

A Guide to Algorithm Design: Paradigms, Methods, and Complexity Analysis

Anne Benoit, Yves Robert, Frédéric Vivien

► **To cite this version:**

Anne Benoit, Yves Robert, Frédéric Vivien. A Guide to Algorithm Design: Paradigms, Methods, and Complexity Analysis. Chapman

Hall/CRC, pp.380, 2013, Applied Algorithms and Data Structures series, 9781439825648. <hal-00908448>

HAL Id: hal-00908448

<https://hal.inria.fr/hal-00908448>

Submitted on 23 Nov 2013

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Anne Benoit, Yves Robert, Frédéric Vivien

A guide to algorithm design

Paradigms, methods and complexity analysis

CRC PRESS

Boca Raton London New York Washington, D.C.

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