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***Abstract*—This paper studies a significant problem in green networking called switching off base stations in case of cooperating service providers by means of stochastic geometric and coalitional game tools. The coalitional game herein considered is played by service providers who cooperate in switching off base stations. When they cooperate, any mobile is associated to the nearest BS of any service provider. Given a Poisson point process deployment model of nodes over an area and switching off base stations with some probability, it is proved that the distribution of signal to interference plus noise ratio remains unchanged while the transmission power is increased up to preserving the quality of service. The coalitional game behavior of a typical player is called to be *hedonic* if the gain of any player depends solely on the members of the coalition to which the player belongs, thus, the coalitions form as a result of the preferences of the players over their possible coalitions' set. We utilize the Nash-stable core for determining the coalitions of service providers.**

I. Introduction

The term "green networking" is described in [1] as *the practice of selecting energy-efficient networking technologies and products, and minimizing resource use whenever possible*. It covers all dimensions of the network such as personal computers, peripherals, switches, routers, etc. Having a green network may allow to reduce CO_2 emissions and thus will help mitigating the global warming [2]. However, having a significant impact on the overall energy consumption call for improving the energy efficiency of all network components.

With a growing awareness to the dangers related to large scale energy consumption and drafting of many international agreements as well as legislation have reduced energy consumption in several sectors [3]. There is also a growing willingness to reduce energy consumption in wireless networks. On the one hand, wireless communication infrastructures, like the ones managed by mobile network operators, are a major contributor to the ever-increasing energy consumption of the ICT industry, which calls for the adoption of energy-efficient solutions in their design and operation. Moreover, the recent explosive growth of smartphones market adoption and the consequent mobile internet traffic requirements have prompted waves of research and standard development activities to meet the expected future demands in an energy-efficient manner. On the other hand, wireless networks will also be a major component of the communication infrastructure required by other "green" solutions for the efficient management of energy, since they enable practices like telecommuting (e.g., traffic reduction) and remote administration (e.g., the Smart Energy Grid), which are expected to significantly help reduce the environmental footprint of many human activities.

Energy consumption can be reduced by dynamically switching off/on cells, base stations (BSs) and other radio resources (e.g. transmit antennas), according to observed traffic load, resource utilization, quality and coverage. This paper studies the problem of cooperation of mobile network operators for switching off BSs in a setting that the energy saving by switching off BSs is sought to maximize. The assumption is that operators share their BSs and any customer can be associated to the BS of any operator. Indeed, we suppose that each operator has information about the average transmitted power of their BSs.

The cellular network model consists of BSs arranged according to some homogeneous Poisson point process in the Euclidean plane. The main weakness of the Poisson model is that because of the independence of the Poisson point process, BSs will in some cases be located very close together but with a significant overlap area. This weakness is balanced by two strengths: the natural inclusion of different cell sizes and shapes and the lack of edge effects, i.e. the network extends indefinitely in all directions [5].

In this work, we use stochastic geometry (refer to [6], [7]) to evaluate the overall gain that can be reached with some cooperation. This work is thus based on optimizing the density of switched on BSs. The cooperation of operators is analyzed using the rules of hedonic coalition formation. We utilize the Nash-stable core [8] for determining the coalitions of service providers.

II. The Model

We assume full frequency reuse. Each mobile is associated with the BS being nearest to it. All BSs being out the nearest one cause interference to the mobile.

We consider a homogenous independently marked Poisson p.p. of BSs represented by $\tilde{\Phi} = \sum_i \delta_{(X_i, M_i)}$ where X_i shows the location of BS i, and M_i denotes the mark corresponding to the BS i. Indeed, a mark shows the energy profile of related SP. Consider a tagged mobile at an arbitrary point on the Euclidean plane, say the origin.

Let p_0 denote the point in $\tilde{\Phi}$ which is the closest to it, and represents the BS to which it is connected. Let d_i be the distance of p_i to the origin. Moreover, suppose that average transmission power of a typical BS is P_t. This power should be understood as resulting from a long-term observation the SP performs. We consider an attenuation due to a path-loss with exponent α as well as the effect of fading denoted by the random variable h. The transmission power received at the tagged mobile from p_0 is thus given by $P_t h_0 d_0^{-\alpha}$. Thus, the total average interference from other BSs is $P_t \sum_{i\in\Phi\backslash p_0} h_i d_i^{-\alpha}$. Hence, the SINR at the mobile is [4]

$$\mathsf{SINR} = \frac{P_t h_0 d_0^{-\alpha}}{P_t \sum_{i\in\Phi\backslash p_0} h_i d_i^{-\alpha} + \sigma^2} = \frac{P_t h r^{-\alpha}}{I+\sigma^2}, \tag{1}$$

where σ^2 stands for additive noise variance.

A. Base Station Energy Profile

The energy consumption model is mandatory to predict the power consumption of a typical BS as a function of the traffic load. In this paper, we adopt a linear energy profile model for a typical BS formulated as

$$P = P_0 + \beta P_{tot}, \tag{2}$$

where P_0 denotes the power consumption for operational tasks, β is the slope of the traffic-dependant part, and P_{tot} is the total transmitted power by the corresponding BS. Further, we assume that the average total transmission power is a function of the traffic intensity, given by

$$P_{tot}(T) = p + wf\left(\frac{T}{\lambda}\right), \tag{3}$$

where p is the minimum average total transmission power, e.g. signaling overhead in common pilot or control signal, w represents the power used per throughput (W/bps) as well as T is the traffic intensity (bps/m^2) and λ is the intensity of BSs ($points/m^2$) of the corresponding SP. In this work, we suppose that p and w are equal of each SP. Note that T/λ is the traffic per BS ($bps/point$). The function $f(T/\lambda)$ represents the power consumption as a function of the traffic.

