Power Allocation in Relay Channels under a Global Power Constraint using Virtual Nodes

Paul Ferrand, Jean-Marie Gorce and Claire Goursaud
Université de Lyon, INRIA
INSA-Lyon, CITI-INRIA, F-69621, Villeurbanne, France
Email : paul.ferrand@insa-lyon.fr

***Abstract*—Relay channels have been extensively studied in the literature since the seminal paper by Cover and El Gamal. Nevertheless, characterizing the capacity of relay channels still presents open issues. While numerous works addressed this problem with constant powers or targeted the sum-rate optimization, computing the capacity in the case of a global power constraint was less studied. In this paper, we introduce the concept of virtual nodes to derive analytical expressions of the relay channel capacity as a function of the total power. This transformation leads to simple closed-form expressions of the upper bound and *decode-and-forward* (DF) lower bound on the capacity of the full- and half-duplex relay channels. The half-duplex study is separated into low and high signal-to-noise ratio (SNR) cases. The impact of these approximations is evaluated and found to achieve a large part of the maximal capacity in the worst case where the equivalent received SNR is neither low nor high, typically between 0-10dB.**

I. Introduction

In the recent years, relay channels and protocols have seen an increase of interest, due to their ability to combat multipath fading in wireless networks through cooperative diversity [1] and provide performance gains while homogeneizing the global network resource consumption [2]. Relay channels have been extensively studied in the litterature, through Van Der Meulen's early work and the comprehensive characterization of both the upper bound of the capacity region as well as several lower bounds by Cover and El Gamal (see [3] and references therein). An extensive survey by Kramer, Gatspar and Gupta for full duplex relay channels and more complex network situations can be found in [4].

While the study of full-duplex channels is simpler and gives insights into the behavior of cooperating protocols, it is not fully comprehensive for wireless channels since wireless nodes cannot send and receive data at the same time. We consider in this paper a time-slotted approach to the half-duplex problem, where nodes share their total network time between listening and transmitting phases. This particular relay channel and associated bounds have been studied in [5].

The general form of the relay channel supposes that the signal sent by the source and the relay may be correlated and add coherently. While non-coherent relay channels do provide performance benefits, the coherent addition of signals leads to a significant increase in their performance. This coherency does require that both nodes cooperate in the creation of their respective codebooks, and that they are able to transmit in a beamformed manner so that the signals add seamlessly at the destination. While few systems and protocols are able to achieve this degree of cooperation, their theoretical study is further complexified by the need to consider the correct amount of correlation between signals in the analysis on top of the transmission power of each node.

Power allocation in relay channels is thus a complex task. In [6], [7] the authors consider individual power constraints on nodes in a full-duplex relay channel and extract exact closed-form expressions for both the cutset upper bound and the DF lower bound. If the individual power constraint is relaxed into a global power constraint, the solution to the problem takes a different form. In [8], Host-Madsen and Zhang express the ergodic bounds on the capacity of the full and half duplex relay channel through an iterative water-filling algorithm. Liang *et al.* further this study by applying a max-min rule derived from detection rules in hypothesis testing to the general power allocation problem, and are thus able to provide insights into the analytical forms of the solution as well as iterative algorithms with high convergence speed [9]. Ng and Goldsmith give a closed-form expression of the DF lower bound for relay channels where the relay node is colocated with either the source or the destination in [10]. This last paper also investigates the impact of channel side information on the capacity of the relay channel.

In this paper, we focus our interest on both the cutset upper-bound of the capacity, and the DF lower bound. These two bounds do not meet in general, which means that the exact capacity of the relay channel is still uncertain. The DF lower bound is still of great interest, because it is the performance limit of cooperation protocols where the relay node performs a complete decoding of the signal received from the source, and as such is directly usable on common hardware. Our contributions in this paper are as follow:

