA is strongly irreversible. We proceed by examining the conditions one by one and by exhibiting for each case a non-reachable (and thus an evanescent) state.

Case 1: Let us show that **0** (with RMT 0 only) is non-reachable. Assume that y is a predecessor of **0**. First $y \neq \mathbf{0}$ as **0** is fully unstable (RMT 0 is active). The CA state y thus contains a single 1 (as the number of ones can only vary by 1 in the fully asynchronous update) and the transition from y to **0** was applied on the single 1 and with RMT 2. However, this is impossible as RMT 2 is passive. The case of RMT 5 and 7 is identical up to the 0/1 exchange.

Case 2: Let us show that **01** (with RMT 2 and 5 only) is non-reachable. First, if RMTs 2 and 5 are active, then this CA state is fully unstable. Now, assume that there is a CA state $x \neq \mathbf{01}$ and an updated cell i such that $F(x,i) = \mathbf{01}$, then, as x and **01** differ on only cell, it is easy to see that either RMT 0 or 7 produced a change of state on i, which is impossible if RMT 0 and RMT 7 are both passive.

Case 3: Let us show that **001** (with RMT 1, 2 and 4 only) is non-reachable. First, we note that this CA state is fully unstable as its RMT sequence is **124**. Again, if **001** had a predecessor $x \neq \mathbf{001}$, then the last update on x is either a 0 changed into a 1 (application of RMT 0) or a 1 changed into a 0 (application of RMT 3 or 6), which in both cases can not happen if RMTs 0, 3 and 6 are all passive. The proof for the RMTs shown into parentheses is identical up to the 0/1 exchange. □

Let us now show that the three conditions above of Th. 1 are also necessary for an ECA to be strongly irreversible.

Proof :

Let us consider an ECA that has a non-reachable state x. We will show that x has only four "forms" (up to the 0/1 exchange) that each brings us to the three conditions of the theorem. Let x be a non-reachable state. First, let us note that x is fully unstable and that no transition can lead to x. As a consequence, we can state an *exclusion rule* : $\tilde{x}$, the RMT sequence of x, can not contain two transitions in one of the following couples of RMTs {0, 2}, {1, 3},

$\{4, 6\}$ and $\{5, 7\}$. To see why, assume for example that RMTs 0 and 2 are both present in $\tilde{x}$, that is, $\exists i, j \in \mathcal{L}, \tilde{x_i} = 0$ and $\tilde{x_j} = 2$. As x is fully unstable, RMT 0 and 2 are both active, then, it can be remarked that two successive updates on i (or j) make the system return to x, that is, $F(F(x, i), i) = x$, which is in contradiction with the fact that x is non-reachable.

Now, let us discuss the various possibilities for x.

Case a: Let us assume that $\tilde{x}$ contains a 0. If x contains at least one 1, that is, if $x \neq \mathbf{0}$, then, we can note that x contains either the sequence 00010 or the sequence 00011, that is, $\tilde{x}$ either contains the sequence 012 or the sequence 013. However, the two cases lead to a contradiction due to the exclusion rule. The only possibility is then $x = \mathbf{0}$, which implies that RMT 0 is active and RMT 2 is passive (due to the exclusion rule). We are thus in case 1 of the theorem. The case with RMTs 5 and 7 is symmetric by 0/1 exchange.

Now, let us assume that $\tilde{x}$ does not contain RMT 0 nor RMT 7. This implies that x contains at least one 01 pattern. We need to distinguish several sub-cases.

Case b: If x does not contain the 00 or 11 pattern, that is $x = \mathbf{01}$ and $\tilde{x} = \mathbf{25}$, we can deduce that RMT 0 and 5 are active and that RMT 1 and 7 are passive (exclusion rule); we are then verifying Case 2 of the theorem.

Case c: Let us now assume that $x \neq \mathbf{01}$, and without loss of generality, that x contains the 00 pattern. Then, x necessarily contains the pattern 1001 (otherwise, it would contain the pattern 000) which means that $\tilde{x}$ thus contains the RMTs 4 and 1, and, because of the exclusion rule, does not contain RMT 3 nor 6. Two possibilities are now offered :

Case d: x contains the pattern 10011 : this is excluded because of the exclusion rule as this would imply that $\tilde{x}$ contains RMT 1 and 3.

Case e: x contains the pattern 10010 but does not contain pattern 000 (RMT 0), nor 011 (RMT 3), nor 110 (RMT 6). This means that $\tilde{x}$ contains 1, 2 and 4 (if $x = \mathbf{001}$) and possibly RMT 5 (if x contains 100101). In both cases, this means that RMT 1, 2 and 4 are active and that RMT 0, 3, 6 are passive and the last case of the theorem is proved.

The parts of the theorem presented into parentheses are symmetric to the cases discussed above by the 0/1 exchange. □

As a consequence, it can be seen that the RMT sequence of a non-reachable state necessarily verifies one of the four following combinations of RMTs: (a) only RMT 0 (or only RMT 7), (b) only RMTs 2 and 5 (c) only RMTs 1, 2 and 4 (or only RMTs 3, 5 and 6), (d) only RMTs 1, 2, 4 and 5 (or only RMTs 2, 3, 5 and 6).

There are 132 such strongly irreversible rules; they are listed in Table 2.

Example 1 *Let us consider ECA 87, a rule which satisfies the conditions of Th. 1 as RMT 7 is active and RMT 5 is passive (see Table 1). Consider the evolution of state* $\mathbf{1}$. *If a cell is updated with RMT 7, then one 0 appears and it is easy to see that afterwards the last 0 cannot disappear as RMT 5 is passive. Hence,* $\mathbf{1}$ *is non-reachable and is evanescent which makes the rule strongly irreversible.*

Table 2: List of the 132 strongly irreversible rules that verify Theorem 1. Bold fonts show the *minimal representative rules* (rules with the smallest code among the group of rules that are obtained by left-right and 0-1 exchange).