Actually, different energy profiles could be proposed [9]–[11]. Here, since we focus to work on the coalitional game, we consider without loss of generality a simple linear model

B. SINR Distribution

By considering only the SINR, it means that we do not take into account any power control at the transmission. Here, the transmission power is constant for all mobiles.

First, we give the definition of coverage probability [5]:

$$p_C = \mathbf{P}\{\mathsf{SINR} > \rho\}, \tag{4}$$

where ρ is the target SINR that ensures the coverage. The distribution of the SINR is thus the complementary probability of the coverage probability, i.e. $p_{\mathsf{SINR}} = 1 - p_C$.

The distance between the origin and the nearest BS has the following probability density function [5]:

$$f(r) = e^{-\lambda\pi r^2} 2\pi\lambda r. \tag{5}$$

Conditioning on the nearest BS being at a distance r from the mobile, the probability of coverage is

$$p_C(\lambda) = \int_0^\infty \mathbf{P}\left\{ h > \frac{\rho(\sigma^2+I)}{P_t r^{-\alpha}} \middle| r \right\} e^{-\lambda\pi r^2} 2\pi\lambda r dr. \tag{6}$$

If we consider the case of Rayleigh fading, the random variable $h \sim \exp(\mu)$ follows an exponential distribution with mean $1/\mu$, and therefore [5]

$$p_C(\lambda) = \int_0^\infty \exp\left(-\frac{\mu\rho\sigma^2}{P_t r^{-\alpha}}\right) \mathcal{L}_I\left(\frac{\mu\rho}{P_t r^{-\alpha}}\right) e^{-\lambda\pi r^2} 2\pi\lambda r dr, \tag{7}$$

where $\mathcal{L}_I(s)$ is the Laplace transform of random variable I given as

$$\begin{aligned}
\mathcal{L}_I(s) &\triangleq \mathbf{E}\{\exp(-sI)\} \\
&= \mathbf{E}_{\Phi,h_i}\left\{ \exp\left(-sP_t \sum_{i\in\Phi\backslash p_0} h_i d_i^{-\alpha}\right)\right\} \\
&= \mathbf{E}_{\Phi}\left\{ \prod_{i\in\Phi\backslash p_0} \mathbf{E}_h\left\{\exp\left(-sP_t h d_i^{-\alpha}\right)\right\}\right\} \\
&= \exp\left(-\int_r^\infty \left(1-\mathbf{E}_h\left\{\exp\left(-sP_t h x^{-\alpha}\right)\right\}\right) 2\pi\lambda dx\right) \\
&= \exp\left(-2\pi\lambda \int_r^\infty \frac{x}{1+\frac{\mu}{sP_t x^{-\alpha}}} dx\right)
\end{aligned} \tag{8}$$

where $s = \frac{\mu\rho}{P_t r^{-\alpha}}$ and the expectation over the fading and the p.p. are independent. Since the all h_i have the same distribution, we are able to calculate the expectation over only one variable denoted as h. Hence, the distribution of SINR can be calculated by

$$\begin{aligned}
p_{\mathsf{SINR}}(\lambda) = 1 - p_C(\lambda) = 1 - 2\pi\lambda \int_0^\infty \exp\Bigg(- \pi r^2 \lambda \Bigg(1 + \\
\frac{2\rho\, {}_2F_1\left(1, 1-\frac{2}{\alpha}; 2-\frac{2}{\alpha}; -\rho\right)}{\alpha-2} \Bigg) - \frac{\mu\rho\sigma^2}{P_t r^{-\alpha}} \Bigg) r dr,
\end{aligned} \tag{9}$$

in which hypergeometric function ${}_2F_1(a,b;c;z)$ is a special function represented by the hypergeometric series defined for $|z| < 1$ by the power series

$${}_2F_1(a,b;c;z) = \sum_{n=0}^\infty \frac{(a)_n (b)_n}{(c)_n} \frac{z^n}{n!} \tag{10}$$

provided that $c \neq 0, -1, -2, \ldots$. Here $(f)_n$ is the Pochhammer symbol defined by

$$(f)_n = \begin{cases} 1, & \text{if } n = 0, \\ f(f+1)\cdots(f+n-1), & \text{if } n > 0. \end{cases} \tag{11}$$

For $\alpha = 4$, the coverage probability can be found to be as following:

$$p_C(\lambda, \alpha = 4) = \frac{\lambda}{2\sigma}\sqrt{\frac{\pi^3 P_t}{\mu\rho}}\text{erfc}\left(\frac{\pi\lambda}{2\sigma}\sqrt{\frac{P_t}{\mu\rho}}\Big(1+ \sqrt{\rho}\tan^{-1}(\sqrt{\rho})\Big)\right)\times \exp\left(\left(\frac{\pi\lambda}{2\sigma}\sqrt{\frac{P_t}{\mu\rho}}\left(1+\sqrt{\rho}\tan^{-1}\left(\sqrt{\rho}\right)\right)\right)^2\right). \quad (12)$$

III. Switching off Base Stations

In this section, we introduce the underlying approach to the problem in which the BSs are turned off. We assume that a SP has an observation that a typical BS is activated with some probability q. According to data traffic the SP turns on or off a BS. It gathers the information of this operation and set the activation probability q during that long enough period observations. However, we look for such an optimum value of q by which the SP maximizes its own energy saving introduced in Section IV-D.

A. Scaling

In this section, we adopt the technic of thinning a p.p which is performed through scaling. We put forward in [4] this approach. Also, here, we give a proof which does not exist in [4]. In the following, we derive the intensity measure of points of $\tilde{\Phi}^q$ being a thinned version of the initial one.