- We consider a global power constraint on the network, rather than a local power constraint on each node. This constraint allows for a *fair* comparison between cooperation protocols as far as power is concerned. It also has practical applications when we aim at minimizing the power radiated – or consumed – by the network as a whole, rather than individually by each node. We show that with such a constraint, the general coherent relay channel capacity can be expressed as the capacity of an equivalent non-coherent relay channel, where a *virtual* relay node handles the coherent cooperation between the

source and the original relay. Thanks to this formulation the coherent relay channel is assimilated to a non coherent relay channel for which bounds expressions are more tractable.
- Based on this transformation, we express a very simple closed form expression for the upper bound and DF lower bound of the capacity of the full-duplex relay channel under a global power constraint, along with the corresponding resource allocations. This expression generalizes the results in [10] where the relay is either near the source or the destination.
- The virtual node transformation can apply to half-duplex relay channels, simplifying the mathematical optimization problem by one degree of freedom. In order to obtain results similar in form to the full-duplex case, we use a fixed time-slot hypothesis along with classical approximations of the $\log$ function at low and high SNR. The proposed bounds are tight for low SNR below 0dB, and for high SNR above 10dB.

II. Full-duplex relay channel

The relay channel is a network model composed of three nodes ; an information source, a destination, and a relay node whose only purpose is to help the source in its transmission and thus has no information of its own. For each node i, we associate random variables representing the complex symbols sent and received, respectively labeled as X_i and Y_i. In the full-duplex mode, we suppose that nodes are able to send and receive data at the same time.

We operate under a classical Gaussian model, where the source and relay transmit their complex symbols using an average power P_i. Under this model, the complex symbols received by the relay and the destination are expressed as:

$$Y_2 = h_2 X_1 + Z' \tag{1}$$

$$Y_3 = h_1 X_1 + h_3 X_2 + Z \tag{2}$$

The signal is corrupted by the channel between the nodes through a static attenuation and a Gaussian noise Z' and Z of power density N, independent for each receiver. We define the normalized transmitted power as $\bar{P}_i = P_i/N$. The relay channel in our model has no individual node power requirement, but rather a total consumed power constraint $\bar{P}_1+\bar{P}_2 = \bar{P}_{\text{tot}}$. This relay channel is represented on Fig.1 with the attenuation coefficients of the channel. To simplify the subsequent equations in this paper, we denote the squared module

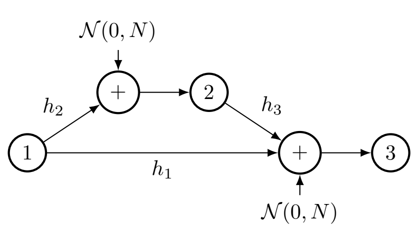

Fig. 1: Full-duplex gaussian relay channel.

of the attenuation coefficient between nodes as $g_j = |h_j|^2$. All capacity results are in nats. For this model, the capacity upper bound and DF lower bound are given respectively by [3]:

$$C \leq \max_{\substack{\rho\in[0,1] \\ \bar{P}_1,\bar{P}_2:\bar{P}_1+\bar{P}_2=\bar{P}_{\text{tot}}}} \min\left\{\log\left(1+(g_1+g_2)(1-\rho^2)\bar{P}_1\right),\right.$$
$$\left.\log\left(1+g_1\bar{P}_1+g_3P_2+2\rho\sqrt{g_1g_3\bar{P}_1\bar{P}_2}\right)\right\} \tag{3}$$

$$C \geq \max_{\substack{\rho\in[0,1] \\ \bar{P}_1,\bar{P}_2:\bar{P}_1+\bar{P}_2=\bar{P}_{\text{tot}}}} \min\left\{\log\left(1+g_2(1-\rho^2)\bar{P}_1\right),\right.$$
$$\left.\log\left(1+g_1\bar{P}_1+g_3\bar{P}_2+2\rho\sqrt{g_1g_3\bar{P}_1\bar{P}_2}\right)\right\} \tag{4}$$

Both equations are almost identical, differing only in the missing g_1 coefficient in the *decode-and-forward* bound. We can also note the presence of the coherency variable ρ. For non-coherent channels, ρ is set to 0, greatly simplifying the analysis of both bounds. In the general case however we have to optimize over the value of ρ to obtain the tightest bounds.