0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
10	**11**	**12**	**13**	**14**	**15**	16	17	**18**	**19**
20	21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**
30	31	**37**	39	**45**	47	53	55	61	63
64	65	66	67	68	69	70	71	**72**	**73**
74	75	**76**	**77**	**78**	79	80	81	82	83
84	85	86	87	88	89	**90**	91	92	93
94	95	101	103	109	111	117	119	**122**	125
127	133	135	141	143	149	151	157	159	**160**
161	**162**	**164**	165	167	**168**	**170**	173	175	176
178	181	183	**184**	186	189	191	197	199	205
207	213	215	218	221	223	224	226	229	231
232	234	237	239	240	242	245	247	248	250
253	255								

V Identification of the recurrent rules

This section identifies the set of rules which are irreversible and, by complementation, those which are recurrent. We proceed by identifying the rules for which particular states are transient.

Theorem 2 *A rule R is irreversible if one of the following conditions is verified:*

1. *RMT 0 (resp. RMT 7) is active and RMT 2 (resp. RMT 5) is passive or RMT 2 (resp. RMT 5) is active and RMT 0 (resp. RMT 7) is passive.*
2. *RMTs 0, 1, 2 and 4 (resp. RMTs 3, 5, 6 and 7) are passive and RMT 3 or 6 (resp. RMT 1 or 4) are active.*
3. *RMTs 0, 2, 3 and 6 (resp. RMTs 1, 4, 5 and 7) are passive and RMT 1 or 4 (resp. RMT 3 or 6) are active.*

Proof :

Case 1: If RMT 0 is active and RMT 2 is passive, as shown the proof of Case 1 of Th. 1, **0** is evanescent and thus transient. If RMT 2 is active and RMT 0 is passive, let us consider the state $x = 00100$. If the third cell is updated, the system reaches **0**, which is a fixed point, and which implies that x transient.

Case 2: Now, consider $x = 001100$; its RMT sequence is $\tilde{x} = 013640$. If RMT 3 is active, $y = 000100$ can be reached by updating the third cell. This a fixed point as its RMT sequence contains only 0, 1, 2 and 4, which all correspond to passive RMTs. Similarly, if RMT 6 is active the fixed point $y = 001000$ can be reached; which shows that x is transient.

Table 3: List of the 46 rules that are conjectured to be recurrent.

33	**35**	**38**	**41**	**43**	**46**	49	**51**	52	**54**
57	59	**60**	**62**	97	99	102	**105**	107	**108**
113	115	116	118	121	123	131	**134**	139	**142**
145	147	148	**150**	153	155	**156**	158	195	198
201	**204**	209	211	212	214				

Table 4: List of the 78 remaining rules: conjectured to be the irreversible ACA that are not strongly irreversible.

32	**34**	**36**	**40**	**42**	**44**	48	**50**	**56**	**58**
96	98	100	**104**	**106**	**110**	112	114	120	124
126	**128**	129	**130**	**132**	**136**	137	**138**	**140**	144
146	**152**	**154**	163	166	169	171	**172**	174	177
179	180	182	185	187	188	190	192	193	194
196	**200**	202	203	206	208	210	216	217	219
220	222	225	227	228	230	233	235	236	238
241	243	244	246	249	251	252	254		

Case 3: We start with $x = 00100$; its RMT sequence is $\tilde{x} = 01240$. As RMT 1 and 4 are active, $y = 00110$ or $y' = 01100$ can be reached. However, from any CA state that contains two or more 1s, it is not possible to return to x as RMTs 2, 3 and 6 are passive. (This implies that a 1 that has at least one 0 next to it can not disappear). Hence, x is transient.

The proofs for the RMTs mentioned in the parentheses is identical by exchanging the cell states 0 and 1. □

By rewriting the conditions of the theorem, it can be verified that the rules for which it does *not* apply verify the following conditions: RMTs 0 and 2 (resp. 5 and 7) are either both active or both passive, and : a) there is at least one couple of active RMTs in the following sets: $\{2,5\}$, $\{1,6\}$, $\{3,4\}$, $\{1,3\}$, $\{4,6\}$ or b) RMTs 1, 3, 4 and 6 are all passive. There are 46 rules which verify these conditions, which are listed in Table 3. Our conjecture is that all these rules are recurrent, that is, all their states are recurrent.

The 210 rules which satisfies at least one condition of Th. 2 are irreversible. We have already identified 132 rules (Tab. 2) as strongly irreversible. The remaining 78 irreversible ACA are listed in Table 4; we conjecture that they are not strongly irreversible, that is, they have at least one transient state but no evanescent state.

VI Conclusion

We reported a classification of the ECA space according to reversibility properties under fully asynchronous update with periodic boundary conditions. The main step now consists in completing this classification by showing that the list of recurrent rules presented are closed. This could be done analysing the communication classes of the state space of these rules. While the classifications based on the convergence time to a fixed point remain mainly open [6], achieving this result would represent an important step in the understanding of the dynamics of asynchronous CA.

As usual in the field of CA, one may ask how to extend the results to other types of asynchronism and to the CA spaces with a higher radius or higher dimension. As suggested by I. Marcovici, the classification can also be refined by considering "escaping states", that is, states where there is a possibility to stay but for which once this state is leaved, it can not be returned to.

Another question is to know if the reversibility issues presented here are similar to other views, for instance the one recently studied by Wacker and Worsch [13].

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