Lemma 1: **Thinning through scaling**: *Choose some $0 \le q \le 1$. Scaling each coordinate by $\sqrt{q}$ in $\mathbb{R}^2$ results in a p.p. Φ^q of intensity measure $q\Lambda$ if the initial p.p. Φ has some intensity measure Λ.*

Proof: Let the coordinates of a typical point on some $E \subset \mathbb{R}^2$ be x and y, respectively. Scaling up each coordinate by q gives the new coordinates $x' = x/q$ and $y' = y/q$, respectively. We know that the distance from the origin before scaling is $d = \sqrt{x^2 + y^2}$. After scaling the distance becomes $d' = \frac{1}{q}\sqrt{x^2 + y^2}$ which means that each point moves away from the origin by

$$d' = \frac{d}{q}. \quad (13)$$

It is straightforward to understand that when scaling up only one coordinate by q results in a new p.p. with intensity measure $q\Lambda$. Then, we are able to state that scaling up each coordinate by q brings out a new p.p. with intensity measure $q^2\Lambda$.

Eventually, a new p.p Φ^q is obtained by scaling each coordinate by $\sqrt{q}$ of the original one Φ which corresponds to the deleting independently points with probability $1-q$. Deleted points should be imagined as the BSs that are switched off. Consequently, the new intensity measure is $\Lambda^q = q\Lambda$. ■

B. SINR Distribution of the Scaled Network

Let us now calculate the new SINR distribution while the initial p.p. is scaled by $\sqrt{q}$. If we replace all d_i in (1) by d'_i and replace P_t by $\bar{P}_t$ then we can interpret the SINR distribution of the original p.p. as the one corresponding to a network where BSs are located according to a new p.p. with intensity parameter $\lambda^q = q\lambda$ where $\bar{P} = P_0 + \beta\bar{P}_{tot}$.

Using the relation of transmission power given in eq. (3), the coverage probability given in eq. (9) can be expressed as

$$p_C(\lambda) = 2\pi\lambda\int_0^\infty \exp\left(-\pi r^2\lambda(1+2f(\alpha,\rho)) - \frac{\mu\rho\sigma^2}{P_t r^{-\alpha}}\right) r dr. \quad (14)$$

The scaling of initial process requires $\lambda \to q\lambda$ then $r \to r/\sqrt{q}$, $dr \to dr/\sqrt{q}$ which gives the following coverage probability

$$p_C(q\lambda) = 2\pi q\lambda\int_0^\infty \exp\left(-\pi\left(\frac{r}{\sqrt{q}}\right)^2 q\lambda(1+2f(\alpha,\rho)) - \frac{\mu\rho\sigma^2}{P_t q^{\alpha/2} r^{-\alpha}}\right)\frac{r dr}{q}. \quad (15)$$

It is straightforward that if we choose $P_t \to P_t q^{-\alpha/2}$, then $p_C(q\lambda) = p_C(\lambda)$. Moreover, the average total transmission power of a BS of the thinned network is obtained by $P_{tot} \to P_{tot}q^{-\alpha/2}$. This result indicates that the SINR distribution remains unchanged while the average total transmission power is increased by $q^{-\alpha/2}$ of a typical BS.

IV. Multiple Service Providers

In this section, we extend the analysis to the multiple SPs case. We derive the formulation for two SPs but the results can be immediately obtained for more than two SPs case.

A. Non-cooperation of SPs

Assume that there are two SPs. We denote by Φ_1 and Φ_2 the location of the BSs of the SP 1 and SP 2 with intensity parameters λ_1 and λ_2, respectively where the energy profiles of SP 1 and SP 2 are given as P_1 and P_2, respectively. Furthermore, let the scaling factors of two SPs be $\sqrt{q_1}$ and $\sqrt{q_2}$. Hence, the p.p. after scaling of SP 1 and SP 2 is represented as $\Phi_1^{q_1}$ and $\Phi_2^{q_2}$, respectively. The points of network which is a result of the sum of thinned version of SP 1 and SP 2 can thus be represented as $\Phi_1^{q_1} + \Phi_2^{q_2}$ with intensity measure $q_1\Lambda_1 + q_2\Lambda_2$.

We assume that the mobile is a customer of SP 1. Provided that SPs do not cooperate, we suppose that the thinning is performed independently by each SP. The SINR of the mobile before scaling can be calculated as following:

$$\mathsf{SINR} = \frac{P_{t,1}h_0 d_0^{-\alpha}}{P_{t,1}\sum_{i\in\Phi_1\setminus p_0} h_i d_i^{-\alpha} + P_{t,2}\sum_{j\in\Phi_2} h_j d_j^{-\alpha} + \sigma^2}, \quad (16)$$

where d_0 denotes the distance of the mobile to the nearest BS of SP 1 as well as $P_{t,i}$ is the transmission power of SP i.

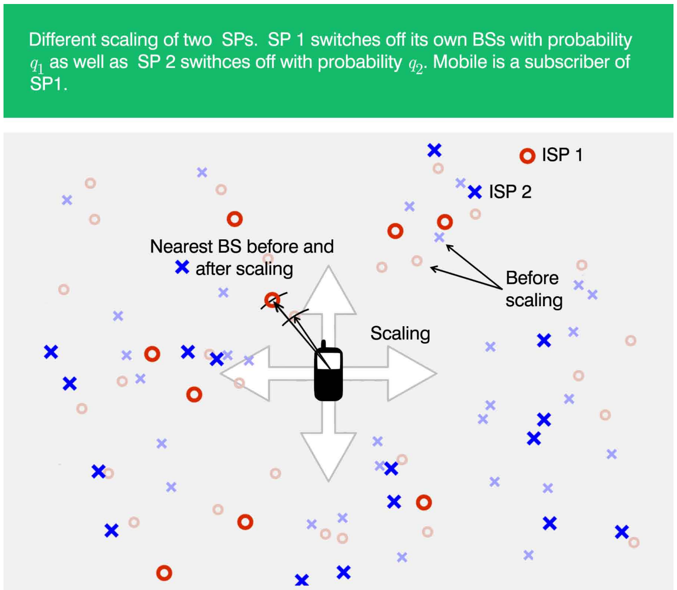

Fig. 1. Scaling in case of non-cooperation of two SPs.