To simplify the analysis, we note $\bar{P}_1 = \bar{P}_{1,1} + \bar{P}_{1,2}$ and identify $\bar{P}_{1,1} = (1-\rho^2)\bar{P}_1$. We can thus write (3) as:

$$C \leq \max_{\substack{\bar{P}_{1,1},\bar{P}_{1,2},\bar{P}_2: \\ \bar{P}_{1,1}+\bar{P}_{1,2}+\bar{P}_2=\bar{P}_{\text{tot}}}} \min\left\{\log\left(1+(g_1+g_2)\bar{P}_{1,1}\right),\right.$$
$$\left.\log\left(1+g_1\bar{P}_{1,1}+\left(\sqrt{g_1\bar{P}_{1,2}}+\sqrt{g_3\bar{P}_2}\right)^2\right)\right\} \tag{5}$$

The correlated signals takes the form of a MISO channel (Multiple Input Single Output), from a "virtual node" combining the cooperative part of the signal from the source and the signal from the relay. If we introduce a new power variable $\bar{P}_{\text{eq}}$ for the power allocated to this virtual node, the optimal power allocation $(\bar{P}^*_{1,2}, \bar{P}^*_2)$ is known to be the [11]:

$$\bar{P}^*_{1,2} = \frac{g_1}{g_1+g_3}\bar{P}_{\text{eq}} \qquad \bar{P}^*_2 = \frac{g_3}{g_1+g_3}\bar{P}_{\text{eq}} \tag{6}$$

Injecting (6) into (5) we obtain an expression similar to the upper bound to the capacity of an equivalent *non-coherent* channel (by considering $\rho = 0$ in (7)). By identification, the cooperation between the original source and relay is captured in a *virtual relay*, transmitting on a channel whose gain is $(g_1 + g_3)$ (Fig.2). In this equivalent channel, the source and the virtual relay do not send correlated information since the correlated part is already integrated in the virtual relay.

$$C \leq \max_{\substack{\bar{P}_{1,1},\bar{P}_{\text{eq}} \\ \bar{P}_{1,1}+\bar{P}_{\text{eq}}=\bar{P}_{\text{tot}}}} \min\left\{\log\left(1+g_1\bar{P}_{1,1}+(g_1+g_3)\bar{P}_{\text{eq}}\right)\right.$$
$$\left.\log\left(1+(g_1+g_2)\bar{P}_{1,1}\right)\right\} \tag{7}$$

The power allocation problem is thus much simpler to solve. The following proposition gives an exact expression for the bounds on the capacity for the full-duplex relay channel:

Proposition 1. *The upper bound on the capacity for the coherent full-duplex relay channel under a total power $\bar{P}_{\text{tot}}$ to*

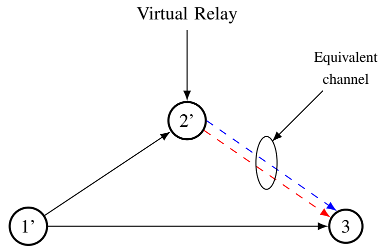

Fig. 2: Equivalent virtual relay model for the full-duplex relay channel. The virtual relay channel towards the destination is an equivalent MISO channel, whose path gains are computed from the gains on the original channel.

be distributed between the source and relay node is expressed as:

$$C \leq \log\left(1 + \frac{(g_1+g_2)\hat{g}_3}{g_2+\hat{g}_3}\bar{P}_{\text{tot}}\right) \tag{8}$$

We now suppose $g_2 \geq g_1$. The decode-and-forward *lower bound in that case is expressed as:*

$$C \geq \log\left(1 + \frac{g_2\hat{g}_3}{g_2+g_3}\bar{P}_{\text{tot}}\right) \tag{9}$$

Proof: In Eq.(7), by rewriting $\bar{P}_{\text{eq}} = \bar{P}_{\text{tot}} - \bar{P}_{1,1}$, we can see that the two terms of the $\min$ function evolve in opposite direction with regard to $P_{1,1}$. The maximum is thus attained when both terms are equal, which directly translates into $g_2\bar{P}_{1,1} = \hat{g}_3\bar{P}_{\text{eq}}$. Under the constraint $P_{1,1} + \bar{P}_{\text{eq}} = \bar{P}_{\text{tot}}$ we thus have the following optimal power allocation:

$$\bar{P}^*_{1,1} = \frac{\hat{g}_3}{g_2+\hat{g}_3}\bar{P}_{\text{tot}} \qquad \bar{P}^*_{\text{eq}} = \frac{g_2}{g_2+\hat{g}_3}\bar{P}_{\text{tot}} \tag{10}$$