B. SINR Distribution in case of Non-cooperation

In order to derive the distribution of SINR when SPs do not cooperate, we first represent the SINR as following:

$$\mathsf{SINR} = \frac{P_{t,1} h r^{-\alpha}}{I_1 + I_2 + \sigma^2}. \tag{17}$$

The coverage probability conditioning to the nearest BS of SP 1 is formulated as

$$p_C(\lambda_1, \lambda_2) = \int_0^\infty \mathbf{P}\left\{ h > \frac{\rho(\sigma^2 + I_1 + I_2)}{P_{t,1} r^{-\alpha}} \middle| r \right\} \times \exp(-\pi\lambda_1 r^2) 2\pi\lambda_1 r dr, \tag{18}$$

where r is the distance between mobile and the nearest BS of SP 1. Then, the coverage probability can be expressed as

$$p_C(\lambda_1, \lambda_2) = \int_0^\infty \exp\left(-\frac{\mu\rho\sigma^2}{P_{t,1} r^{-\alpha}}\right) \mathcal{L}_{I_1}\left(\frac{\mu\rho}{P_{t,1} r^{-\alpha}}\right) \times \mathcal{L}_{I_2}\left(\frac{\mu\rho}{P_{t,1} r^{-\alpha}}\right) \exp(-\pi\lambda_1 r^2) 2\pi\lambda_1 r dr. \tag{19}$$

The Laplace transform of the interferences arising due to SP 1 and SP 2 are given as

$$\mathcal{L}_{I_1}(s) = \exp\left(-2\pi\lambda_1 \int_r^\infty \frac{x}{1 + \frac{\mu}{s P_{t,1} x^{-\alpha}}} dx\right), \tag{20}$$

$$\mathcal{L}_{I_2}(s) = \exp\left(-2\pi\lambda_2 \int_0^\infty \frac{x}{1 + \frac{\mu}{s P_{t,2} x^{-\alpha}}} dx\right), \tag{21}$$

respectively, where $s = \frac{\mu\rho}{P_{t,1} r^{-\alpha}}$. Note that the lower limit of the Laplace transform integral of I_2 is zero which takes into account the interference that occurs from the points of SP 2 being nearer than the nearest BS of SP 1. Thus, the integral in eq. (22) gives the coverage probability of non-cooperation case. For specific values of α, closed form coverage probability expressions can be found. For example, let $\alpha = 4$. The coverage probability is given in eq. (23).

The coverage probability of the networks of SP 1 and SP 2 scaled by $\sqrt{q_1}$ and $\sqrt{q_2}$, respectively can be calculated by the integral in eq. (24). By increasing the transmission power of SP 1 and SP 2 as $P_{t,1} \to P_{t,1} q_1^{-\alpha/2}$, $P_{t,2} \to P_{t,2} q_2^{-\alpha/2}$, respectively, the SINR distributions of two SPs do not change in the non-cooperation case (see eq. (24)).

C. Cooperation of SPs

In the case of cooperation among the SPs, any mobile is associated to the nearest BS of any SP. Thus, SPs share their resources in order to obtain a better SINR level for each customer. By this way, the power consumption can be lowered providing a "green" approach to the BS deployment.

Let $S = (1, 2)$ show the coalition of SP 1 and SP 2. We assume that in case of a cooperation the operators control jointly the network such that the activation probability q_S of a BS is determined in order to maximize the energy saving density. Formally, the traffic and the network intensity that reveals by cooperation are supposed to be additive, i.e. $\sum_{i \in S} T_i$ and $\sum_{i \in S} \lambda_i$, respectively. It is also considered that the network formed by cooperation has the equal average total transmission power per BS given by

$$P_{tot}^S = p + wf\left(\frac{\sum_{i \in S} T_i}{\sum_{i \in S} \lambda_i}\right). \tag{25}$$

Thus, the SINR can be expressed as following:

$$\mathsf{SINR} = \frac{P_t^S h_0 d_0^{-\alpha}}{P_t^S \sum_{i \in \Phi_S} h_i d_i^{-\alpha} + \sigma^2}, \tag{26}$$

where $\Phi_S = \Phi_1 + \Phi_2$ denotes the p.p. which is a result of sum of the p.p. of SP 1 and SP 2 having the intensity $\lambda_1 + \lambda_2$ which is a direct result of superposition property, and P_t^S is the transmission power between mobile and the tagged BS in case of cooperation.

Scaling each coordinate by $\sqrt{q_S}$ results in a network which corresponds to a thinned one as a consequence of cooperation. The points of the network resulting from the scaling is denoted as Φ^{q_S} which has the intensity measure $q_S(\lambda_1 + \lambda_2)$. Thus, we can adopt the same result obtained for the single operator where the transmission power is adjusted as $P_t^S \to P_t^S q_S^{-\alpha/2}$. Moreover, the energy profile of a typical BS of SP i is given by

$$P_i^S = P_{0,i} + \beta_i P_{tot}^S. \tag{27}$$

We also denote by $\bar{P}_i^S = P_{0,i} + \beta_i P_{tot}^S q_S^{-\alpha/2}$ the energy profile of SP i corresponding to the thinned network.

D. Energy Saving

We are interested to see what is the energy saving by switching off BSs (independently) with probability $1 - q_S$, given that at the same time we increase the transmission energy to compensate for decreasing the resources in a way that the probability distribution of the SINR is unchanged.

Now, we introduce the energy saving density when SPs form a coalition S. The power consumption density of SP

$$p_C(\lambda_1,\lambda_2) = 2\pi\lambda_1\int_0^\infty \exp\left(-\pi r^2\lambda_1\left(1+\frac{2\rho\,{}_2F_1\left(1,\frac{\alpha-2}{\alpha};2-\frac{2}{\alpha};-\rho\right)}{\alpha-2}\frac{2\pi\frac{\lambda_2}{\lambda_1}\left(\frac{P_{t,1}}{\rho P_{t,2}}\right)^{-2/\alpha}}{\alpha\sin\left(\frac{2\pi}{\alpha}\right)}\right)-\frac{\mu\rho\sigma^2}{P_{t,1}r^{-\alpha}}\right)rdr. \quad (22)$$

$$p_C(\lambda_1,\lambda_2,\alpha=4)=\frac{\lambda_1}{2\sigma}\sqrt{\frac{\pi^3P_{t,1}}{\mu\rho}}\mathrm{erfc}\left(\frac{\pi^2\ \lambda_2\sqrt{\rho P_{t,2}}+2\pi\lambda_1\sqrt{P_{t,1}}\left(\sqrt{\rho}\tan^{-1}\left(\sqrt{\rho}\right)-1\right)\right)}{4\sigma\sqrt{\mu\rho}}\right)\times$$
$$\exp\left(\left(\frac{\pi^2\lambda_2\sqrt{\rho P_{t,2}}+2\pi\lambda_1\sqrt{P_{t,1}}\left(\sqrt{\rho}\tan^{-1}\left(\sqrt{\rho}\right)-1\right)}{4\sigma\sqrt{\mu\rho}}\right)^2\right). \quad (23)$$

$$p_C(q_1\lambda_1,q_2\lambda_2)=2\pi q_1\lambda_1\int_0^\infty \exp\left(-\pi\left(\frac{r}{\sqrt{q_1}}\right)^2 q_1\lambda_1\left(1+\frac{2\rho\,{}_2F_1\left(1,\frac{\alpha-2}{\alpha};2-\frac{2}{\alpha};-\rho\right)}{\alpha-2}+\right.\right.$$
$$\left.\left.\frac{2\pi\frac{q_2\lambda_2}{q_1\lambda_1}\left(\frac{P_{t,1}}{\rho P_{t,2}}\right)^{-2/\alpha}}{\alpha\sin\left(\frac{2\pi}{\alpha}\right)}\right)-\frac{\mu\rho\sigma^2}{P_{t,1}\left(\frac{r}{\sqrt{q_1}}\right)^{-\alpha}}\right)\frac{rdr}{q_1}. \quad (24)$$

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i can be calculated by $\lambda_i P_i^S$. We can also calculate the power consumption density by $q^S\lambda_i\bar{P}_i^S$ when considering the thinned network. So, in case of coalition S, the energy saving density is characterized as $\sum_{i\in S}\lambda_i P_i^S - q_S\sum_{i\in S}\lambda_i\bar{P}_i^S$ resulting in following function

$$G(S)=(1-q_S)\sum_{i\in S}\lambda_i P_{0,i}+\left(1-q_S^{1-\alpha/2}\right)P_{tot}^S\sum_{i\in S}\lambda_i\beta_i. \quad (28)$$

Let us represent by $U_S = \sum_{i\in S}\lambda_i P_{0,i}$ and $V_S = P_{tot}^S\sum_{i\in S}\lambda_i\beta_i$. The meaning of these variables can be interpreted as following: U_S corresponds to the energy saving density of the operational power costs arising due to the BSs of coalition S as well as V_S is the energy saving density due to transmission power in case of coalition S. Then, the energy saving density can be expressed as

$$G(S)=(1-q_S)U_S+\left(1-q_S^{1-\alpha/2}\right)V_S,\quad \left[\frac{W}{m^2}\right]. \quad (29)$$

V. Cooperation Behavior in the context of Coalitional Games

Cooperation among agents (here, those correspond to SPs) is significant because interaction between them would lead to different factions (coalitions among agents). The analysis of cooperation is performed using coalitional games in general. At the end of cooperation, the agents obtain some gain (utility). The allocation of the gain is important in determining the stability of coalition partition. Coalitional game theory studies mainly two cases: (i) how to find such gain allocation methods that would guarantee the stability of grand coalition (all players are in the same coalition), (ii) how to determine the coalition partition when grand coalition is not possible (refer to [12]). The latter is called as *coalition formation games* introduced in Section VI-B. In most cases, the grand coalition cannot be reached. Therefore, coalition formation games are used to analyze the cooperation behavior of agents. Then, the question arises: *which coalition should be joined?* Applying selfish decisions to coalition formation is called *hedonic coalition formation*.

In this paper, we study the case in which the SPs determine their decisions in a hedonic setting. To this end, the coalition partitions are found by using the tools of hedonic coalition formation games. We utilize *the Nash-stable core* as a solution concept which includes those gain allocation methods that always result in the partitions from which none deviates [8].