Plugging these values into (7) gives (8). The *decode-and-forward* lower bound's proofs follows a similar procedure, and the optimal power allocation is in that case:

$$\bar{P}^*_{1,1} = \frac{\hat{g}_3}{g_2+g_3}\bar{P}_{\text{tot}} \quad \bar{P}^*_{\text{eq}} = \frac{g_2-g_1}{g_2+g_3}\bar{P}_{\text{tot}} \tag{11}$$

The power allocation for the decode and forward bound is only valid if $g_2 \geq g_1$. This particularity is expected from the general behavior of decode and forward schemes, who perform well when the source-relay channel is of higher quality than the source-destination channel [4], [8]. In both cases, the optimal $(\bar{P}^*_{1,2}, \bar{P}^*_2)$ are given by (6). ■

Remark. – All the results in this paper are also valid for non-coherent relay channels under a global power constraint, by changing $\hat{g}_3$ into g_3 in both the power allocations and the capacity results.

III. Half-duplex relay channel

In wireless channels, nodes are usually unable to receive and transmit at the same time. Full-duplex results are thus insightful for theoretical studies, but do not provide realistic performance evaluations. In the case of the relay channel, this means that we have basically a cooperation in two phases ; the relay will first listen to the source for the first part of the transmission and then transmit its cooperative signal. The total network time is thus shared between these two phases.

In the general case, it is possible to allocate arbitrarily a time share t_1 to phase one and t_2 to phase two, such that $t_1 + t_2 = 1$. In the first phase, the source node transmits alone using power P_1, and in our model we allow the source to transmit at $P'_1 \neq P_1$ in the second phase. Using results from [5] for coherent half-duplex relays and the virtual source transformation from the preceding section, we can write the upper and DF lower bounds as follow, with $\bar{P}'_{1,1} + \bar{P}'_{1,2} = \bar{P}'_1$ and $\bar{P}_{\text{eq}} = \bar{P}'_{1,2} + \bar{P}_2$:

$$\begin{aligned} C \leq \min\{&t_1\log\left(1+(g_1+g_2)\bar{P}_1\right) + t_2\log\left(1+g_1\bar{P}'_{1,1}\right), \\ &t_1\log\left(1+g_1\bar{P}_1\right) + t_2\log\left(1+g_1\bar{P}'_{1,1}+\hat{g}_3\bar{P}_{\text{eq}}\right)\} \end{aligned} \tag{12}$$

$$\begin{aligned} C \geq \min\{&t_1\log\left(1+g_2\bar{P}_1\right) + t_2\log\left(1+g_1\bar{P}'_{1,1}\right), \\ &t_1\log\left(1+g_1\bar{P}_1\right) + t_2\log\left(1+g_1\bar{P}'_{1,1}+\hat{g}_3\bar{P}_{\text{eq}}\right)\} \end{aligned} \tag{13}$$

The optimization is over $(\bar{P}_1, \bar{P}'_{1,1}, \bar{P}_{\text{eq}})$ verifying the global mean power constraint $t_1\bar{P}_1 + t_2(\bar{P}'_{1,1} + \bar{P}_{\text{eq}}) = \bar{P}_{\text{tot}}$. Even with the virtual relay model this optimization problem is extremely hard to solve analytically on both the power and time variables. This problem can directly be expressed as a convex optimization problem and thus efficiently solved, by considering the half-duplex case as an energy distribution problem under a total energy constraint, rather than a power distribution problem. This transformation is described in [12] and will be used as the comparison for our power allocation in the remainder of the section.

In order to obtain closed-form results, we first restrict ourselves to the case $t_1 = t_2 = 1/2$. This approach matches practical protocols where an equal time share is assigned to each network phase, but is likely to induce some degradation in the capacity region. On Fig.3, we quantify this degradation by considering the relative performance as the ratio of the capacity attained with the constraint versus the unconstrained case. We can see that the added constraint has little to no impact on low and high values of $\bar{P}_{\text{tot}}$. The degradation at medium SNR is mild when the relative coefficients of the source-destination and relay-destination are close and increases in the case of strong assymetry in the links – the cross-dotted curve on Fig.3.