VI. Coalitional Game Preliminaries

We represent a coalitional game in utility function form as $\langle N,u\rangle$ where $N=(1,2,\dots n)$ is a non-empty finite set of players who consider different cooperation possibilities, and $u:2^N\to\mathbb{R}$ is the utility function. Each subset $S\subset N$ is referred to as a *crisp coalition*. The set N is called the *grand coalition* and $\emptyset$ is called the *empty coalition* where $u(\emptyset)=0$. We denote the collection of coalitions, i.e. the set of all subsets of N by 2^N. These games are usually called coalitional games with transferable utility (TU games, for short) where its members can jointly guarantee themselves and which can be transferred without loss between them [15].

A. Utility Allocation

The utility of player $i\in S$ is denoted by ϕ_i^S. The meaning is that what player i gains being in coalition S. The sum of utilities in a coalition S must be equal to the total utility which is called *efficiency*: $\sum_{i\in S}\phi_i^S=u(S)$. The gain vector of player i for all possible coalitions is denoted by $\phi_i\in\mathbb{R}^{2^{n-1}}$. For example, let $N=(1,2)$ then $\phi_1=\left\{\phi_1^{(1)},\phi_1^{(1,2)}\right\}$. Moreover, we call as *allocation method* $\phi\in\mathbb{R}^{n2^{n-1}}$ the gains of all possible coalitions of all players, i.e., $\phi=\{\phi_1,\phi_2,\dots,\phi_n\}$.

B. Coalition Formation Games

In some cases acting together may be difficult, costly or illegal, or the players may for various personal reasons not wish to do so [14]. Then, the question arises: how the coalitions does have to be formed in order that the players do not deviate from?

A coalition formation game is given by a pair $\langle N, \succ \rangle$, where $\succ = (\succeq_1, \succeq_2, \ldots, \succeq_n)$ denotes the preference profile, specifying for each player $i \in N$ his preference relation $\succeq_i$, i.e. a reflexive, complete and transitive binary relation.

Definition 1: A coalition structure or a *coalition partition* is defined as the set $\Pi = \{S_1, \ldots, S_l\}$ which partitions the players set N, i.e., $\forall k$, $S_k \in N$ are disjoint coalitions such that $\bigcup_{k=1}^{l} S_k = N$. Given Π and i, let $S_\Pi(i)$ denote the set $S_k \in \Pi$ such that $i \in S_k$ [13].

Definition 2: **Hedonic Coalition Formation**: A coalition formation game is classified as hedonic if [13]

1) *The gain of any player depends solely on the members of the coalition to which the player belongs.*
2) *The coalitions form as a result of the preferences of the players over their possible coalitions set.*

Definition 3: **Nash Stability**: A partition Π is said to be Nash stable if no player can benefit from moving from his coalition $S_\Pi(i)$ to another existing coalition S_k, i.e., $\forall i, S_\Pi(i) \succeq_i S_k \cup \{i\}$ for all $S_k \in \Pi \cup \{\emptyset\}$ [13].
Nash-stable partitions are immune even to those movements of individuals when a player who wants to change does not need permission to join an existing coalition [13].

C. Properties of Preferences

The preference relation of a player can be defined over a *preference function*. Let us denote by $\pi_i : 2^N \to \mathbb{R}$ the preference function of player i. Thus, player i prefers the coalition S to T iff,

$$\pi_i(S) \geq \pi_i(T) \Leftrightarrow S \succeq_i T. \tag{30}$$

Furthermore, we are able to define the preference relation by means of a function which characterizes how a player prefers another player when they share the same coalition. In the following, we define this function. The preferences of player i is said to be *additively separable* if there exists a function $v_i : N \to \mathbb{R}$ such that $\forall S, T \subseteq N$

$$\sum_{j \in S} v_i(j) \geq \sum_{j \in T} v_i(j) \Leftrightarrow S \succeq_i T, \tag{31}$$

where we normalize by setting $v_i(i) = 0$ [13].

A profile of additively separable preferences, represented by $(v_i, \ldots, v_n)$, satisfies *symmetry* if $v_i(j) = v_j(i), \forall i, j$.

D. The Nash-Stable Core

Let us assume that the preference function of player i is the gain obtained in the corresponding coalition, i.e., $\pi_i(S) = \phi_i^S$. Algorithmically, for any partition $\Pi \in \mathcal{P}$ where $\mathcal{P}$ is the set of all possible partitions, if the following linear program is feasible, then the Nash-stable core is non-empty.

$$\min_{\phi \in \Re^{|\phi|}} \left\{ \sum_{\forall S \in 2^N} \sum_{\forall i \in S} \phi_i^S \middle| \phi_i^{S_\Pi(i)} \geq \phi_i^T, \forall T \in \Pi \cup \emptyset, \forall i \in N \right.$$
$$\left. \text{and} \sum_{j \in S} \phi_j^S = u(S), \forall S \in 2^N \right\}, \tag{32}$$

where $|\phi| = n(2^{n-1} - 1)$ as well as we denote as $\mathcal{C}_\Pi := \{\phi_i^{S_\Pi(i)} \geq \phi_i^T, \forall T \in \Pi \cup \emptyset, \forall i \in N\}$ the set of constraints arising due to partition Π. Combining all possible constraint sets provides the sufficient condition of the non-emptiness of Nash-stable core:

$$\mathcal{N}\text{-core} = \left\{ \sum_{\forall S \in 2^N} \sum_{\forall i \in S} \phi_i^S \middle| \bigcup_{\Pi \in \mathcal{P}} \mathcal{C}_\Pi \right.$$
$$\left. \text{and} \sum_{j \in S} \phi_j^S = u(S), \forall S \in 2^N \right\}, \tag{33}$$

where $\bigcup_{\Pi \in \mathcal{P}} \mathcal{C}_\Pi$ is the union of all possible constraint sets. Note that it is a non-trivial problem as well as the union could result in a non-convex set.

In case of *relaxed efficiency*, the sum of allocated utilities in a coalition is not strictly equal to the utility of the coalition, i.e. $\sum_{i \in S} \phi_i^S \leq u(S)$ which results in

$$\sum_{i,j \in S : j > i} v(i,j) \leq \frac{1}{2} \Delta(S), \tag{34}$$

where $\Delta(S) = u(S) - \sum_{i \in S} u(i)$ is the *marginal utility* due to coalition S. The motivation behind relaxed efficiency is the following: in case of the individual deviations, the efficiency principle is not important since there is no group interest; therefore, we can relax this condition (thus, we call it relaxed efficiency) [8].

VII. The Coalitional Switch off Game

In this section, we examine the cooperation of SPs in terms of hedonic coalition formation games. We denote by $\langle N, \succ, u \rangle$ the coalitional switch off game in which $N = (1, 2, \ldots, n)$ is the set of SPs, $\succ$ is the preference profile of SPs, and u is the utility function. First, we determine the utility function of cooperation, then we study the properties of it.

A. The Utility Function of Cooperation

Above, we mentioned that it is necessary to define a gain or cost function that characterizes the problem of cooperation. In our context, we need to analyze such a characteristic that should explain the total switch off gain, denoted by u, of a coalition. Precisely, we formalize this utility as in terms of maximization of energy saving density given in eq. (29), i.e.,

$$u(S) = f\left(\max_{q_S} G(S)\right) \quad \text{subject to} \quad 0 \leq q_S \leq 1. \tag{35}$$

The physical meaning of the utility function is to measure the total gain (it could some amount of money) when the switching off probability gives the global maximum of the energy saving density. However, in the sequel we shall study when $f\left(\max_{q_S} G(S)\right) = \max_{q_S} G(S)$. Let us find the optimal value of q_S which can be calculated as

$$\frac{\partial G(S)}{\partial q_S} = -U_S + \left(\frac{\alpha}{2} - 1\right) q_S^{-\alpha/2} V_S = 0. \quad (36)$$

Then, the following gives the optimal value

$$q_S^* = \min\left\{ \left(\frac{\left(\frac{\alpha}{2} - 1\right) V_S}{U_S} \right)^{2/\alpha}, 1 \right\} \quad (37)$$

by which the utility function can be expressed as

$$u(S) = \begin{cases} U_S + V_S + \frac{V_S^{2/\alpha}}{U_S^{2/\alpha - 1}} \left(\frac{\alpha}{2} - 1\right)^{1-2/\alpha}, & \text{if } q_S^* < 1, \\ 0, & \text{if } q_S^* = 1. \end{cases} \quad (38)$$

Further, we could find the limit which guaranties that $u(S) > 0$,

$$\left(\frac{\left(\frac{\alpha}{2} - 1\right) V_S}{U_S} \right)^{2/\alpha} < 1 \quad \Rightarrow \quad U_S > \left(\frac{\alpha}{2} - 1\right) V_S. \quad (39)$$

What we infer from this result is that as long as energy saving density of operational power costs is higher than the total transmission energy saving multiplied by $\frac{\alpha}{2} - 1$, there exists a non-zero utility of a typical coalition S.

Furthermore, recall that the allocation of the utility $u(S)$ to player i being in coalition S is denoted as ϕ_i^S. This gain corresponds to the energy saving allocated to player i. Thus, we say that player i obtains ϕ_i^S gain when the BSs are activated by q_S of the joint network formed by coalition S.

B. Properties of the Utility Function

In the following, we enumerate the properties of the utility function of the coalitional switch off game.

Lemma 2: Monotonicity: The coalitional switch off game is not always monotonic.

Proof: Assume that there are two SPs, i.e. $S = (1, 2)$, and each SP has equal λ, P_0, and β as well as different traffics T_1, T_2. Also, suppose that $\alpha = 4$, $p = 0$ and $w = 1$. Then, we are able to find the following:

$$u(1) = \lambda P_0 + \beta T_1 - 2\sqrt{\lambda P_0 \beta T_1}, \quad (40)$$

$$u(1, 2) = 2\lambda P_0 + \beta\left(T_1 + T_2\right) - 2\sqrt{2\lambda P_0 \beta\left(T_1 + T_2\right)}. \quad (41)$$

If we can prove that $u(1, 2) < u(1)$, then we could conclude that the monotonicity does not hold. To this end, let us denote the difference of these utilities as

$$\Delta = u(1, 2) - u(1) = \lambda P_0 + \beta T_2 - 2\sqrt{2\lambda P_0 \beta\left(T_1 + T_2\right)} + 2\sqrt{\lambda P_0 \beta T_1}, \quad (42)$$

which means that if $\Delta < 0$, then the utility function has no monotonicity property. Assuming that $\lambda P_0 = 1$ and $\beta = 1$, let us look at the limit of this difference by converging the traffic of SP 1 to infinity,

$$\lim_{T_1 \to \infty} \Delta = \lim_{T_1 \to \infty} \left(1 + T_2 - 2\sqrt{2(T_1 + T_2)} + 2\sqrt{T_1}\right) = -\infty \quad (43)$$

Consequently, we are able to state that when traffic of SP 1 increases to high levels then the monotonicity might not hold. ■

We can conclude that the SPs have an incentive to deviate from grand coalition if they play the coalitional switch off game. Because, non-monotonicity implies that sometimes the utility might not increase when a player joins the game. Therefore, we come up with the coalition formation problem. We consider the hedonic approach to the coalition formation of SPs in this work.