Under this constraint, it is possible to derive the optimal values of $(\bar{P}^*_1, \bar{P}'^*_{1,1}, \bar{P}^*_{\text{eq}})$ as fourth order polynomial roots, which is still far from practical. To further ease the manipulation of the sum of logarithm functions in half-duplex relay channels, we split the problem in two approaches and use the usual approximations $\log(1+x) \approx x$ as $x \to 0$ and $\log(1+x) \approx \log(x)$ as $x \to \infty$. Using these simplifications, we can enunciate the following result:

Proposition 2. *The upper bound on the capacity of the general half-duplex relay channel is closely approximated for high values of $\bar{P}_{\text{tot}}$ by:*

$$C \leq \frac{1}{2}\left(\log\left(1+g_1\bar{P}_{\text{tot}}\right) + \log\left(1 + \frac{\hat{g}_3(g_1+g_2)}{g_2+\hat{g}_3}\bar{P}_{\text{tot}}\right)\right) \tag{14}$$

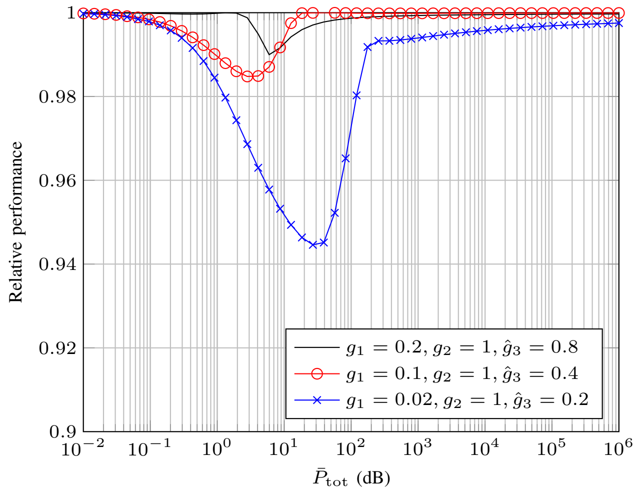

Fig. 3: Performance loss induced by introducing the constraint $t_1 = t_2 = 1/2$ in the general half-duplex relay optimization problem.

The upper bound on the capacity of the general half-duplex relay channel is closely approximated for low values of P_{tot} by:

$$C \leq \frac{1}{2}\log\left(1 + 2\frac{\hat{g}_3(g_1+g_2)}{g_2+g_3}\bar{P}_{\text{tot}}\right) \tag{15}$$

In the high SNR case, the optimal power allocation is as follow, from (12)*:*

$$\bar{P}_1 = \bar{P}_{\text{tot}} \quad \bar{P}'_{1,1} = \frac{\hat{g}_3}{g_2+\hat{g}_3}\bar{P}_{\text{tot}} \quad \bar{P}_{\text{eq}} = \frac{g_2}{g_2+\hat{g}_3}\bar{P}_{\text{tot}} \tag{16}$$

In the low SNR case:

$$\bar{P}_1 = 2\frac{\hat{g}_3}{g_2+\hat{g}_3}\bar{P}_{\text{tot}} \quad \bar{P}'_{1,1} = 0 \quad \bar{P}_{\text{eq}} = 2\frac{g_2}{g_2+\hat{g}_3}\bar{P}_{\text{tot}} \tag{17}$$

Proof: For high values of $\bar{P}_{\text{tot}}$, we have to solve the following optimization problem:

$$\begin{aligned}
&\min_{R,\bar{P}_1,\bar{P}'_{1,1},\bar{P}_{\text{eq}}} && -R\\
&\text{s.c.} && 2R \leq \log\left((g_1+g_2)\bar{P}_1\right) + \log\left(g_1\bar{P}'_{1,1}\right)\\
& && 2R \leq \log\left(g_1\bar{P}_1\right) + \log\left(g_1\bar{P}'_{1,1} + \hat{g}_3\bar{P}_{\text{eq}}\right)\\
& && 2\bar{P}_{\text{tot}} = \bar{P}_1 + \bar{P}'_{1,1} + \bar{P}_{\text{eq}}
\end{aligned}$$