TABLE I
MATHEMATICAL MODEL PARAMETERS

$\alpha = 3$
$\lambda_1 = \frac{1 point}{km^2}$, $\lambda_2 = \frac{2 points}{km^2}$, $\lambda_3 = \frac{1.5 points}{km^2}$
$P_{0,i} = 40$ W, $i = (1, 2, 3)$
$p = 125 \times 10^{-3}$ W, $w = 1 \times 10^{-13}$ W
$T_i = 10^8$ Hz, $i = (1, 2, 3)$
$\beta_i = 4$, $i = (1, 2, 3)$

VIII. Example Scenario

In this section, we study an example scenario in which the introduced concepts are explained practically. We compare the results for different allocation methods.

Assume there are three SPs, $N = (1, 2, 3)$. The parameters concerning the network of each SP are given in Table I. Thus, the utility function yield the following results for all possible coalitions

$$\begin{aligned} &u(S_1) = 36.24 \times 10^{-6}, u(S_2) = 60.25 \times 10^{-6}, \\ &u(S_3) = 36.29 \times 10^{-6}, u(S_4) = 88.52 \times 10^{-6}, \\ &u(S_5) = 73.67 \times 10^{-6}, u(S_6) = 92.70 \times 10^{-6}, \\ &u(S_7) = 125.84 \times 10^{-6}, \end{aligned} \quad (44)$$

where $S_1 = (1)$, $S_2 = (2)$, $S_3 = (3)$, $S_4 = (1, 2)$, $S_5 = (1, 3)$, $S_6 = (2, 3)$, $S_7 = (1, 2, 3)$. These utilities denote the maximal energy saving density that can be obtained with cooperation.

A. Finding the Nash-stable Core

Here, we would like to obtain an efficient allocation method that will result in a Nash-stable partition. Clearly, we need a distribution method arranging the gains by such a way that the players will form a coalition formation which they will not deviate from.

Relaxed efficiency provides to calculate the symmetric gain of the players which can be given by

$$\begin{aligned}&\max v(1,2)+v(1,3)+v(2,3) \text{ subject to}\\&v(1,2)\leq -3.98\times 10^{-6},\\&v(1,3)\leq 0.57\times 10^{-6},\\&v(2,3)\leq -1.92\times 10^{-6},\\&v(1,2)+v(1,3)+v(2,3)\leq -3.47\times 10^{-6}.\end{aligned} \tag{45}$$

The solution of this linear program results in $v(1,2) = -3.98\times 10^{-6}$, $v(1,3) = 0.57\times 10^{-6}$, and $v(2,3) = -1.92\times 10^{-6}$. According to that solution, the utilities of each player are $\phi_1^{12} = 32.26\times 10^{-6}$, $\phi_1^{13} = 36.81\times 10^{-6}$, $\phi_1^{123} = 34.32\times 10^{-6}$, $\phi_2^{12} = 56.27\times 10^{-6}$, $\phi_2^{23} = 58.33\times 10^{-6}$, $\phi_2^{123} = 54.35\times 10^{-6}$, $\phi_3^{13} = 36.86\times 10^{-6}$, $\phi_3^{23} = 34.37\times 10^{-6}$, $\phi_3^{123} = 34.94\times 10^{-6}$, which produces the following preference profile:

$$\begin{aligned}&(1,3)\succ_1 (1)\succ_1 (1,2,3)\succ_1 (1,2)\\&(2)\succ_2 (2,3)\succ_2 (1,2)\succ_2 (1,2,3)\\&(1,3)\succ_3 (3)\succ_3 (1,2,3)\succ_3 (2,3).\end{aligned} \tag{46}$$

Thus, the Nash-stable partition is $\Pi = \{(1,3),(2)\}$. The total utility related to the Nash-stable partition is given by $u_{\text{tot}} = \sum_{S\in\Pi}\sum_{i\in S}\phi_i^S = 133.92\times 10^{-6}$.

B. Social Optimum

The social optimum is formulated as a set partitioning optimization problem which can be given by

$$u^*_{\text{tot}} = \max_{\Pi}\sum_{S\in\Pi} u(S), \tag{47}$$

by which we find a partition Π^* maximizing the global utility. Note that the total social utility in case of a Nash-stable partition will always be lower or equal to the one obtained by social optimum, i.e. $u_{\text{tot}}\leq u^*_{\text{tot}}$.

The social optimum in the considered example is found to be $u^*_{\text{tot}} = 133.92\times 10^{-6}$ and related partition $\Pi^* = \{(1,3),(2)\}$ which is the equivalent partition corresponding to the Nash-stable one.

Remark 1: In this example, we show that even though the SPs behave selfishly in deciding their partners, the global utility might still exists if the gain allocation method is chosen to be the Nash-stable core. Notice the this setting corresponds to maximizing the total energy saving density which is a *green* solution to the problem.

IX. Conclusion

We analyzed the cooperation of SPs on switch off operation of BSs in the context of green networking. The homogenous Poisson p.p. approach to the deployment of BSs has been used in order to study the SINR distribution of SPs. It was proven that scaling the coordinates of $\mathbb{R}^2$ by $\sqrt{q}$ from the origin of a homogenous Poisson point process result in a thinned homogenous Poisson point process with intensity modified by q. The SINR distribution of the original network was derived and by increasing the transmission power by some factor of q, it was proven that the SINR distribution of the thinned network (obtained by scaling the locations of BSs) remains unchanged. Furthermore, in the case of non-cooperating SPs, the SINR distribution is obtained of the original and thinned network of SPs, respectively. We also found the SINR distribution of cooperation case used in the context of coalition formation of SPs. The operations on the network formed by cooperation are assumed to be run jointly by SPs meaning that they share their resources such that any mobile is tagged to the nearest BS of any SP. The maximal energy saving density of a cooperation is supposed to be the utility of the coalition. We derive the closed form results of the utility.

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