We associate the Lagrangian multipliers λ_1 and λ_2 to the inequality constraints, and μ to the equality constraint.The partial derivatives of the Lagrangian function $\mathcal{L}$ w.r.t. the optimization variables are:

$$\begin{aligned}
&\frac{\partial\mathcal{L}}{\partial R} = 1 - 2\lambda_1 - 2\lambda_2 \qquad \frac{\partial\mathcal{L}}{\partial\bar{P}_1} = -\frac{1}{2\bar{P}_1} + \mu\\
&\frac{\partial\mathcal{L}}{\partial\bar{P}'_{1,1}} = -\frac{\lambda_1}{\bar{P}'_{1,1}} - \frac{\lambda_2 g_1}{g_1\bar{P}'_{1,1} + \hat{g}_3\bar{P}_{\text{eq}}} + \mu\\
&\frac{\partial\mathcal{L}}{\partial\bar{P}_{\text{eq}}} = -\frac{\lambda_2\hat{g}_3}{g_1\bar{P}'_{1,1} + \hat{g}_3\bar{P}_{\text{eq}}} + \mu
\end{aligned} \tag{18}$$

We study this problem under the Karush-Kuhn-Tucker (KKT) conditions, allowing us to derive analytical results on the optimal set of power allocations [13]. From these we can directly see that $\lambda_2 = 0 \implies \mu = 0$, which violates the KKT conditions. Furthermore, having $\lambda_1 = 0$ leads to the condition $g_3 = 0$, a degenerate case. At the optimum, we thus have $\lambda_1 \neq 0$ and $\lambda_2 \neq 0$ which means that both inequalities are verified with equality, leading to a new relation between the power variables. We are left with a 4 equation system with 4 unknowns to solve, as described below:

$$2\lambda_2\hat{g}_3\bar{P}_1 = g_1\bar{P}'_{1,1} + \hat{g}_3\bar{P}_{\text{eq}} \tag{19}$$

$$\lambda_2 g_3\bar{P}'_{1,1} = (\frac{1}{2} - \lambda_2)\left(g_1\bar{P}'_{1,1} + \hat{g}_3\bar{P}_{\text{eq}}\right) \tag{20}$$

$$(g_1+g_2)\bar{P}'_{1,1} = g_1\bar{P}'_{1,1} + \hat{g}_3\bar{P}_{\text{eq}} \tag{21}$$

$$\bar{P}_1 + \bar{P}'_{1,1} + \bar{P}_{\text{eq}} = 2\bar{P}_{\text{tot}} \tag{22}$$

Combining the first two equations w.r.t. the λ_2 term leads to the relation $\bar{P}_1 = \bar{P}'_{1,1} + \bar{P}_{\text{eq}}$. Along with the power constraint and the remaining equation, we can thus deduce the power allocation from (16).

At low values of $\bar{P}_{\text{tot}}$, we proceed in a similar way and we are left with the following equation and the total power constraint:

$$(g_1+g_2)\bar{P}_1 + g_1\bar{P}'_{1,1} = g_1\bar{P}_1 + g_1\bar{P}'_{1,1} + \hat{g}_3\bar{P}_{\text{eq}} \tag{23}$$

This equation simplifies into $\hat{g}_3\bar{P}_{\text{eq}} = g_2\bar{P}_1$, which does not lead to constraints on $\bar{P}'_{1,1}$. The optimal power allocation thus reduces to (17). Injecting the power allocations into (12) gives (14) and (15). ■

On Fig.4, we plot the relative performance of both high and low SNR approximations, and the $t_1 = 1/2$ constraint, w.r.t. the general problem. We can see that both are well-behaved in their respective range, and degrades rapidly in the medium SNR range. The lowest performance point position will shift depending on the channels considered ; channels with a low coefficient will lead the low SNR approximation to be valid for higher values of $\bar{P}_{\text{tot}}$, while the high SNR approximation behaves in the opposite way. If we set an acceptable relative performance of 95%, there is a 10 dB range at medium SNR where our proposition crosses the threshold. Depending on the application, the tradeoff may be acceptable with regard to the computation simplicity of the power allocations. Increasing the performances in that range would require much more complicated methods of resolutions, be it root finding on high order polynomials or complete convex optimization problems.

Since both problems share a similar analytical form, extending these results to the *decode-and-forward* lower bound is straightforward. The capacity bounds and power allocations are described in Prop.3, and the relative performance is plotted in Fig.5. As seen on Fig.4 and Fig.5, the suboptimality of the choice $t = 1/2$ and the high/low SNR approximations in the half-duplex case lead to the same relative loss in performance in both the upper bound (Prop.2) and the lower bound (Prop.3).

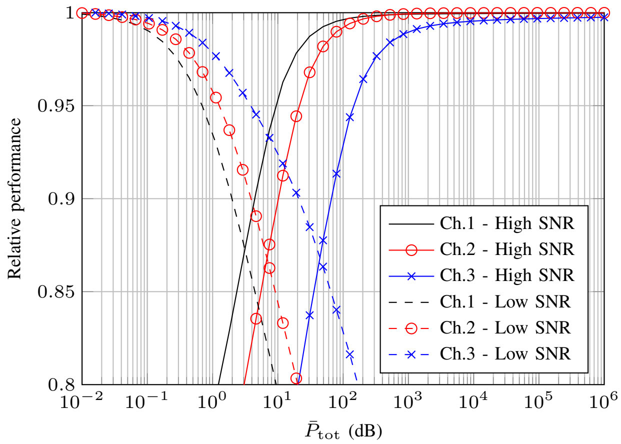

Fig. 4: Relative performance of the power allocation in proposition 2 w.r.t. the general optimization problem. Values for g_1, g_2 and $\hat{g}_3$ are the same as the ones in Fig.3.

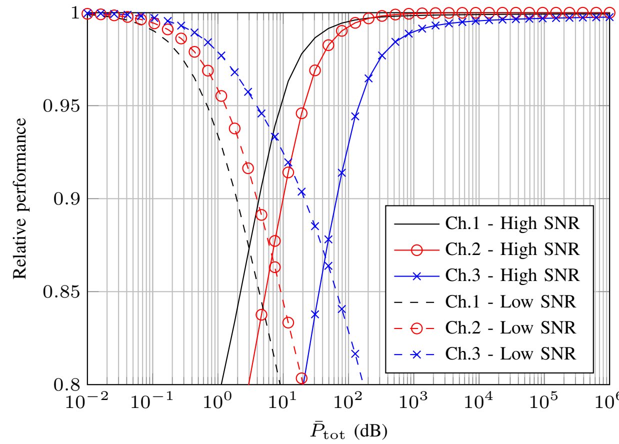

Fig. 5: Relative performance of the power allocation in proposition 3 w.r.t. the general optimization problem for the *decode-and-forward* lower bound on the capacity. Values for g_1, g_2 and $\hat{g}_3$ are the same as the ones in Fig.3.

Proposition 3. *We suppose $g_2 \geq g_1$. The DF lower bound on the capacity of the general half-duplex relay channel is closely approximated at high SNR by:*

$$C \leq \frac{1}{2}\left(\log\left(1 + g_1 \bar{P}_{\text{tot}}\right) + \log\left(1 + \frac{g_2 \hat{g}_3}{g_2 + g_3}\bar{P}_{\text{tot}}\right)\right) \quad (24)$$

The DF lower bound on the capacity is closely approximated at low SNR by:

$$C \leq \frac{1}{2}\log\left(1 + 2\frac{g_2 \hat{g}_3}{g_2 + g_3}\bar{P}_{\text{tot}}\right) \quad (25)$$

In the high SNR case, the optimal power allocation is as follow, from (13)*:*

$$\bar{P}_1 = \bar{P}_{\text{tot}} \quad \bar{P}'_{1,1} = \frac{\hat{g}_3}{g_2 + g_3}\bar{P}_{\text{tot}} \quad \bar{P}_{\text{eq}} = \frac{g_2 - g_1}{g_2 + g_3}\bar{P}_{\text{tot}} \quad (26)$$

In the low SNR case:

$$\bar{P}_1 = 2\,\frac{\hat{g}_3}{g_2 + g_3}\bar{P}_{\text{tot}} \quad \bar{P}'_{1,1} = 0 \quad \bar{P}_{\text{eq}} = 2\,\frac{g_2 - g_1}{g_2 + g_3}\bar{P}_{\text{tot}} \quad (27)$$

IV. Conclusion

In this paper, we described a network model transformation, allowing us to treat coherent relay channels analytically as non-coherent ones. Using this transformation, we were able to derive a closed-form expression for bounds on the capacity of the full-duplex relay channel along with the associated power allocation. Half-duplex relay channels are harder to analyze, due to the presence of sums of logarithms in their capacity expressions. Applying successive approximations leads to closed-form expressions similar to the full-duplex case, although the proposed power allocations induce a performance degradation at medium SNR.

The virtual relay transformation presented in this paper is an interesting way of quickly simplifying the analysis of coherent communication networks operating under a global power constraint. We expect in further contribution to be able to present such an approach on more complex network models, such as the cooperative multiple-access channel. Links between this approach and superposition coding should also be investigated, since both use a power-splitting paradigm. The proposed power allocations also suppose perfect channel side information (CSI). It would thus be interesting to study the resiliency of these results under imperfect CSI.

References

[1] V. Stankovic, A. Host Madsen, and Zixiang Xiong, "Cooperative diversity for wireless ad hoc networks," *IEEE Sig. Process. Mag.*, vol. 23, no. 5, pp. 37–49, 2006.

[2] F. Parzysz, M. H. Vu, and F. Gagnon, "A half-duplex relay coding scheme optimized for energy efficiency," in *Proc. IEEE Information Theory Workshop (ITW)*, 2011, pp. 306–310.

[3] T. Cover and A. Gamal, "Capacity theorems for the relay channel," *IEEE Trans. Inf. Theory*, vol. 25, no. 5, pp. 572 – 584, sep 1979.

[4] G. Kramer, M. Gastpar, and P. Gupta, "Cooperative Strategies and Capacity Theorems for Relay Networks," *IEEE Trans. Inf. Theory*, vol. 51, no. 9, pp. 3037 – 3063, sept. 2005.

[5] M. Khojastepour, A. Sabharwal, and B. Aazhang, "Lower bounds on the capacity of gaussian relay channel," in *Proc. Annu. Conf. Information Sci. Syst. (CISS)*, 2004, pp. 17–19.

[6] A. El Gamal, M. Mohseni, and S. Zahedi, "Bounds on capacity and minimum energy-per-bit for AWGN relay channels," *IEEE Trans. Inf. Theory*, vol. 52, no. 4, pp. 1545 –1561, april 2006.

[7] Abbas El Gamal and Young-Han Kim, *Network Information Theory*, Cambridge University Press, Ed., 2011.

[8] A. Host Madsen and J. Zhang, "Capacity bounds and power allocation for wireless relay channels," *IEEE Trans. Inf. Theory*, vol. 51, no. 6, pp. 2020 –2040, june 2005.

[9] Yingbin Liang, V. V. Veeravalli, and H. V. Poor, "Resource Allocation for Wireless Fading Relay Channels: Max-Min Solution," *IEEE Trans. Inf. Theory*, vol. 53, no. 10, pp. 3432–3453, 2007.

[10] C. T. K. Ng and A. Goldsmith, "The impact of CSI and power allocation on relay channel capacity and cooperation strategies," *IEEE Trans. Wireless Commun.*, vol. 7, no. 12, pp. 5380–5389, 2008.

[11] John G. Proakis and Masoud Salehi, *Digital Communications*, 5th ed. McGraw-Hill, 2008.

[12] Paul Ferrand, Claire Goursaud, and Jean-Marie Gorce, "Common rate maximization in cooperative multiple access channels," in *Proc. IEEE Wireless Commun. Networking Conf. (WCNC)*, 2013.

[13] Stephen Boyd and Lieven Vanderberghe, *Convex Optimization*. Cambridge University Press, 2004.